Testing gravity with the two-body problem

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30/03/2021

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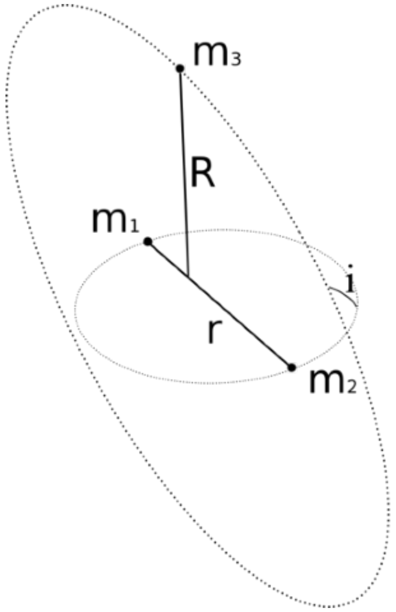
ABOUT MYSELF

- 2017-2020: PhD in CPT Marseille, with Federico Piazza

Theme : Tests of gravity with GWs

- 2020-present: Postdoc at Scuola Normale Superiore (Pisa), with Enrico Trincherini

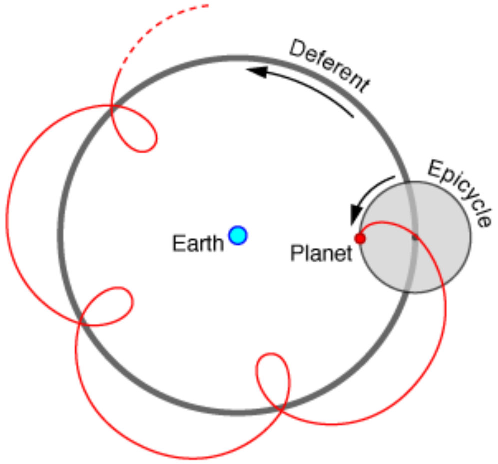
Theme: Effective field theories in gravity (with application to the relativistic three-body problem)



- Orbit circularization
- 3-body GW
- Exoplanets
- PBH formation
- ...

A LONG AND RICH HISTORY…

THE GREEKS…



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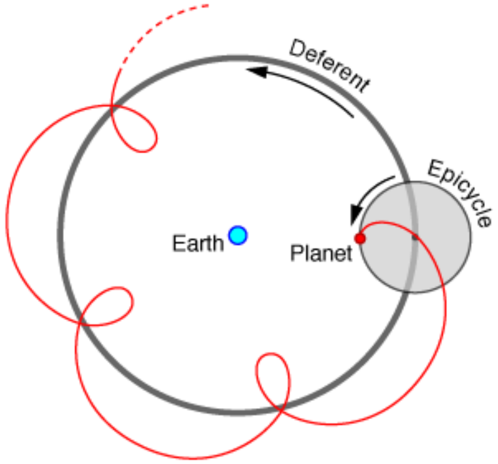
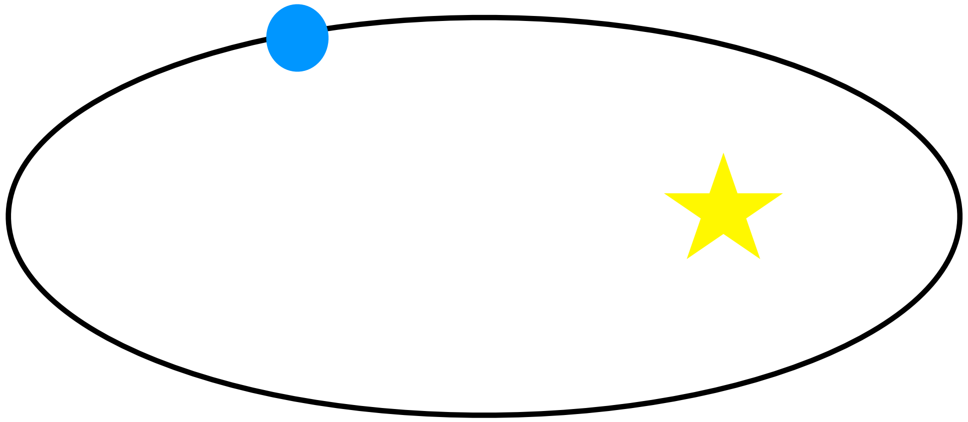


KEPLER…



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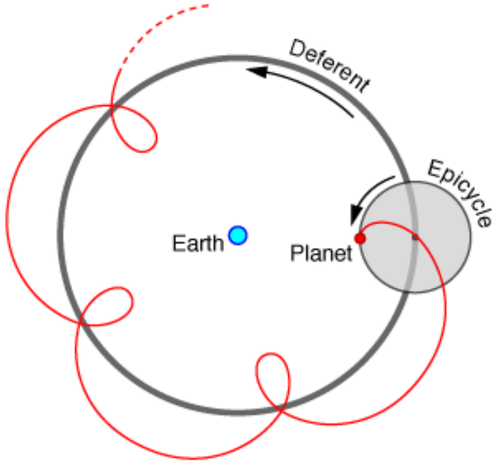

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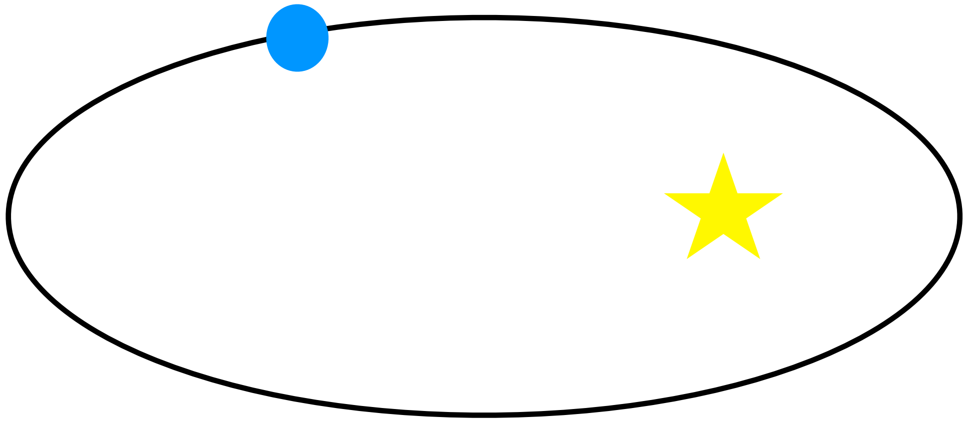
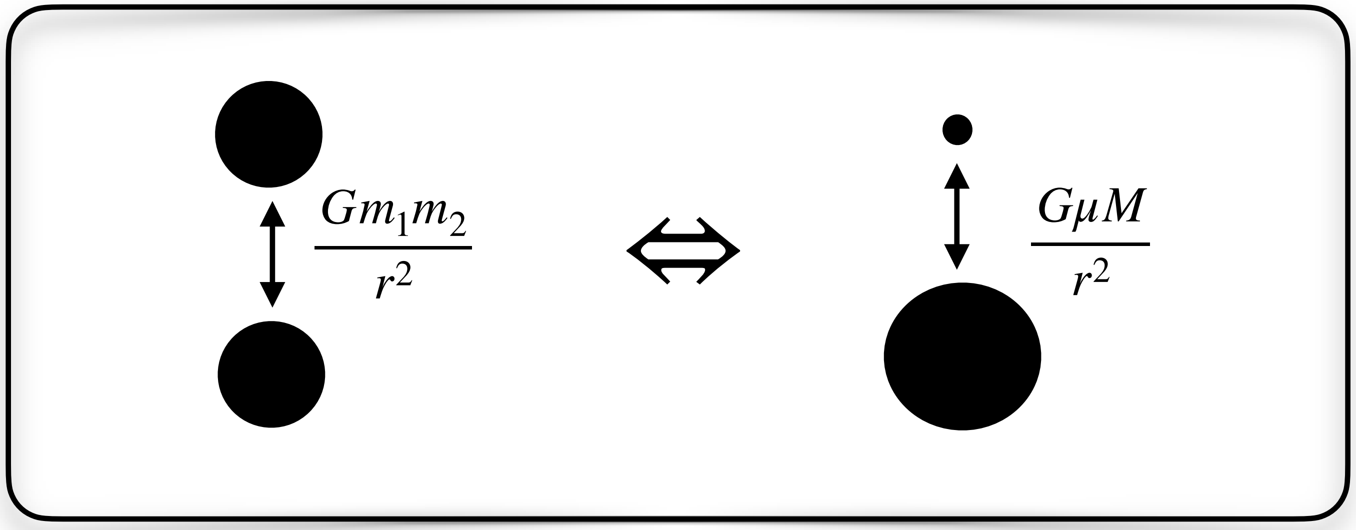


NEWTON…

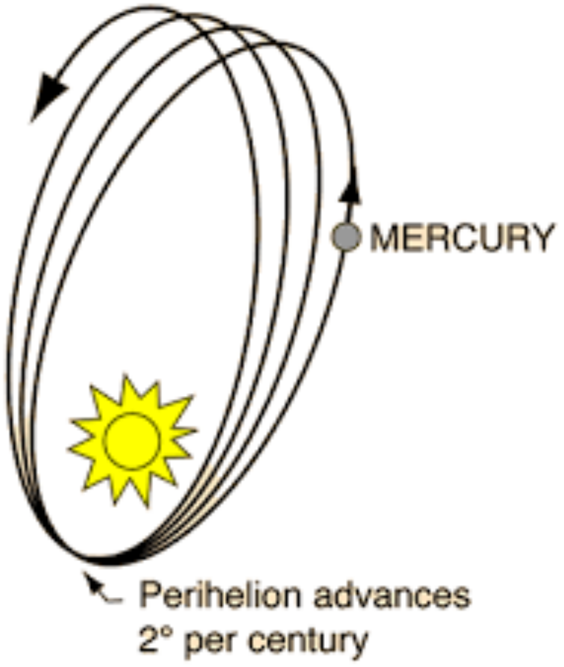


A LONG AND RICH HISTORY…

EINSTEIN'S GENERAL RELATIVITY (GR)

Post-Newtonian force

$$\mathbf{F} = -\frac{Gm_1m_2}{r^3}\mathbf{r}\left[1 + \mathcal{O}\left(\frac{v^2}{c^2}\right)\right]$$



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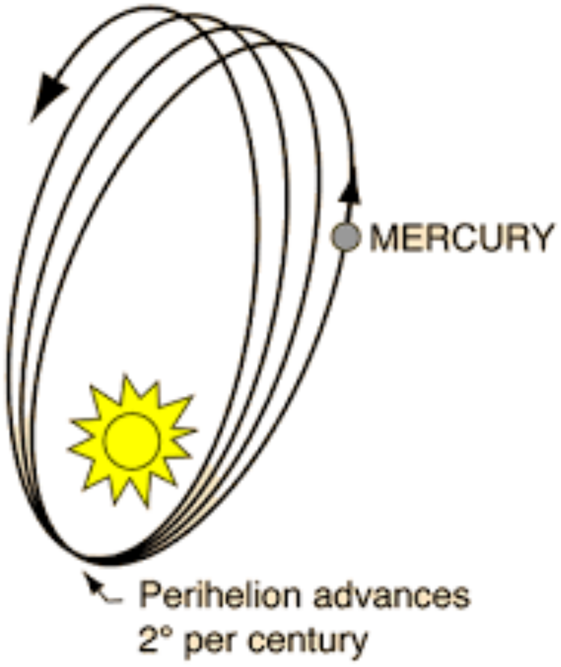
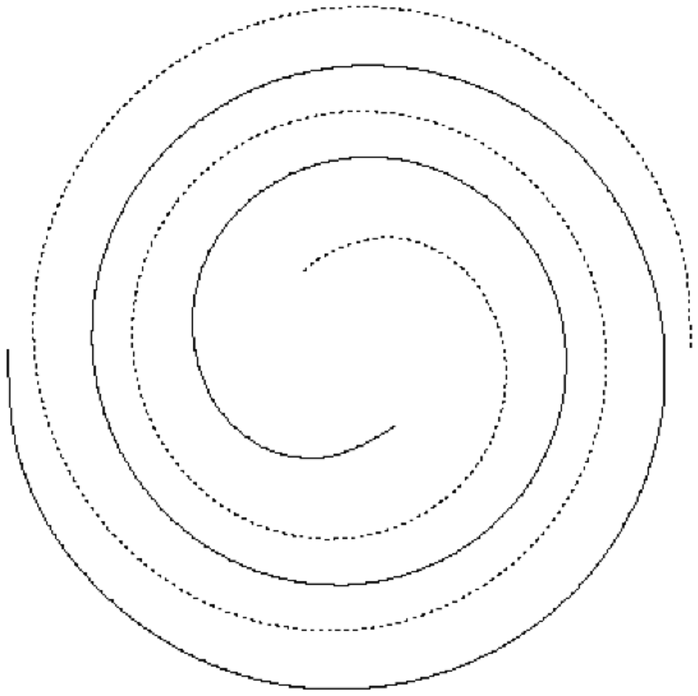


Gravitational Waves (GW)

$$P = \frac{G}{5}\left\langle \dddot{Q}^{kl}\dddot{Q}_{kl}\right\rangle$$



PROBLEMATIC

What can we learn from gravity using two-body trajectories ? Where should we look for new physics, and which parameters control possible deviations from GR ?

$\Rightarrow$ EFFECTIVE FIELD THEORY (EFT) ideas are crucial

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$\Rightarrow$ EFFECTIVE FIELD THEORY (EFT) ideas are crucial

A simple example : Eddington parameters

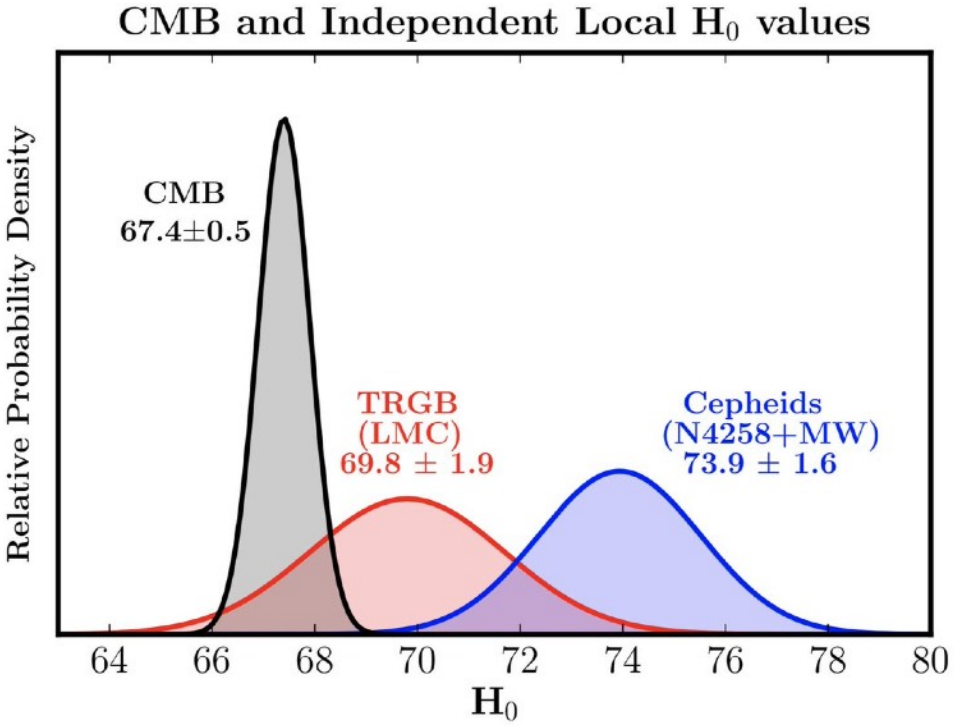
$$g_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu \simeq -\left(1-\frac{2GM}{r}+\beta\frac{2G^2M^2}{r^2}+\ldots\right)\mathrm{d}t^2+\left(1+\gamma\frac{2GM}{r}+\ldots\right)(\mathrm{d}x^2+\mathrm{d}y^2+\mathrm{d}z^2)\ .$$

Today's constraints : $|\gamma-1|\lesssim 2\times 10^{-5}$ $|\beta-1|\lesssim 8\times 10^{-5}$

WHY MODIFY GRAVITY ?

COSMOLOGICAL CONSTANT PROBLEM

HUBBLE TENSION



SCALAR-TENSOR THEORIES: $g_{\mu\nu} \quad + \quad \varphi$

ST THEORIES AND COSMOLOGY

A second EFT example: EFT OF INFLATION/DARK ENERGY

A unifying and effective description of cosmological perturbations

$$\phi(t,\vec{x}) \rightarrow \phi_0(t) \quad (\delta\phi = 0) \qquad -\frac{1}{2}\partial\phi^2 \rightarrow -\frac{1}{2}\dot{\phi}_0^2(t)\; g^{00}$$

Creminelli et al. '06
Cheung et al. '07
Gubitosi et al. '12

ST THEORIES AND COSMOLOGY

The Action

$$S = \int d^4x \sqrt{-g} \left[\frac{M^2}{2} f(t) R - \Lambda(t) - c(t) g^{00} \right] + S_{DE}^{(2)}$$

The most generic action written in terms of $g_{\mu\nu}$ compatible with the residual symmetry of spatial diffeomorphisms

Dictionary between theories (Quintessence, Brans-Dicke, Galileons…) and effective parameters (measured in observations):

$$w(t) \ , \quad \mu(t) = \frac{\dot{f}}{f}$$

ST THEORIES AND COSMOLOGY

GW170817: GWs propagate at the speed of light $\left| c_T - c \right| \lesssim 10^{-15}$

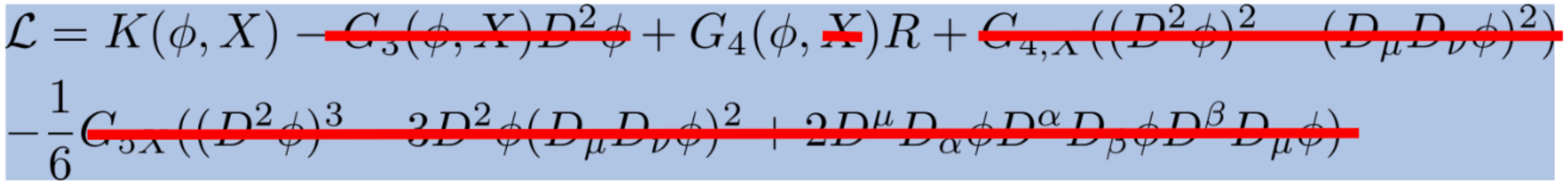
$$S^{(2)}_{\rm DE} \supset \underset{\uparrow \atop 0}{m_4^2}\delta K_2 + \tilde{m}_4^2 \delta g^{00} R - m_5^2 \delta g_{00} \delta K_2$$

Equal

The great massacre:

Quintessence and K-essence

Coupling to matter



$$\mathcal{L} = K(\phi, X) \cancel{- G_3(\phi, X) D^2\phi} + G_4(\phi, \cancel{X})R + \cancel{G_{4,X}((D^2\phi)^2 - (D_\mu D_\nu \phi)^2)}$$
$$-\frac{1}{6}\cancel{G_{5X}((D^2\phi)^3 - 3D^2\phi(D_\mu D_\nu\phi)^2 + 2D^\mu D_\alpha\phi D^\alpha D_\beta\phi D^\beta D_\mu\phi)}$$

GW observations can be extremely powerful probes of fundamental physics!

TESTING GR WITH GW OBSERVATIONS

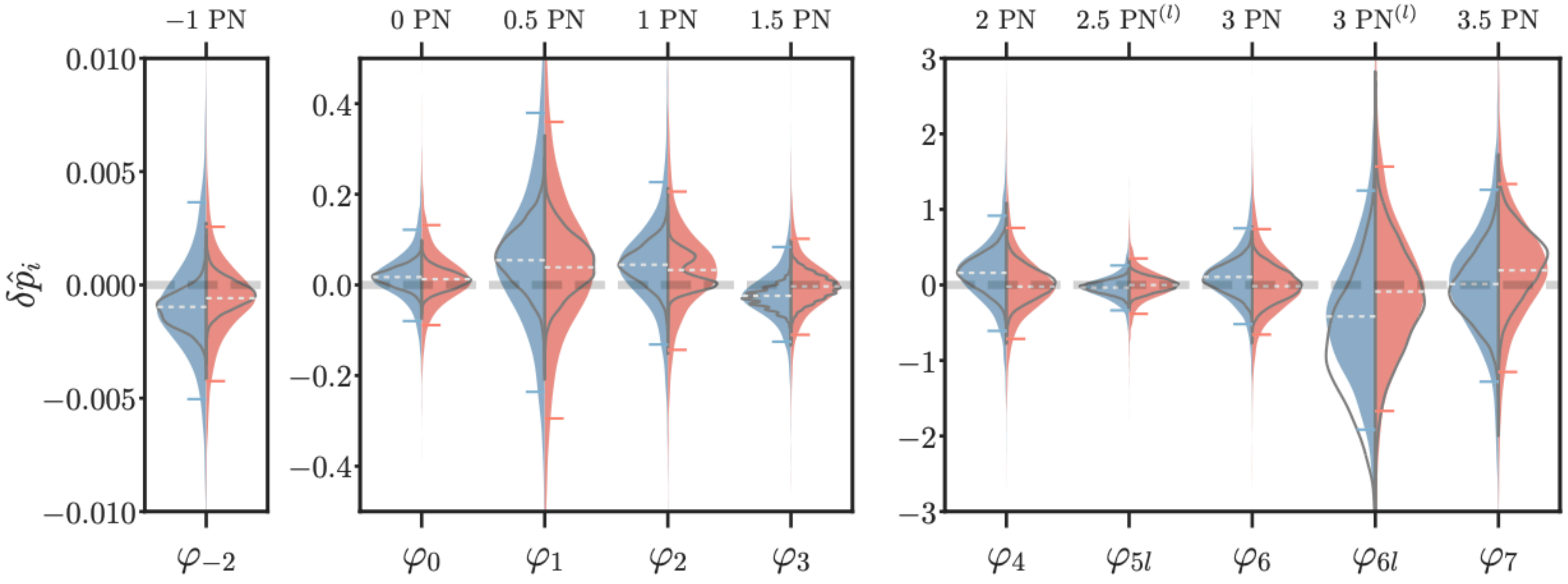
Interferometers give access to the phase of GWs

$$\tilde{h}(f) = A(f)e^{i\Psi(f)}$$

$$\Psi(f) = 2\pi f t_0 + \phi_0 + \sum_k \phi_{\mathrm{PN}}^k \, (\pi \mathcal{M} f)^{(k-5)/3}$$

Parametrized Post-Einsteinian (ppE): vary the PN parameters ϕ_{PN}^k

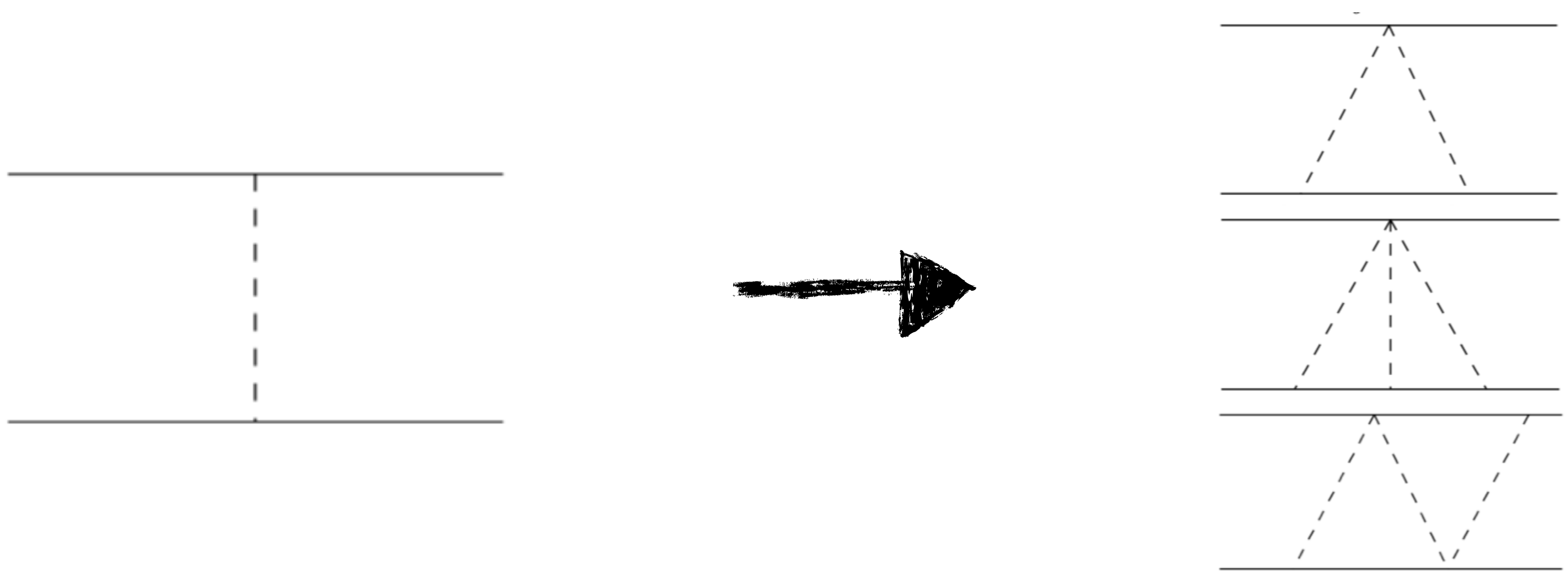
N. Yunes, F. Pretorius 09



LIGO-Virgo collaboration

PLAN

1. THE TWO-BODY PROBLEM IN GR : AN EFT APPROACH



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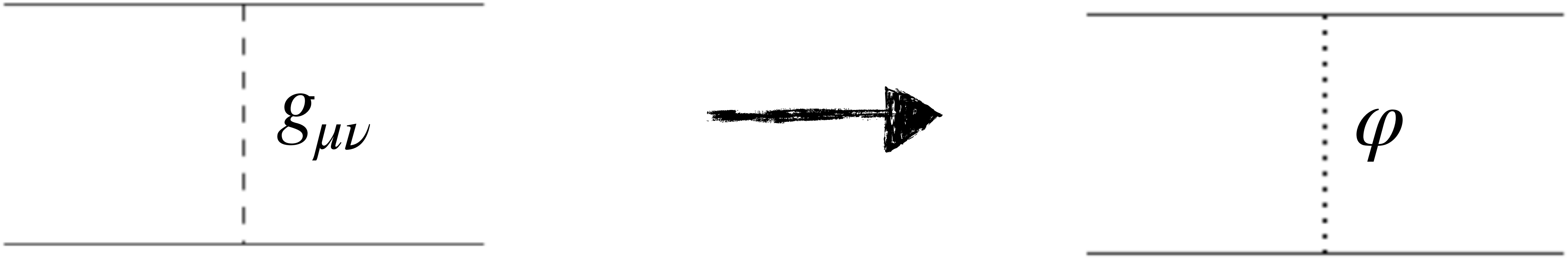
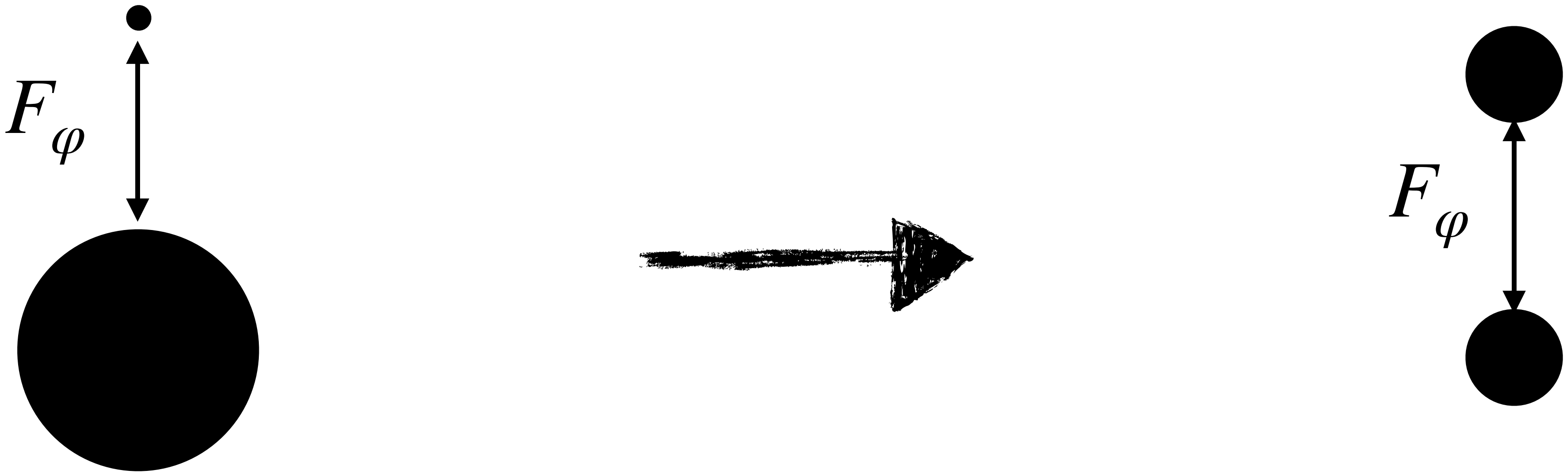


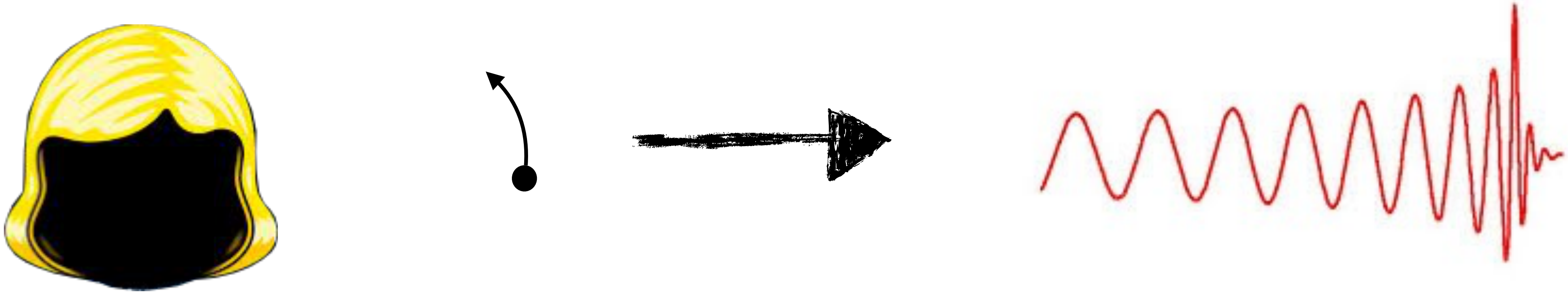
Plan

1. The two-body problem in GR : an EFT approach
2. The two-body problem in Scalar-Tensor theories
3. Two-body problem and screening mechanisms



PLAN

1. THE TWO-BODY PROBLEM IN GR : AN EFT APPROACH
2. THE TWO-BODY PROBLEM IN SCALAR-TENSOR THEORIES
3. TWO-BODY PROBLEM AND SCREENING MECHANISMS
4. EXTREME MASS RATIO INSPIRALS AND SCALAR HAIR

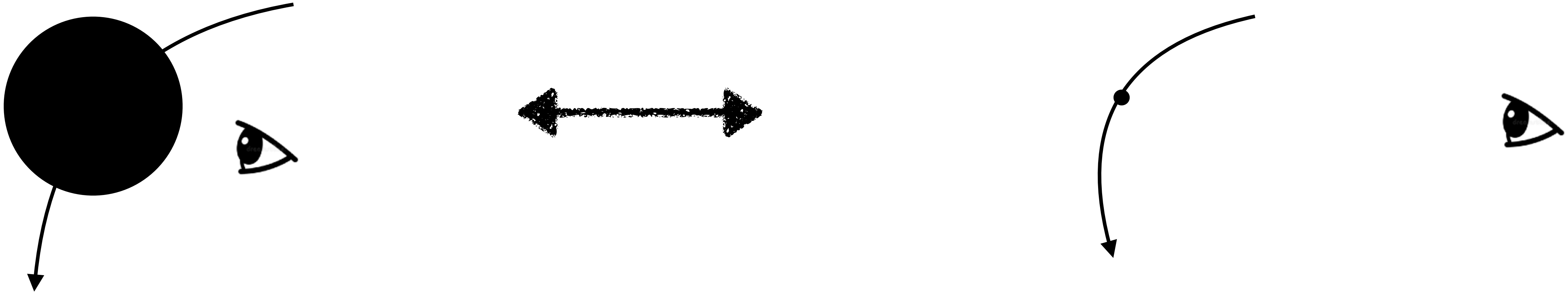


THE TWO-BODY PROBLEM IN GR

Basic ingredient of GR : the METRIC $g_{\mu\nu}$

Action principle (in vacuum) : $S_{\rm EH} = \frac{M_P^2}{2}\int d^4x\sqrt{-g}R \quad \Rightarrow \quad G_{\mu\nu} = 0$

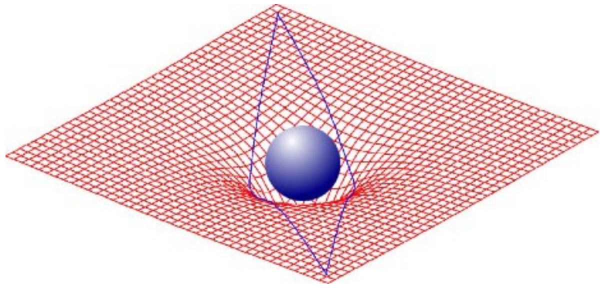
Model black holes/neutron stars with POINT-PARTICLES



$$S_{\rm pp,A} = -\, m_A \int d\tau_A \, = -\, m_A \int dt \sqrt{-g_{\mu\nu}\frac{dx_A^\mu}{dt}\frac{dx_A^\nu}{dt}}$$

FINITE-SIZE EFFECTS

$$-m_A \int d\tau_A \qquad \Rightarrow \qquad \frac{d^2 x_A^\mu}{d\tau^2} + \Gamma^\mu_{\nu\rho} \frac{dx_A^\nu}{d\tau} \frac{dx_A^\rho}{d\tau} = 0$$



Finite-size effects can be added through non-minimal operators:

$$\mathcal{O}_1 = c_R \int \mathrm{d}\tau_A \, R \qquad \qquad \mathcal{O}_2 = c_V \int \mathrm{d}\tau_A \, R_{\mu\nu} v^\mu v^\nu$$

$$\Rightarrow \qquad \frac{d^2 x_A^\mu}{d\tau^2} + \Gamma^\mu_{\nu\rho} \frac{dx_A^\nu}{d\tau} \frac{dx_A^\rho}{d\tau} \neq 0$$

These are quite high-order effects

THE TWO-BODY PROBLEM IN GR

EFT approach : use field theory tools

Goldberger and Rothstein 06
Porto 06
+ many others…

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

$$\mathcal{O}(v^2) = \mathcal{O}\left(\frac{GM}{r}\right) \ll 1$$

THE TWO-BODY PROBLEM IN GR

EFT approach : use field theory tools

Goldberger and Rothstein 06
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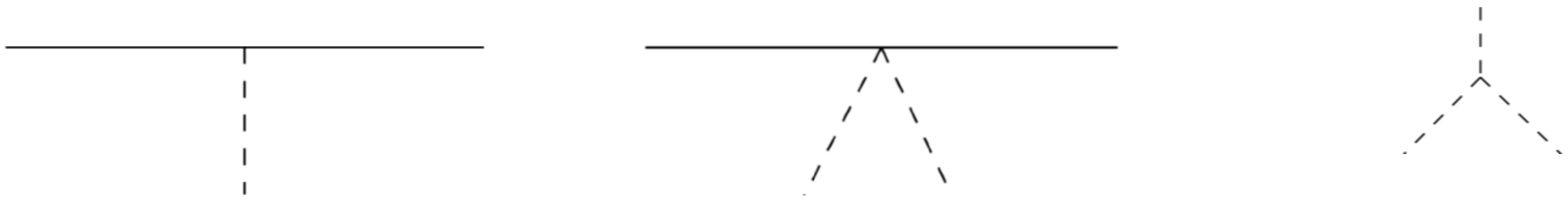
$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \quad \Rightarrow \quad S = S^{(2)} + S_{\rm int}$$

GREEN FUNCTION or PROPAGATOR: from $\int d^4x\sqrt{-g}R$

$$S^{(2)} = -\frac{1}{8}\int d^4x\left[-\frac{1}{2}(\partial_\mu h^\alpha_\alpha)^2 + (\partial_\mu h_{\nu\rho})^2\right]$$

INTERACTION VERTEX: from $-m_A\int dt\sqrt{-g_{\mu\nu}\frac{dx^\mu_A}{dt}\frac{dx^\nu_A}{dt}}$

$$S_{\rm int} \supset m\int {\rm d}t\ h_{00}\ , \qquad m\int {\rm d}t\ h^2_{00}\ , \qquad \int {\rm d}^4x\ \partial^2 h^3$$



THE TWO-BODY PROBLEM IN GR

The two-body dynamics is encoded in the EFFECTIVE ACTION :

$$e^{iS_{\text{eff}}[\mathbf{x}_1(t),\mathbf{x}_2(t)]} = \int \mathscr{D}h_{\mu\nu}\, e^{iS[\mathbf{x}_1(t),\mathbf{x}_2(t),h_{\mu\nu}]}$$

REAL PART: CONSERVATIVE

IMAGINARY PART: DISSIPATIVE

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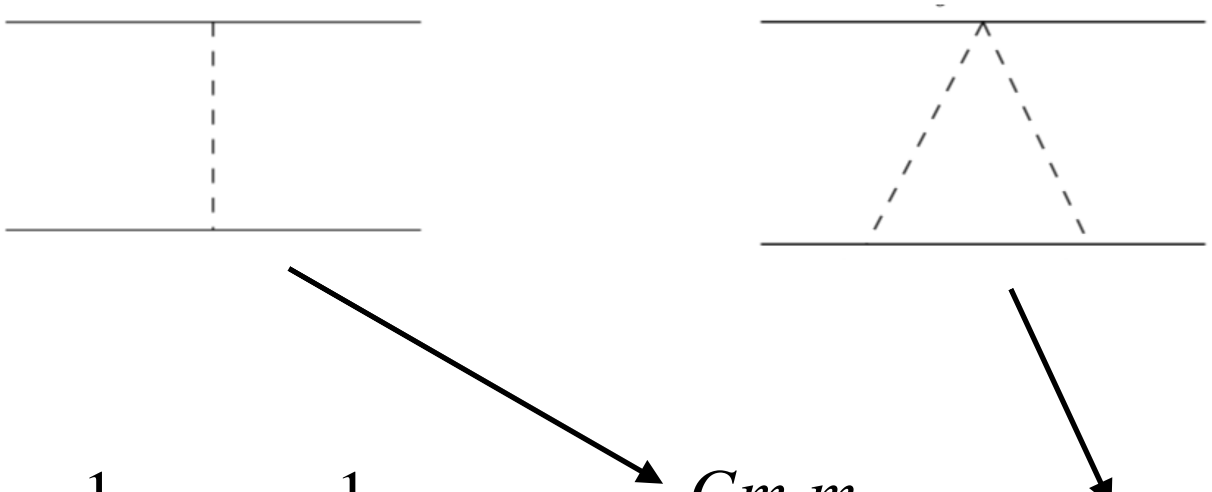
$$\Re(S_{\rm eff}) = \int dt L[\mathbf{x}_A, \mathbf{v}_A]$$



$$L \quad = \frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2 + \frac{Gm_1 m_2}{|\mathbf{x_1}(t) - \mathbf{x_2}(t)|} + L_{\rm 1PN} + \dots$$

IMAGINARY PART: DISSIPATIVE

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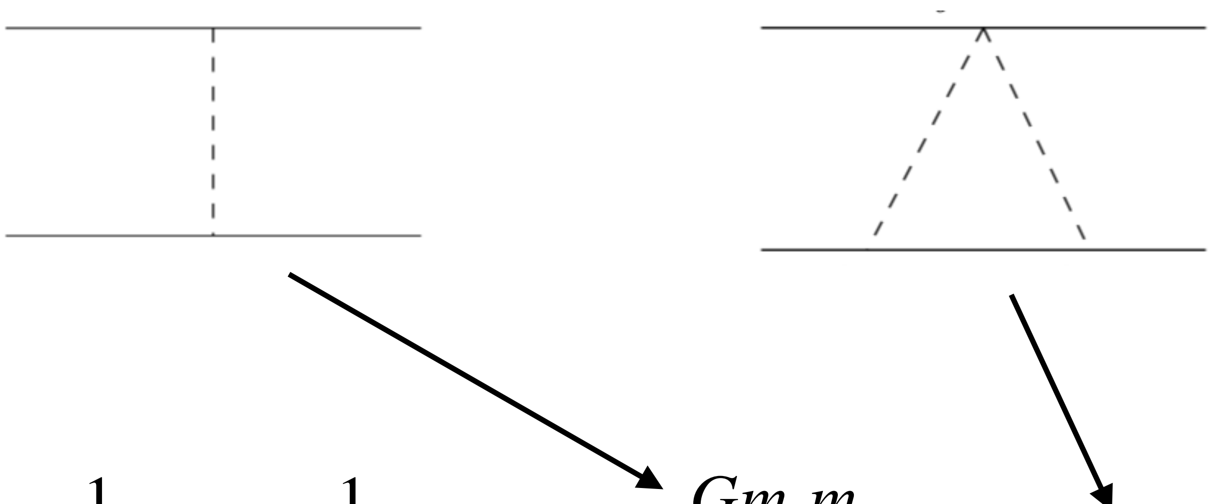
REAL PART: CONSERVATIVE

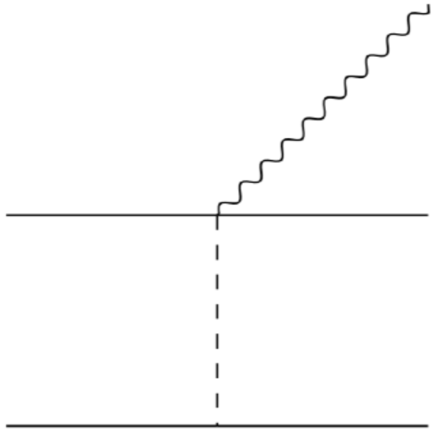
$$\mathfrak{R}(S_{\text{eff}}) = \int dtL[\mathbf{x}_A, \mathbf{v}_A]$$



$$L \quad = \frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2 + \frac{Gm_1m_2}{|\,\mathbf{x_1}(t) - \mathbf{x_2}(t)\,|} + L_{1\text{PN}} + \ldots$$

IMAGINARY PART: DISSIPATIVE

$$\mathfrak{I}(S_{eff}) = \frac{T}{2}\int dEd\Omega \frac{d^2\Gamma}{dEd\Omega}$$



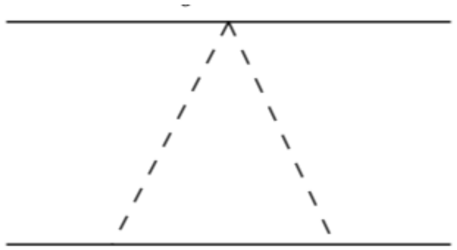
$$P = \frac{G}{5}\left\langle \dddot{Q}^{kl}\dddot{Q}_{kl}\right\rangle + \ldots$$

A RESUMMATION TECHNIQUE

A. Kuntz (PRD) 20

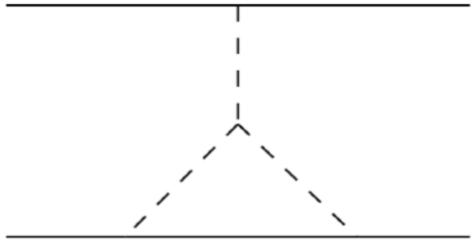
In the 1PN potential enter two types of vertex

$$S_{\text{pp}} \supset \int \mathrm{d}t\, h_{00}^2$$



$$= \frac{G^2 m_1 m_2^2}{2r^2}$$

$$S_{\text{EH}} \supset \int \mathrm{d}^4x\, \partial^2 h^3$$



$$= -\frac{G^2 m_1 m_2^2}{r^2}$$

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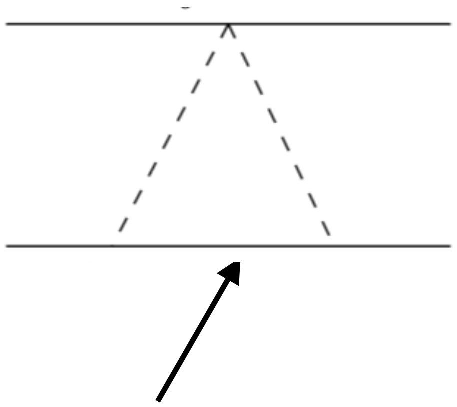

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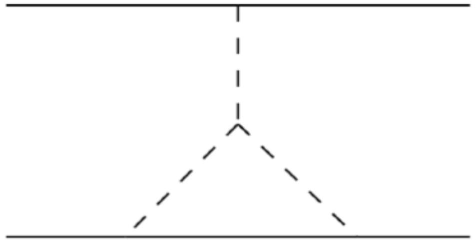

$$= -\frac{G^2 m_1 m_2^2}{r^2}$$

The first one can be resummed exactly !

$$S_{\rm pp,A} = -m_A \int dt \sqrt{-g_{\mu\nu} v_A^\mu v_A^\nu} \quad \Leftrightarrow \quad S_{\rm pp,A} = -\frac{m_A}{2} \int \mathrm{d}t \left[e_A - \frac{g_{\mu\nu} v_A^\mu v_A^\nu}{e_A} \right]$$

with $e_A = \sqrt{-g_{\mu\nu} v_A^\mu v_A^\nu}$

. . . The worldline couplings are now LINEAR

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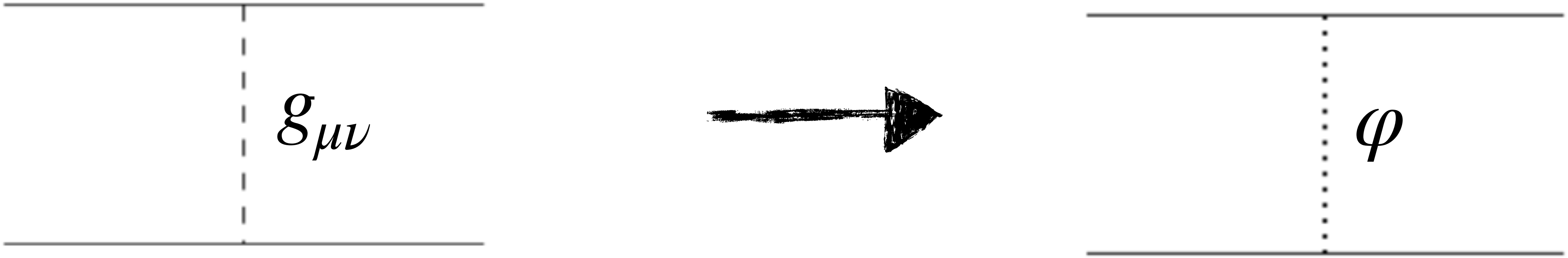


MODIFYING GR : SCALAR-TENSOR THEORIES

GR action: $$S = \frac{M_P^2}{2}\int d^4x\sqrt{-g}R + S_m[g_{\mu\nu},\psi_i]$$

A simple alternative to GR: $g_{\mu\nu} \quad + \quad \varphi$

$$S_\varphi = -\frac{1}{2}\int d^4x\sqrt{-g}\,g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi$$

Coupling of φ with matter, compatible with causality and equivalence principle:

$$S_m[\tilde{g}_{\mu\nu},\psi_i] \quad \text{with} \quad \tilde{g}_{\mu\nu} = \underset{\substack{\uparrow \\ \text{Conformal}}}{A(\varphi,X)}g_{\mu\nu} + \underset{\substack{\nwarrow \\ \text{Disformal}}}{B(\varphi,X)}\partial_\mu\varphi\partial_\nu\varphi \qquad X = g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi$$

Bekenstein 92

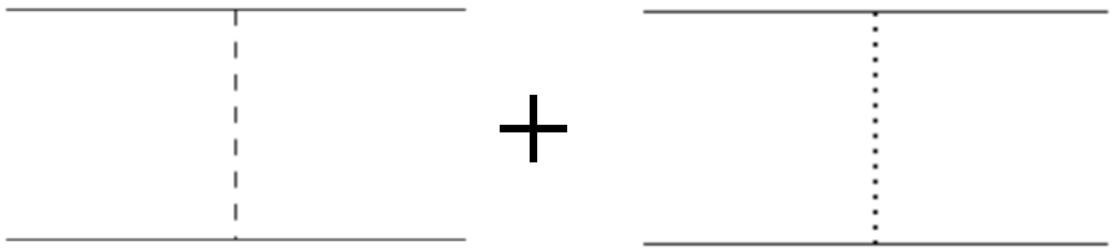
CONFORMAL COUPLING

Focus on $\tilde{g}_{\mu\nu} = A(\varphi) g_{\mu\nu}$

A. Kuntz, F. Piazza, F. Vernizzi (JCAP) 19

$$S_{\rm pp} = -\int d\tau_A\, m_A(\varphi) = m_A \int d\tau \left(-1 + \alpha_A \frac{\varphi}{M_P} + \delta_A \left(\frac{\varphi}{M_P} \right)^2 + \ldots \right)$$

CONSERVATIVE



$$\tilde{G}_N = G_N \left(1 + 2\alpha_1 \alpha_2 \right)$$

DISSIPATIVE

Scalar-tensor theories

CONFORMAL COUPLING

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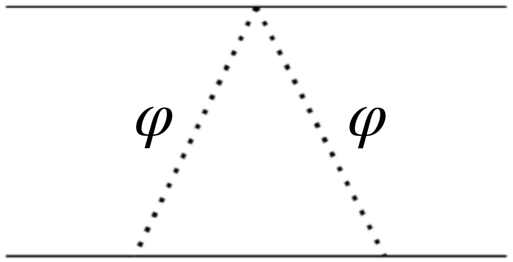
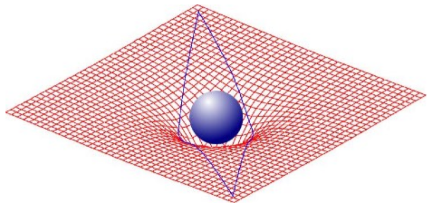
CONSERVATIVE



$+\ldots$

PPN parameters :

$$\gamma_{AB} = 1 - 4 \frac{\alpha_A \alpha_B}{1 + 2\alpha_A \alpha_B}$$



$$\beta_{AB} = 1 - 2 \frac{\alpha_A^2 \alpha_B^2 + f_{AB}}{(1 + 2\alpha_A \alpha_B)^2}$$

DISSIPATIVE

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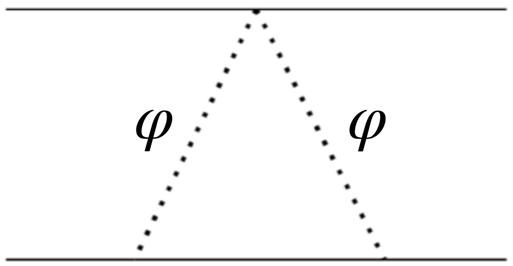

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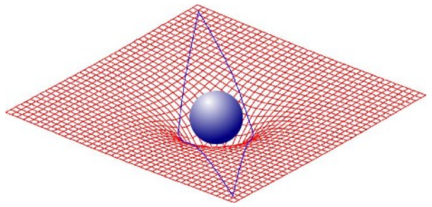
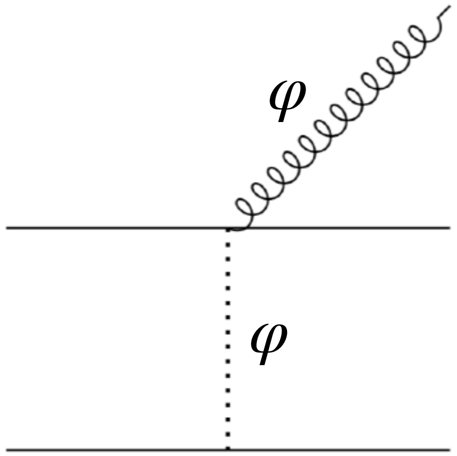


DISSIPATIVE



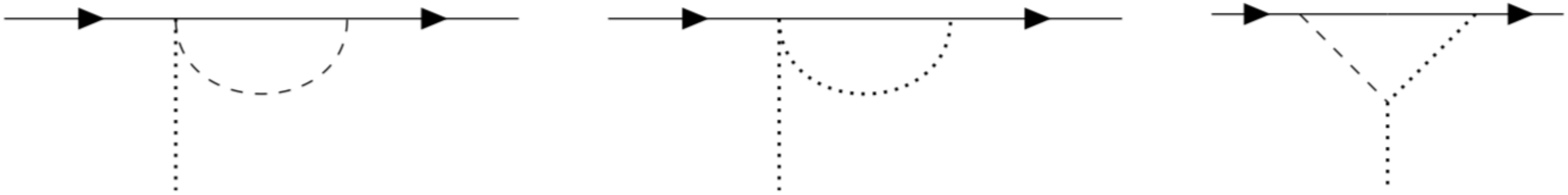
$$P_\phi = 2G_N \left(\left\langle \dot{I}_\phi^{\,2} \right\rangle + \frac{1}{3} \left\langle \ddot{I}_\phi^{i\,2} \right\rangle + \frac{1}{30} \left\langle \dddot{I}_\phi^{ij\,2} \right\rangle + \ldots \right)$$

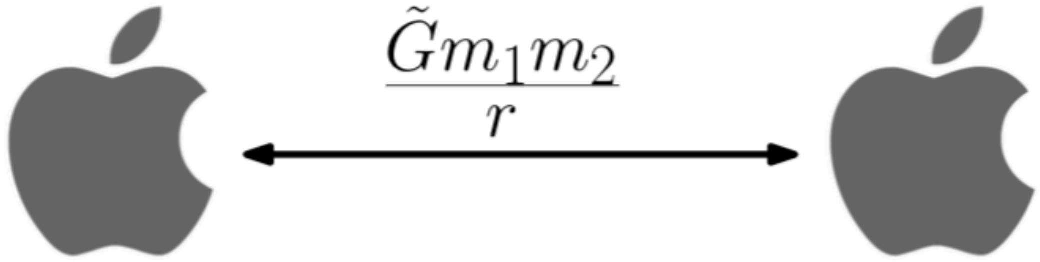
Monopole Dipole Quadrupole

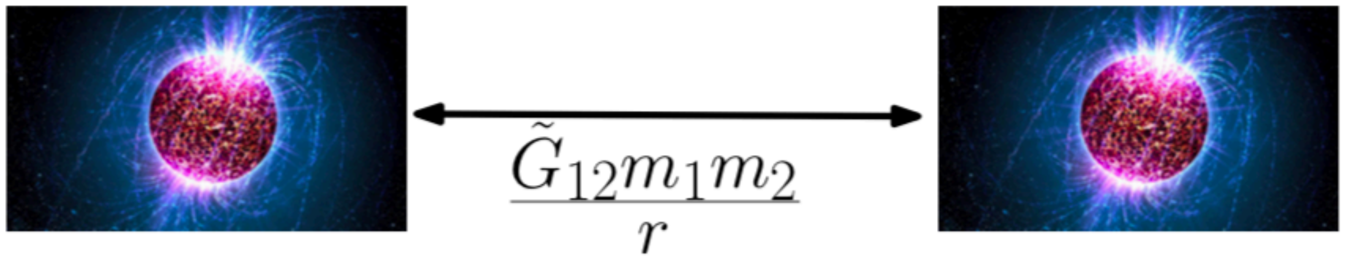
CHARGE RENORMALISATION

A. Kuntz, F. Piazza, F. Vernizzi (JCAP) 19

$$S_{\text{int}} = -\int d\tau_A\, m_A(\varphi) = m_A \int d\tau \left(-1 + \alpha \frac{\varphi}{M_P} + \delta \left(\frac{\varphi}{M_P} \right)^2 + \ldots \right)$$





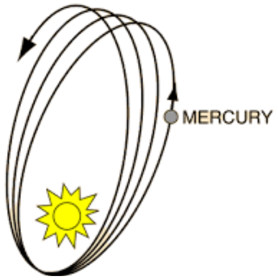


$$\tilde{G}_{12} = G_N \left(1 + 2\alpha_1 \alpha_2 \right)$$

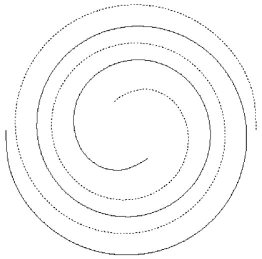
CONCLUSION PART 2

MAIN ASPECTS OF SCALAR-TENSOR THEORIES, WITH RESPECT TO GR:

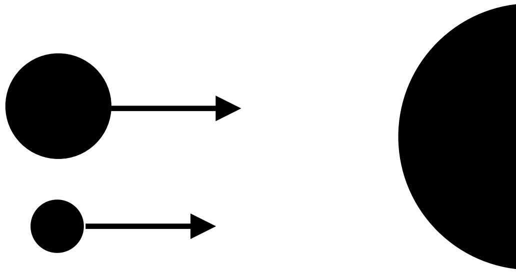
- Bending of light and perihelion is different



- Dipolar radiation



- Violations of the strong equivalence principle



CONCLUSION PART 2

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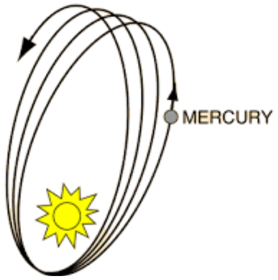

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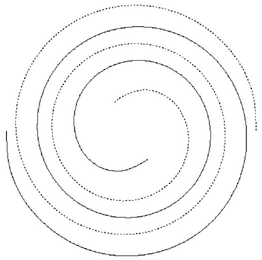

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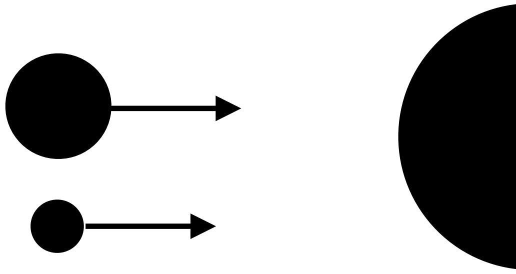


Experimental tests are very stringent: the scalar coupling is small

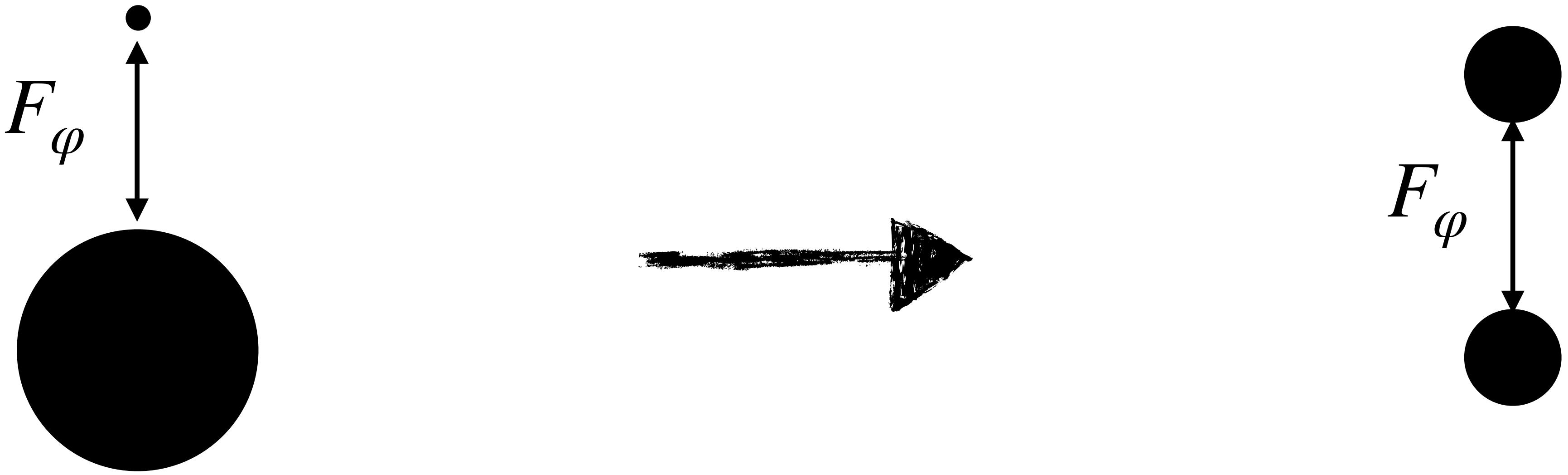
$$\alpha \lesssim 10^{-2}$$

A screening mechanism could explain such a small value

HOW TO FORMULATE THE TWO-BODY PROBLEM WITH A SCREENING MECHANISM?

PLAN

1. THE TWO-BODY PROBLEM IN GR : AN EFT APPROACH
2. THE TWO-BODY PROBLEM IN SCALAR-TENSOR THEORIES
3. TWO-BODY PROBLEM AND SCREENING MECHANISMS



K-MOUFLAGE SCREENING

$$S = \int d^4x \left[-\frac{(\partial\varphi)^2}{2} - \frac{1}{4\Lambda^4}(\partial\varphi)^4 + \frac{\varphi T}{M_P} \right]$$

For cosmological applications

$$\Lambda^2 \sim H M_P$$

Equation of motion around a static source:

$$\varphi_0' + \frac{\left(\varphi_0'\right)^3}{\Lambda^4} = \frac{M}{4\pi M_P r^2}$$

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$$\varphi_0'(r) \simeq \left(\frac{\Lambda^4 M}{4\pi M_P r^2}\right)^{1/3}$$

$$\varphi_0'(r) \simeq \frac{M}{4\pi M_P r^2}$$

$$\frac{\varphi_0}{\varphi_{\rm Newt}} \simeq \left(\frac{r}{r_*}\right)^{4/3}$$

$$\frac{\varphi_0}{\varphi_{\rm Newt}} \simeq 1$$

$$r_* = \left(\frac{M}{4\pi M_P \Lambda^2}\right)^{1/3}$$

(0.1 parsecs for the Sun)

K-MOUFLAGE SCREENING

$$S = \int d^4x \left[-\frac{(\partial\varphi)^2}{2} - \frac{1}{4\Lambda^4}(\partial\varphi)^4 + \frac{\varphi T}{M_P} \right]$$

For cosmological applications

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Equation of motion around a static source:

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EFFECT ON THE PERIHELION

$$\frac{\varphi_0}{\varphi_{\text{Newt}}} \sim \left(\frac{r}{r_*}\right)^{4/3} \leq 10^{-11}$$

L. Iorio 12

TWO-BODY PROBLEM

PERTURBATIVE EXPANSION BREAKS DOWN…

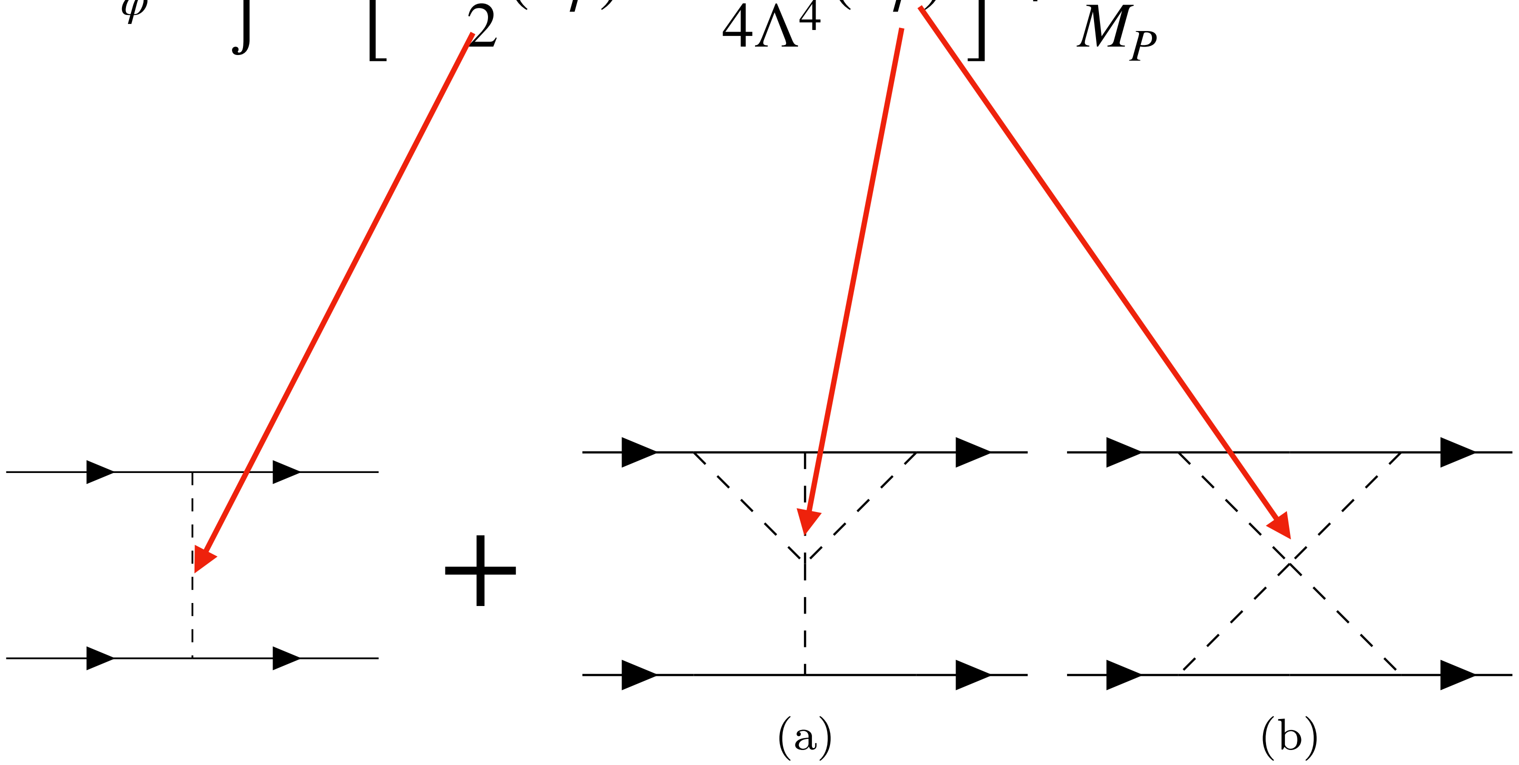
$$e^{iS_{\text{eff}}[\mathbf{x}_1,\mathbf{x}_2]} = \int \mathscr{D}[\varphi] e^{iS[\mathbf{x}_1,\mathbf{x}_2,\varphi]}$$

$$S_\varphi = \int \mathrm{d}^4x \left[-\frac{1}{2}(\partial\varphi)^2 - \frac{1}{4\Lambda^4}(\partial\varphi)^4 \right] + \frac{\varphi T}{M_P}$$

but

$$r < r_* \Leftrightarrow \frac{(\partial\varphi)^2}{\Lambda^4} \gg 1$$

$S_{\text{eff}} =$



$+ \ldots$ DIVERGES

TWO-BODY PROBLEM

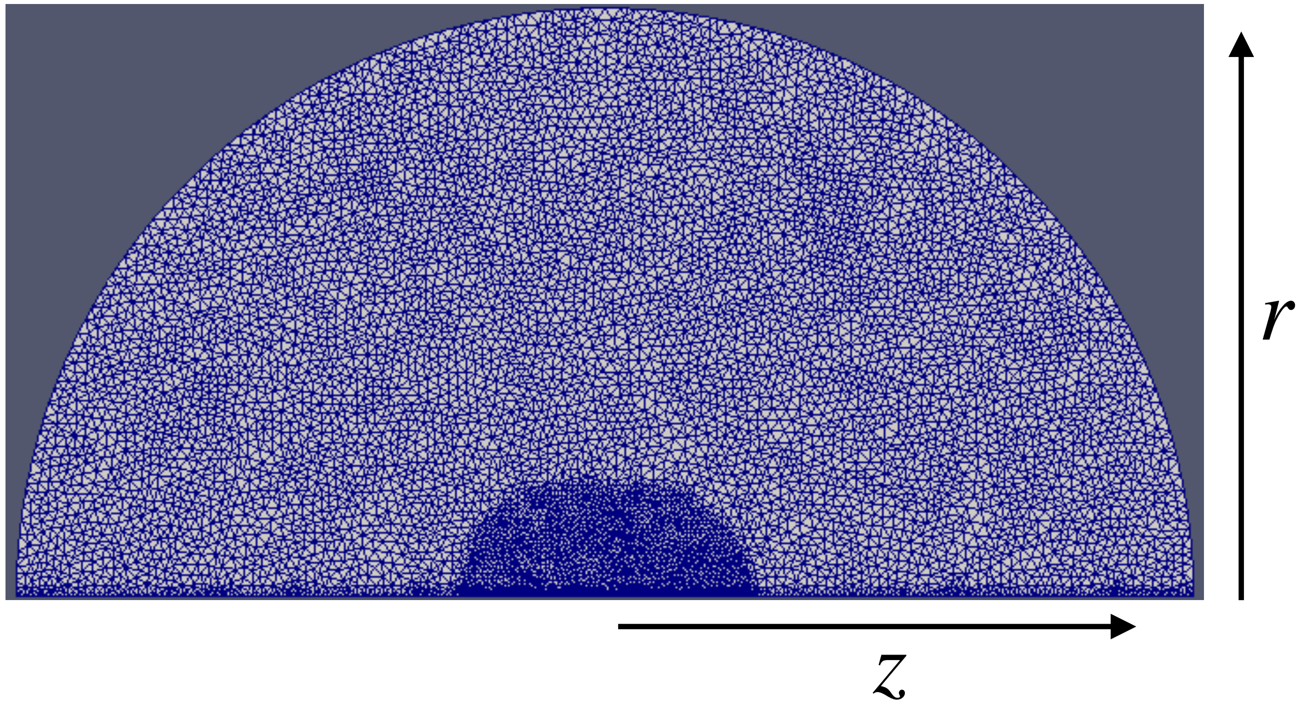
A. Kuntz (PRD) 19

A NUMERICAL SOLUTION:

$$\partial_i \left[\partial^i \varphi + \frac{1}{\Lambda^4} \left(\partial_i \partial^i \varphi \right)^2 \partial^i \varphi \right] = \frac{T}{M_P}$$

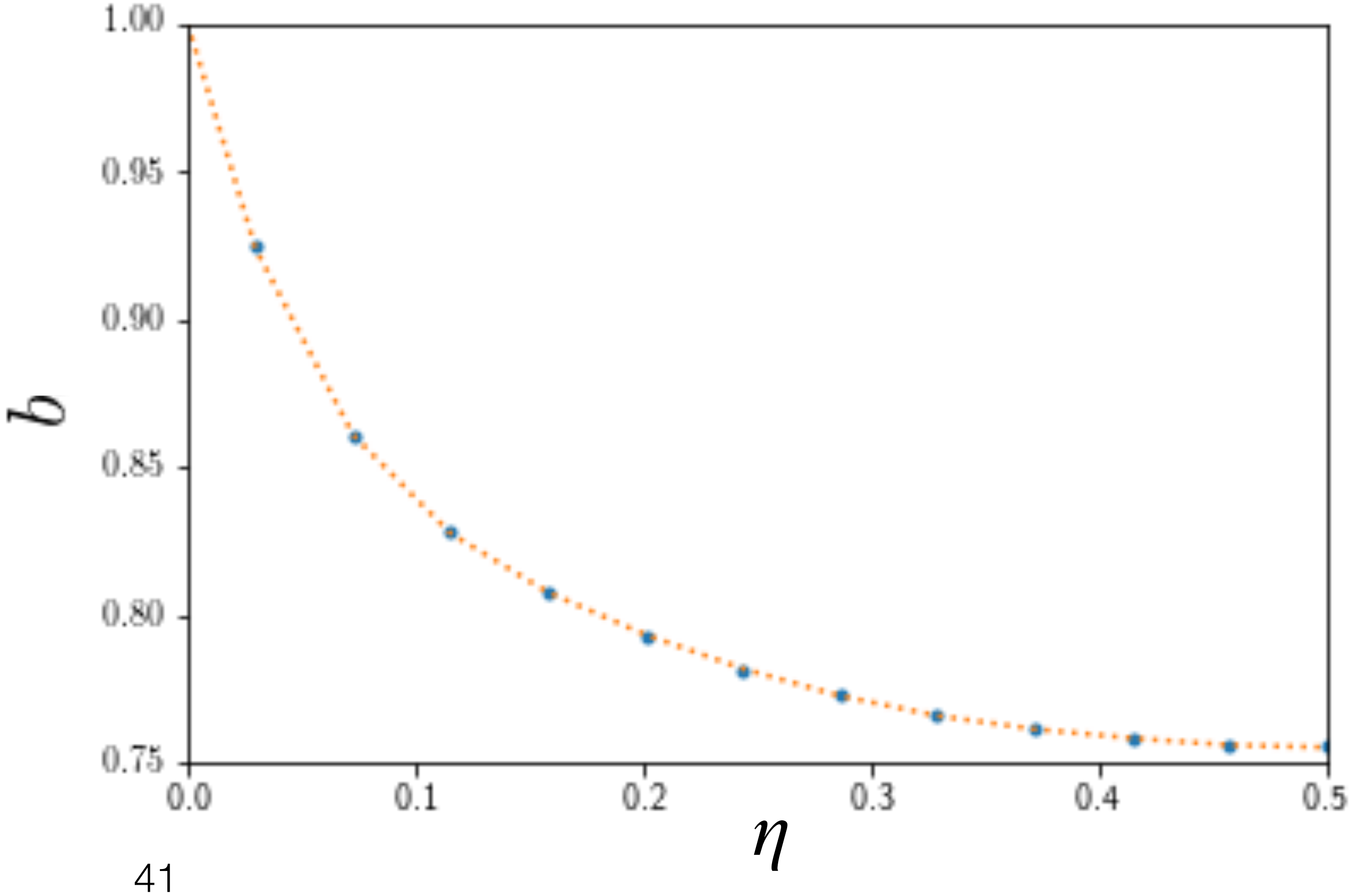
$$\Downarrow$$

$$\frac{E_{2\text{ body}}}{\mu \varphi_0(r)} = b(\eta) \quad (= 1 \text{ in Newton's gravity})$$



$$\mu = \frac{m_1 m_2}{m_1 + m_2}$$

$$\eta = \frac{m_1}{m_1 + m_2}$$



EP VIOLATION

A. Kuntz (PRD) 19

$$E = \mu\, b(\eta)\, \varphi_0(r)$$

$$\Rightarrow \quad \vec{a} = b(\eta)\, \vec{\nabla}\, \varphi_0(r)$$

EP VIOLATION

A. Kuntz (PRD) 19

$$E = \mu\, b(\eta)\, \varphi_0(r)$$

$$\Rightarrow \quad \vec{a} = b(\eta)\, \vec{\nabla}\, \varphi_0(r)$$

$$\delta r_{EM} \simeq 3 \times 10^{12} \left| \eta_{SE} \left(\frac{r}{r_*} \right)^{4/3} \right| \text{ cm}$$

This gives a constraint :

$$\eta_{SE} \left(\frac{r}{r_*} \right)^{4/3} \lesssim 10^{-13}$$

Since $\eta_{\mathrm{SE}} \simeq 10^{-6}$, the perihelion constraint is better:

$$\left(\frac{r}{r_*} \right)^{4/3} \lesssim 10^{-11}$$

CONCLUSIONS PART 3

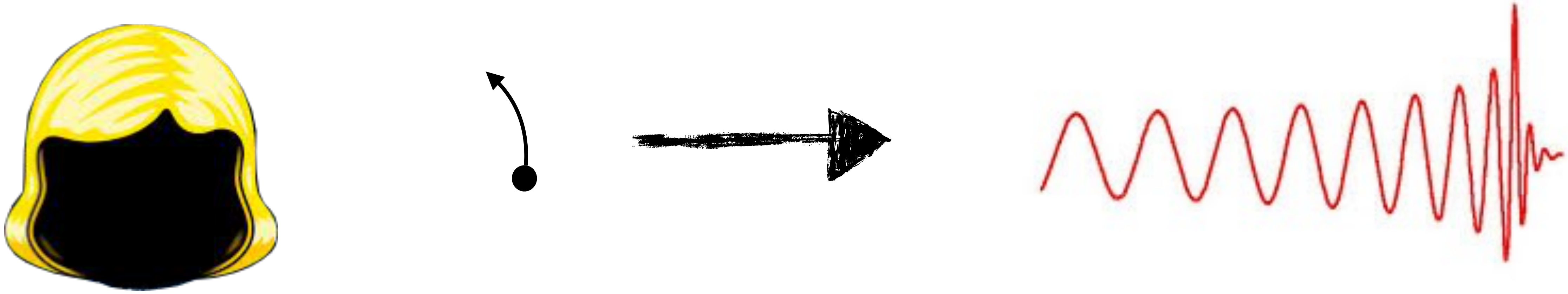
- Screening mechanisms naturally recover GR inside the solar system
- They lead to violations of the Equivalence Principle

There remains an important question:

HOW IS THE (TWO-BODY) MOTION OF BLACK HOLES MODIFIED IN SCALAR-TENSOR THEORIES ?

PLAN

1. THE TWO-BODY PROBLEM IN GR : AN EFT APPROACH
2. THE TWO-BODY PROBLEM IN SCALAR-TENSOR THEORIES
3. TWO-BODY PROBLEM AND SCREENING MECHANISMS
4. EXTREME MASS RATIO INSPIRALS AND SCALAR HAIR



TESTING GR WITH GW OBSERVATIONS

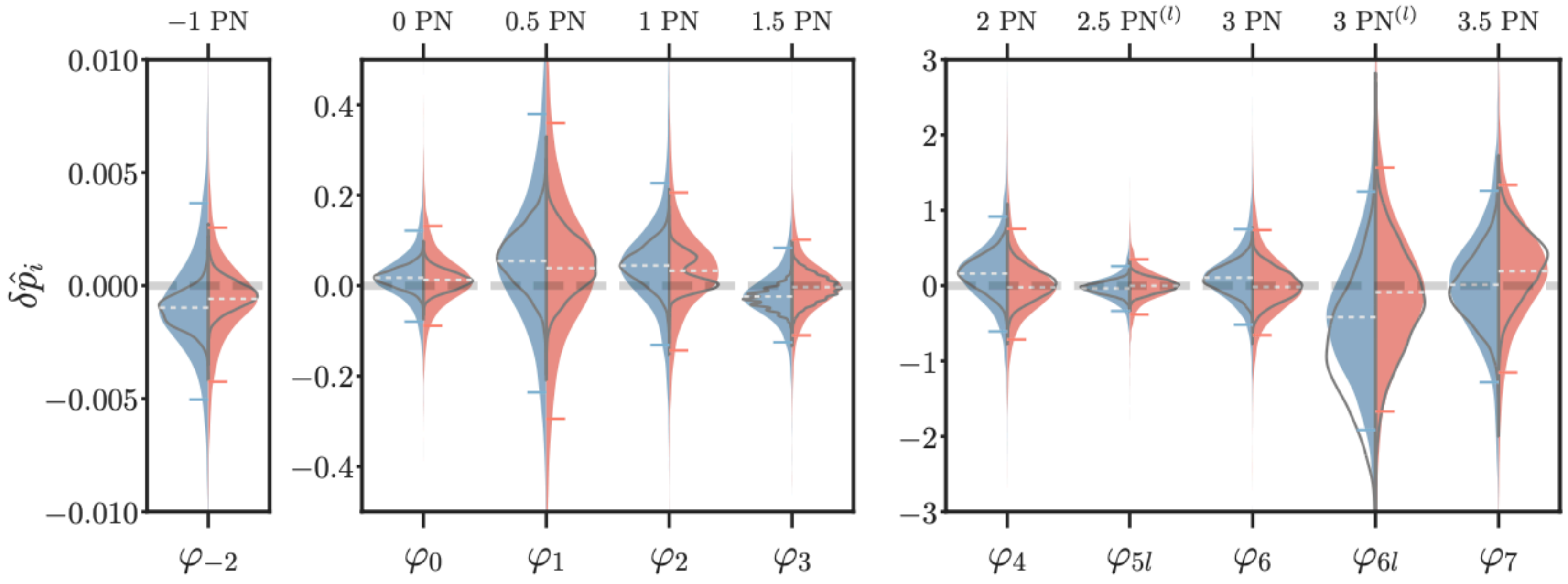
Interferometers give access to the phase of GWs

$$\tilde{h}(f) = A(f)e^{i\Psi(f)}$$

$$\Psi(f) = 2\pi f t_0 + \phi_0 + \sum_k \phi^k_{\rm PN} \, (\pi \mathcal{M} f)^{(k-5)/3}$$

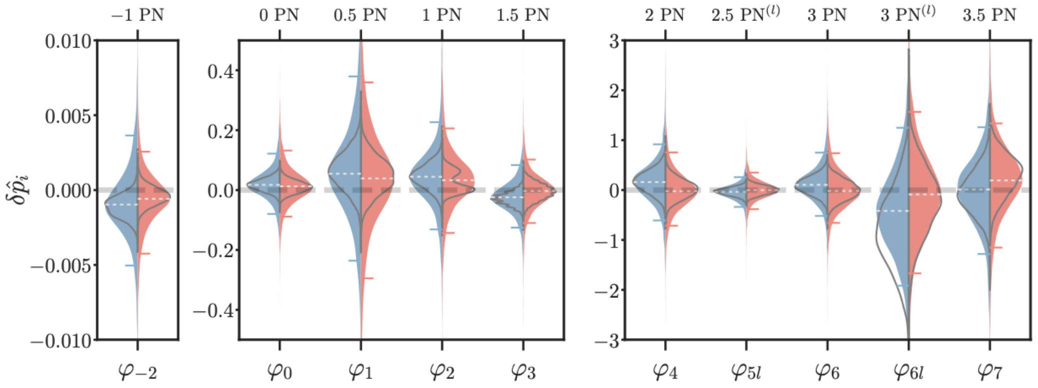
Parametrized Post-Einsteinian (ppE): vary the PN parameters $\phi^k_{\rm PN}$

N. Yunes, F. Pretorius 09



LIGO-Virgo collaboration

TESTING GR WITH GW OBSERVATIONS



Main drawbacks of this analysis:

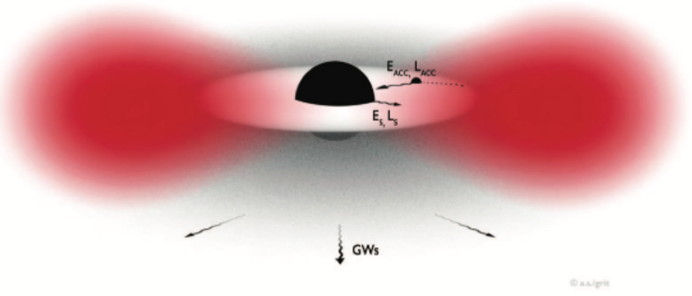
- Too many free parameters
- Neglects correlations between different PN coefficients
- It’s a lot of work to translate these values into constraints on fundamental physics parameters !

An EFT formalism will address all of these three points

Let’s consider theories with one supplementary non-GR parameter :

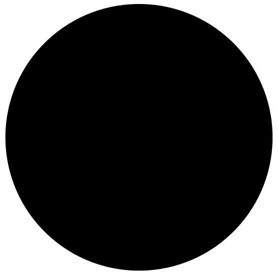
SCALAR CHARGE $q \Leftrightarrow \phi$

- Fundamental force
- Dark matter profile
- Superradiant cloud



THE NO-HAIR THEOREM

In GR, BH are very simple objects!



M, J, Q

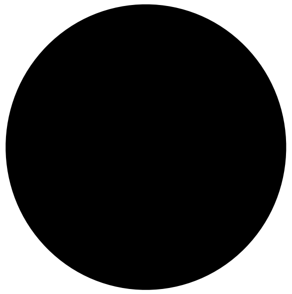
VS

A ton of complicated physics
(composition, EoS…)

This can be generalised to modified gravity:

$$L = \frac{M_P^2}{2} R - (\partial\varphi)^2 - V(\varphi)$$

$$\Downarrow$$



$$\bar{\varphi}(r) = 0$$

(also valid for more complicated Lagrangians)

THE NO-HAIR THEOREM

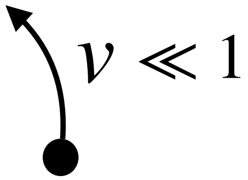
However, it is easy to circumvent the assumptions of the theorem

	I Jacobson '99	II Babichev Esposito-Farèse '13	III Sotiriou et al. '14
Hair type	Environmental	Environmental	Secondary
Lagrangian	$L_1 = \frac{M_P^2}{2} R - \frac{1}{2}(\partial\varphi)^2$	$L = L_1 - \frac{1}{2\Lambda^3}(\partial\varphi)^2 \Box\varphi$	$L = L_1 + \bar{\alpha}\phi\big(R_{\mu\nu\rho\lambda}R^{\mu\nu\rho\lambda} - 4R_{\mu\nu}R^{\mu\nu} + R^2\big)$
Field	$\varphi(t,r) = qt + \beta_{\rm eff}\,\varphi_0(r)$	$\varphi(t,r) = qt + \beta_{\rm eff}\,\varphi_0(r)$	$\varphi(r) = \frac{Q}{r} + \mathcal{O}\left(\frac{1}{r^2}\right)$

The GW signals would then be quite different than in GR!

HAIR EXAMPLE II: CUBIC GALILEON

P. Brax, L. Heisenberg, A. Kuntz (JCAP) 20



$$\varphi = qt + \bar{\varphi}(r) + \delta\varphi$$

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + \delta g_{\mu\nu}$$

QUADRATIC ACTION for fluctuations:

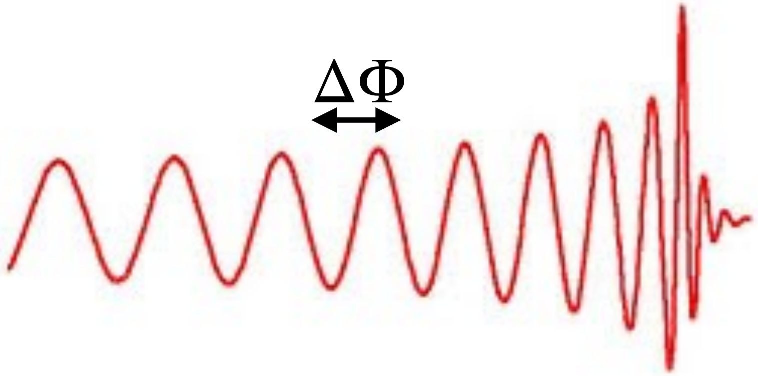
$$K_t = 3\left(\frac{r_*}{r}\right)^{3/2}$$

$$K_r = 4\left(\frac{r_*}{r}\right)^{3/2}$$

$$K_\Omega = \left(\frac{r_*}{r}\right)^{3/2}$$

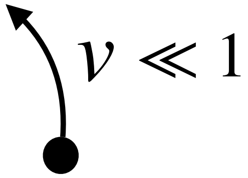
$$S = \int d^4x \frac{1}{2}\left[K_t(\partial_t\delta\varphi)^2 - K_r(\partial_r\delta\varphi)^2 - K_\Omega(\partial_\Omega\delta\varphi)^2\right] + \frac{\beta_{\text{eff}}}{M_P}\delta\varphi T$$

Solve for the field using Green's function



$$\Delta\Phi \simeq 3.5 \times 10^{-7}\beta_{\text{eff}}^{3/2}\left(\frac{\Lambda}{10^{-12}\text{eV}}\right)^{3/2}\left(\frac{m_1}{50M_\odot}\right)^{-1}\left(\frac{m_0}{10^6M_\odot}\right)^{-3/2}\left(\frac{\Omega_{\text{in}}}{10^{-3}\text{Hz}}\right)^{-21/6}$$

A SYSTEMATIC APPROACH



$$\varphi = \bar{\varphi}(r) + \delta\varphi$$

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + \delta g_{\mu\nu}$$

UNITARY GAUGE : $\varphi(t,x) \rightarrow \bar{\varphi}(r)$ i.e $\delta\varphi = 0$

A SYSTEMATIC APPROACH

$$\varphi = \bar{\varphi}(r) + \delta\varphi$$

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + \delta g_{\mu\nu}$$

UNITARY GAUGE : $\varphi(t,x) \to \bar{\varphi}(r)$ i.e $\delta\varphi = 0$

EFFECTIVE ACTION : $S = \int \mathrm{d}^4x \sqrt{-g}\left[\frac{1}{2}M_1^2(r)R - \Lambda(r) - f(r)g^{rr} - \alpha(r)\bar{K}^{\mu}{}_{\nu}K^{\nu}{}_{\mu}\right] + S^{(2)}$

G. Franciolini et al. 19

- Λ, f and α uniquely determined by the background $\bar{g}_{\mu\nu}$
- M_1^2 removable by a conformal transformation

$$g^{(\mathrm{E})}_{\mu\nu}(x) = g^{(\mathrm{J})}_{\mu\nu}(x)M_1^2(r)$$

$$S_{\mathrm{pp}} = -\int dt\, \mu \sqrt{-\bar{g}_{\mu\nu}v^{\mu}v^{\nu}} \to -\int dt\, \mu(r) \sqrt{-\bar{g}_{\mu\nu}v^{\mu}v^{\nu}}$$

THE METRIC

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + \delta g_{\mu\nu}$$

Background:

$$\bar{g}_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu = -a^2(r)\mathrm{d}t^2 + \frac{\mathrm{d}r^2}{b^2(r)} + c^2(r)\left(\mathrm{d}\theta^2 + \sin^2\theta \mathrm{d}\phi^2\right)$$

E.g. for Gauss-Bonnet:

$$a^2(r) = 1 - \frac{2M}{r} + \frac{MQ^2}{6r^3} + \mathcal{O}(r^{-4}) \quad b^2(r) = 1 - \frac{2M}{r} + \frac{Q^2}{2r^2} + \mathcal{O}(r^{-3}) \qquad c^2(r) = r^2 \quad \text{(gauge choice)}$$

THE METRIC

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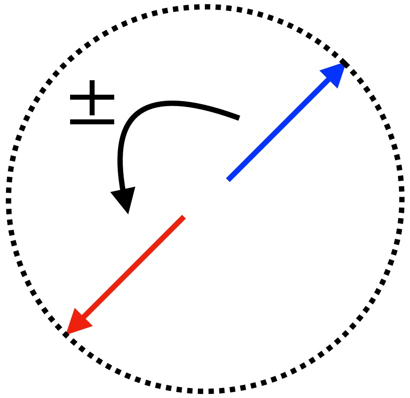
Perturbations:

$\delta g_{\mu\nu}$ transforms under $(i, j) = (\theta, \phi)$ diffs and under PARITY:



$$\delta g_{\mu\nu}^{\mathrm{odd}} \qquad \Leftrightarrow \qquad \Psi$$

THE ODD SECTOR

A. Kuntz, R. Penco, F. Piazza (JCAP) 20

GENERALIZED RW EQUATION

$$\frac{d^2\Psi}{d\tilde{r}^2} + (\omega^2 + V(\tilde{r}))\Psi = S$$

$$\frac{d\tilde{r}}{dr} = 1 + (\ldots)\frac{M}{r} + \mathcal{O}\left(\frac{M}{r}\right)^2$$ GENERALIZED TORTOISE COORDINATE

$$V(\tilde{r}(r)) = -\frac{1}{r^2}\left(1 - \frac{2M}{r}\right)\left(l(l+1) - (\ldots)\frac{M}{r} + \mathcal{O}\left(\frac{M}{r}\right)^2\right)$$ GENERALIZED RW POTENTIAL

THE ODD SECTOR

A. Kuntz, R. Penco, F. Piazza (JCAP) 20

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$$\frac{d^2\Psi}{d\tilde{r}^2} + (\omega^2 + V(\tilde{r}))\Psi = S$$

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GENERALIZED RW POTENTIAL

Solution to the RW equation:

Poisson 93
Sasaki 94

$$\Psi(r) = \Psi_0(r) + (M\omega)\Psi_1(r) + (M\omega)^2\Psi_2(r) + \ldots$$

$$P \propto \sum_{l,m} \left|\frac{d\Psi}{dt}\right|^2$$

DISSIPATED POWER

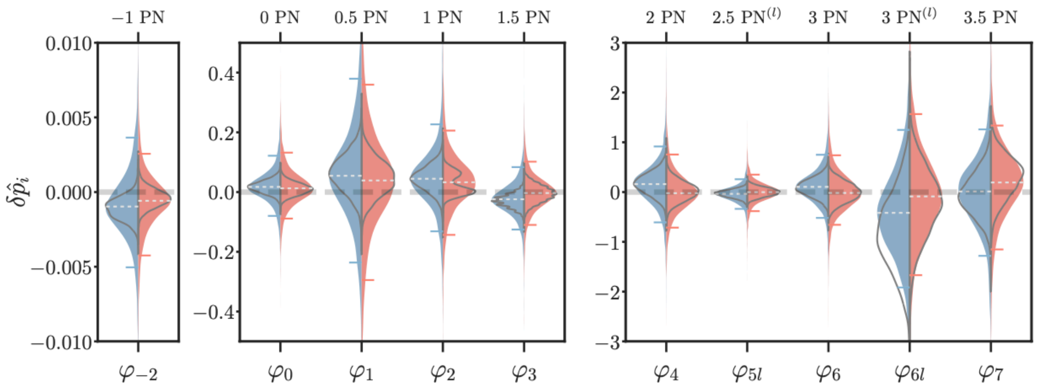
A. Kuntz, R. Penco, F. Piazza (JCAP) 20

$$\frac{P}{P_N} = p_0 + p_1 v^2 + p_2 v^4 + \dots$$

GR Quadrupole

ppE parameters

We go up to 3.5PN !



DISSIPATED POWER

A. Kuntz, R. Penco, F. Piazza (JCAP) 20

$$\frac{P}{P_N} = p_0 + p_1 v^2 + p_2 v^4 + \dots$$

GR Quadrupole

ppE parameters

We go up to 3.5PN !

Our approach bridges the gap between ppE and theory:

→ GIVE ME YOUR METRIC, I WILL GIVE YOU YOUR WAVEFORM !

→ MODELED SEARCH WITH ADDITIONAL NON-GR COEFFICIENTS !

The even sector now needs to be done…

OUTLOOK

- We have investigated on the TWO-BODY PROBLEM in several types of SCALAR-TENSOR THEORIES, often adopting an EFFECTIVE FIELD THEORY viewpoint.
- GRAVITATIONAL WAVES astronomy still in infancy. Interesting physics ahead !
- EFFECTIVE FIELD THEORIES are fantastic tools to compare theory & experiment

THE OLDEST ACADEMIC PROBLEM OF PHYSICS IS STILL A SOURCE OF INSPIRATION!

THANK YOU !