Hairy Extreme Mass Ratio Inspirals

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Action Dark Energy
19/05/2020

Introduction

The Effective Field Theory of Inflation/Dark Energy :

A unifying and effective description of cosmological perturbations

$$\phi(t,\vec{x}) \to \phi_0(t) \quad (\delta\phi = 0) \qquad -\frac{1}{2}\partial\phi^2 \to -\frac{1}{2}\dot{\phi}_0^2(t)\; g^{00}$$

Creminelli et al. ’06
Cheung et al. ’07
Gubitosi et al. ‘12

Introduction

The Action

$$S = \int d^4x\sqrt{-g}\left[\frac{M^2}{2}f(t)R - \Lambda(t) - c(t)g^{00}\right] + S_{DE}^{(2)}$$

The most generic action written in terms of $g_{\mu\nu}$ compatible with the residual symmetry of spatial diffeomorphisms

Dictionary between theories (Quintessence, Brans-Dicke, Galileons…) and effective parameters (measured in observations) :

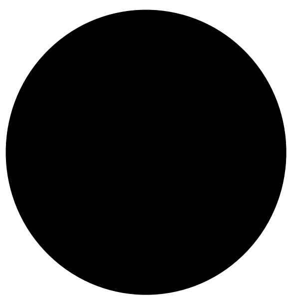
$$w(t)\ , \quad \mu(t) = \frac{\dot{f}}{f}$$

Hairy black holes

Can we devise an EFT for BH inspiral in scalar-tensor theories ?

Focus on one important subclass : hairy black holes

No hair



$$\bar{\phi}(r) = 0$$

$$L_B = \frac{M_P^2}{2} R - (\partial\phi)^2 - V(\phi)$$

Hair

$$\bar{\phi}(r) = \frac{Q}{r}$$

$$L = L_B + \alpha\phi(R_{\mu\nu\rho\lambda}R^{\mu\nu\rho\lambda} - 4R_{\mu\nu}R^{\mu\nu} + R^2)$$

Sotiriou et al. '14

EMRI in the PN regime

$$\phi = \bar{\phi}(r) + \delta\phi$$
$$g_{\mu\nu} = \bar{g}_{\mu\nu} + \delta g_{\mu\nu}$$

Unitary gauge : $\phi(t, x) \to \bar{\phi}(r)$ i.e $\delta\phi = 0$

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Effective action : $S = \int \mathrm{d}^4x \sqrt{-g}\left[\frac{1}{2}M_1^2(r)R - \Lambda(r) - f(r)g^{rr} - \alpha(r)\bar{K}^{\mu}{}_{\nu}K^{\nu}{}_{\mu}\right] + S^{(2)}$

Franciolini et al. '19

- Free functions of r
- Free 'r' indices (contractions with $n_\mu \propto \partial_\mu \phi$)
- K_μ^ν extrinsinc curvature of $r = \mathrm{Cst}$ hypersurfaces
- Λ, f and α uniquely determined by the background $\bar{g}_{\mu\nu}$

Background metric

$$S = \int \mathrm{d}^4x\sqrt{-g}\left[\frac{1}{2}M_1^2(r)R - \Lambda(r) - f(r)g^{rr} - \alpha(r)\bar{K}^{\mu}{}_{\nu}K^{\nu}{}_{\mu}\right] + S^{(2)} + S_{\mathrm{pp}}$$

The effective action is determined by :

- The background metric

$$\bar{g}_{\mu\nu}\mathrm{d}x^{\mu}\mathrm{d}x^{\nu} = -a^2(r)\mathrm{d}t^2 + \frac{\mathrm{d}r^2}{b^2(r)} + c^2(r)\left(\mathrm{d}\theta^2 + \sin^2\theta\mathrm{d}\phi^2\right)$$

Background metric

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The effective action is determined by :

- The background metric

$$\bar{g}_{\mu\nu} dx^\mu dx^\nu = -a^2(r) dt^2 + \frac{dr^2}{b^2(r)} + c^2(r) \left(d\theta^2 + \sin^2\theta d\phi^2 \right)$$

- An effective Planck mass $M_1(r)$ removable by a conformal transformation :

$$g_{\mu\nu}^{(\mathrm{E})}(x) = g_{\mu\nu}^{(\mathrm{J})}(x) M_1^2(r)$$

$\Rightarrow$ Effective mass for the point-particle :

$$S_{\mathrm{pp}} = -\int dt\, \mu \sqrt{\bar{g}_{\mu\nu} v^\mu v^\nu} \to -\int dt\, \mu(r) \sqrt{\bar{g}_{\mu\nu} v^\mu v^\nu}$$

Background example

$$L = \frac{M_P^2}{2}R - (\partial\phi)^2 + \bar{\alpha}\phi(R_{\mu\nu\rho\lambda}R^{\mu\nu\rho\lambda} - 4R_{\mu\nu}R^{\mu\nu} + R^2)$$

$$a^2(r) = 1 - \frac{2M}{r} + \frac{MQ^2}{6r^3} + \frac{M^2Q^2 + 24\bar{\alpha}MQ}{3r^4} + \mathcal{O}(r^{-5})$$

$$b^2(r) = 1 - \frac{2M}{r} + \frac{Q^2}{2r^2} + \frac{MQ^2}{2r^3} + \frac{48\bar{\alpha}MQ + 2M^2Q^2}{3r^4} + \mathcal{O}(r^{-5})$$

$$c^2(r) = r^2$$

$$\phi(r) = \frac{Q}{r} + \frac{MQ}{r^2} + \frac{16M^2Q - Q^3}{12r^3} + \frac{6M^3Q - 12\bar{\alpha}M^2 - MQ^3}{3r^4} + \mathcal{O}(r^{-5})$$

$$M_1^2(r) = 1 - \frac{16\bar{\alpha}Q}{r^3} - \frac{8\bar{\alpha}MQ}{r^4} - \frac{4\bar{\alpha}(4M^2Q + Q^3)}{r^5} + \mathcal{O}(r^{-6})$$

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Secondary hair :

$$b^2(r) = 1 - \frac{2M}{r} + \frac{Q^2}{2r^2} + \frac{MQ^2}{2r^3} + \frac{48\bar{\alpha}MQ + 2M^2Q^2}{3r^4} + \mathcal{O}(r^{-5})$$

$$c^2(r) = r^2$$

Q depends on M

$$\phi(r) = \frac{Q}{r} + \frac{MQ}{r^2} + \frac{16M^2Q - Q^3}{12r^3} + \frac{6M^3Q - 12\bar{\alpha}M^2 - MQ^3}{3r^4} + \mathcal{O}(r^{-5})$$

E.g small coupling :

$$Q \simeq \frac{2\bar{\alpha}}{M}$$

$$M_1^2(r) = 1 - \frac{16\bar{\alpha}Q}{r^3} - \frac{8\bar{\alpha}MQ}{r^4} - \frac{4\bar{\alpha}(4M^2Q + Q^3)}{r^5} + \mathcal{O}(r^{-6})$$

Generic parametrization : $a^2(r) = 1 - \frac{2M}{r} + a_2\left(\frac{2M}{r}\right)^2 + a_3\left(\frac{2M}{r}\right)^3 + \mathcal{O}\left(\frac{M}{r}\right)^4$ etc

Perturbations

- Classify $\delta g_{\mu\nu}$ according to its transformation properties with respect to $(i, j) = (\theta, \phi)$ diffeomorphisms :
  - Scalars : $\delta g_{tt}\ ,\quad \delta g_{rr}\ ,\quad \delta g_{tr}$
  - Vectors : $\delta g_{ti}\ ,\quad \delta g_{ri}$
  - Tensor : δg_{ij}

Perturbations

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  - Scalars : δg_{tt} , δg_{rr} , δg_{tr}
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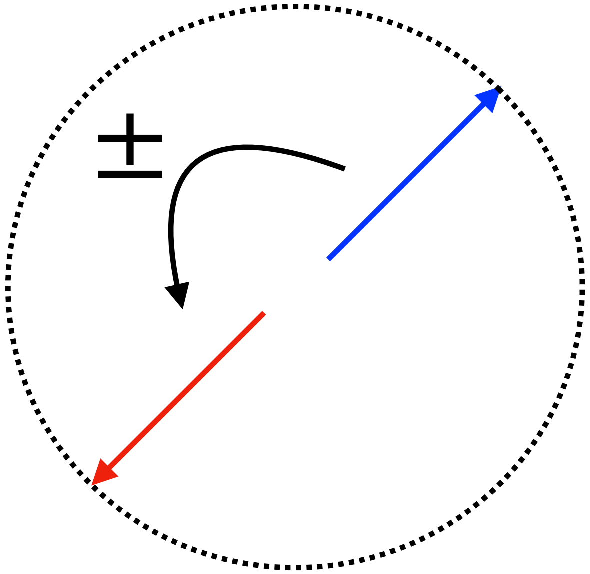
  - Tensor : δg_{ij}
- Further decompose in even and odd modes :



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  - Tensor : δg_{ij}

- Further decompose in even and odd modes :

$t \qquad r \qquad \theta, \phi$

$$\delta g^{\rm odd}_{\mu\nu} = \begin{pmatrix} 0 & 0 & \epsilon^k{}_j \nabla_k h_0 \\ 0 & 0 & \epsilon^k{}_j \nabla_k h_1 \\ \epsilon^k{}_i \nabla_k h_0 & \epsilon^k{}_i \nabla_k h_1 & \frac{1}{2}(\epsilon_i{}^k \nabla_k \nabla_j + \epsilon_j{}^k \nabla_k \nabla_i) h_2 \end{pmatrix}$$

Tensorial structure on the 2-sphere

Pseudo-scalars

The odd sector

$$S = \int \mathrm{d}^4x \sqrt{-g} \left[\frac{1}{2} M_1^2(r) R - \Lambda(r) - f(r) g^{rr} - \alpha(r) \bar{K}^{\mu}{}_{\nu} K^{\nu}{}_{\mu} \right] + S^{(2)} + S_{\mathrm{pp}}$$

and

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Generalized RW equation

$$\frac{d^2\Psi}{d\tilde{r}^2} + (\omega^2 + V(\tilde{r}))\Psi = S$$

$$\frac{d\tilde{r}}{dr} = 1 + (\ldots)\frac{M}{r} + \mathcal{O}\left(\frac{M}{r}\right)^2 \qquad V(\tilde{r}(r)) = -\frac{1}{r^2}\left(1 - \frac{2M}{r}\right)\left(l(l+1) - (\ldots)\frac{M}{r} + \mathcal{O}\left(\frac{M}{r}\right)^2\right)$$

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↓

Related to h_0, h_1

Generalized RW equation

Point-particle source

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Related to the expansion of a, b

The odd sector

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↓

Generalized RW equation

$$\frac{d^2\Psi}{d\tilde{r}^2} + (\omega^2 + V(\tilde{r}))\Psi = S$$

In GR :

$$\frac{d\tilde{r}}{dr} = \frac{1}{1 - \frac{2M}{r}}$$

$$V(\tilde{r}(r)) = -\frac{1}{r^2}\left(1 - \frac{2M}{r}\right)\left(l(l+1) - \frac{6M}{r}\right)$$

Solution of the RW equation

$$\frac{d^2\Psi}{d\tilde{r}^2} + (\omega^2 + V(\tilde{r}))\Psi = S$$

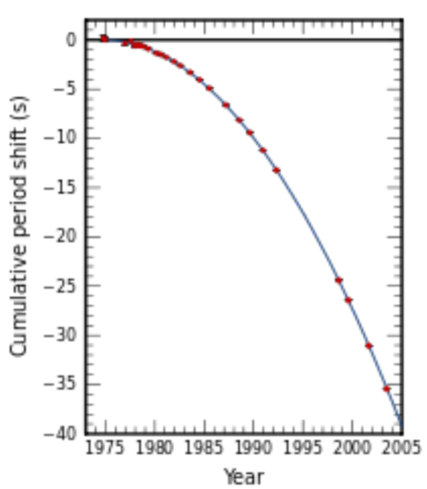
One can perturbatively solve this equation in the PN regime :

$$\Psi(r) = \Psi_0(r) + (M\omega)\Psi_1(r) + (M\omega)^2\Psi_2(r) + \ldots$$

Poisson '93
Sasaki '94

This gives access to the dissipated power and the GW phase

$$P \propto \sum_{l,m} \left| \frac{d\Psi}{dt} \right|^2 \quad \Rightarrow \quad \tilde{h}(f) = A(f) e^{i\Phi(f)}$$



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which reads

$$\frac{P}{P_N} = (\dots)v^2 + (\dots)v^4 + \dots$$

« Newtonian » quadrupole

$$P_N = \frac{32}{5}\left(\frac{\mu_0}{M}\right)^2 v^{10}$$

Odd sector

We go up to 3.5PN !

Comparison with ppE

$$\tilde{h}(f) = A(f)e^{i\Phi(f)}$$

ppE framework : $$\Phi = \Phi_0 + 2\pi f t_0 + \sum_k \phi_k \, v^{b_k}$$

Yunes et al. '09

GR : $b_k = (k-5)/3$

$\phi_k = \phi_k^{\mathrm{PN}}$

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Drawbacks :
- Starting from a generic theory, it is quite a lot of work to obtain ϕ_k, b_k even at leading order

→ Give me your metric, I will give you your waveform !

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- No modeled search, only a posteriori constraints

→ Modeled search with additional non-GR coefficients !
($\bar{\alpha}$ for GB)

Conclusions

- We extend the EFT of Inflation/Dark Energy to gravitational waves
- Our EFT provides waveform templates for hairy black holes in ST theories and in the test-mass limit
- Next steps :
  - Include the even sector
  - Extend to spinning solutions
  - Reconsider no-hair theorems