

Hairy Extreme Mass Ratio Inspirals

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Introduction

The **Effective Field Theory** of Inflation/Dark Energy :

A **unifying** and **effective** description of cosmological perturbations

$$\phi(t, \vec{x}) \rightarrow \phi_0(t) \quad (\delta\phi = 0) \quad -\frac{1}{2}\partial\phi^2 \rightarrow -\frac{1}{2}\dot{\phi}_0^2(t) g^{00}$$



Creminelli et al. '06
Cheung et al. '07
Gubitosi et al. '12

Introduction

The Action

$$S = \int d^4x \sqrt{-g} \left[\frac{M^2}{2} f(t) R - \Lambda(t) - c(t) g^{00} \right] + S_{DE}^{(2)}$$

The most generic action written in terms of $g_{\mu\nu}$ compatible with the residual symmetry of spatial diffeomorphisms

Dictionary between theories (Quintessence, Brans-Dicke, Galileons...) and effective parameters (measured in observations) :

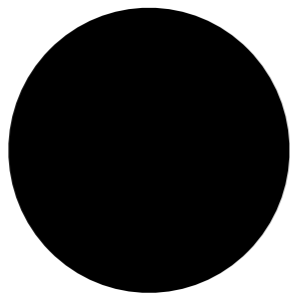
$$w(t) , \quad \mu(t) = \frac{\dot{f}}{f}$$

Hairy black holes

Can we devise an EFT for BH inspiral in scalar-tensor theories ?

Focus on one important subclass : hairy black holes

No hair



$$\bar{\phi}(r) = 0$$

$$L_B = \frac{M_P^2}{2} R - (\partial\phi)^2 - V(\phi)$$

Hair

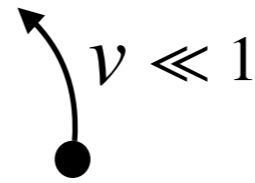
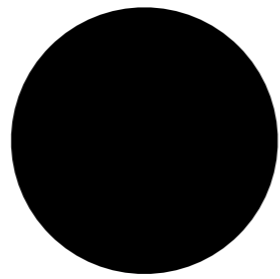


$$\bar{\phi}(r) = \frac{Q}{r}$$

$$L = L_B + \alpha\phi(R_{\mu\nu\rho\lambda}R^{\mu\nu\rho\lambda} - 4R_{\mu\nu}R^{\mu\nu} + R^2)$$

Sotiriou et al. '14

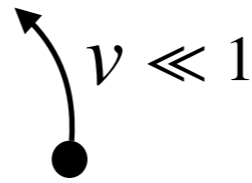
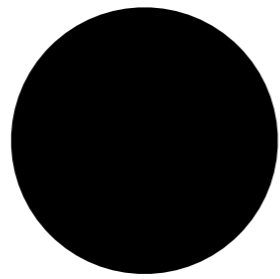
EMRI in the PN regime



$$\begin{aligned}\phi &= \bar{\phi}(r) + \delta\phi \\ g_{\mu\nu} &= \bar{g}_{\mu\nu} + \delta g_{\mu\nu}\end{aligned}$$

Unitary gauge : $\phi(t, x) \rightarrow \bar{\phi}(r)$ i.e $\delta\phi = 0$

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Effective action :
$$S = \int d^4x \sqrt{-g} \left[\frac{1}{2} M_1^2(r) R - \Lambda(r) - f(r) g^{rr} - \alpha(r) \bar{K}^\mu{}_\nu K^\nu{}_\mu \right] + S^{(2)}$$

Franciolini et al. '19

- Free functions of r
- Free 'r' indices (contractions with $n_\mu \propto \partial_\mu \phi$)
- $K^\nu{}_\mu$ extrinsic curvature of $r = \text{Cst}$ hypersurfaces
- Λ , f and α uniquely determined by the background $\bar{g}_{\mu\nu}$

Background metric

$$S = \int d^4x \sqrt{-g} \left[\frac{1}{2} M_1^2(r) R - \Lambda(r) - f(r) g^{rr} - \alpha(r) \bar{K}^\mu{}_\nu K^\nu{}_\mu \right] + S^{(2)} + S_{\text{pp}}$$

The **effective action** is determined by :

- The **background metric**

$$\bar{g}_{\mu\nu} dx^\mu dx^\nu = - a^2(r) dt^2 + \frac{dr^2}{b^2(r)} + c^2(r) (d\theta^2 + \sin^2 \theta d\phi^2)$$

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- An **effective Planck mass** $M_1(r)$ removable by a conformal transformation :

$$g_{\mu\nu}^{(\text{E})}(x) = g_{\mu\nu}^{(\text{J})}(x) M_1^2(r)$$

⇒ Effective mass for the point-particle :

$$S_{\text{pp}} = - \int dt \mu \sqrt{\bar{g}_{\mu\nu} v^\mu v^\nu} \rightarrow - \int dt \mu(r) \sqrt{\bar{g}_{\mu\nu} v^\mu v^\nu}$$

Background example



$$L = \frac{M_P^2}{2} R - (\partial\phi)^2 + \bar{\alpha}\phi(R_{\mu\nu\rho\lambda}R^{\mu\nu\rho\lambda} - 4R_{\mu\nu}R^{\mu\nu} + R^2)$$

$$a^2(r) = 1 - \frac{2M}{r} + \frac{MQ^2}{6r^3} + \frac{M^2Q^2 + 24\bar{\alpha}MQ}{3r^4} + \mathcal{O}(r^{-5})$$

$$b^2(r) = 1 - \frac{2M}{r} + \frac{Q^2}{2r^2} + \frac{MQ^2}{2r^3} + \frac{48\bar{\alpha}MQ + 2M^2Q^2}{3r^4} + \mathcal{O}(r^{-5})$$

$$c^2(r) = r^2$$

$$\phi(r) = \frac{Q}{r} + \frac{MQ}{r^2} + \frac{16M^2Q - Q^3}{12r^3} + \frac{6M^3Q - 12\bar{\alpha}M^2 - MQ^3}{3r^4} + \mathcal{O}(r^{-5})$$

$$M_1^2(r) = 1 - \frac{16\bar{\alpha}Q}{r^3} - \frac{8\bar{\alpha}MQ}{r^4} - \frac{4\bar{\alpha}(4M^2Q + Q^3)}{r^5} + \mathcal{O}(r^{-6})$$

Background example



$$L = \frac{M_P^2}{2} R - (\partial\phi)^2 + \bar{\alpha}\phi(R_{\mu\nu\rho\lambda}R^{\mu\nu\rho\lambda} - 4R_{\mu\nu}R^{\mu\nu} + R^2)$$

Secondary hair :

$$a^2(r) = 1 - \frac{2M}{r} + \frac{MQ^2}{6r^3} + \frac{M^2Q^2 + 24\bar{\alpha}MQ}{3r^4} + \mathcal{O}(r^{-5})$$

$$b^2(r) = 1 - \frac{2M}{r} + \frac{Q^2}{2r^2} + \frac{MQ^2}{2r^3} + \frac{48\bar{\alpha}MQ + 2M^2Q^2}{3r^4} + \mathcal{O}(r^{-5})$$

Q depends on M

E.g small coupling :

$$Q \simeq \frac{2\bar{\alpha}}{M}$$

$$c^2(r) = r^2$$

$$\phi(r) = \frac{Q}{r} + \frac{MQ}{r^2} + \frac{16M^2Q - Q^3}{12r^3} + \frac{6M^3Q - 12\bar{\alpha}M^2 - MQ^3}{3r^4} + \mathcal{O}(r^{-5})$$

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Generic parametrization : $a^2(r) = 1 - \frac{2M}{r} + a_2 \left(\frac{2M}{r}\right)^2 + a_3 \left(\frac{2M}{r}\right)^3 + \mathcal{O}\left(\frac{M}{r}\right)^4$ etc

Perturbations

- Classify $\delta g_{\mu\nu}$ according to its transformation properties with respect to $(i, j) = (\theta, \phi)$ diffeomorphisms :
 - ▶ **Scalars** : δg_{tt} , δg_{rr} , δg_{tr}
 - ▶ **Vectors** : δg_{ti} , δg_{ri}
 - ▶ **Tensor** : δg_{ij}

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- Further decompose in even and odd modes :

$$\delta g_{\mu\nu}^{\text{odd}} = \begin{pmatrix} 0 & 0 & \epsilon^k_j \nabla_k h_0 \\ 0 & 0 & \epsilon^k_j \nabla_k h_1 \\ \epsilon^k_i \nabla_k h_0 & \epsilon^k_i \nabla_k h_1 & \frac{1}{2}(\epsilon_i^k \nabla_k \nabla_j + \epsilon_j^k \nabla_k \nabla_i) h_2 \end{pmatrix}$$

Pseudo-scalars

Tensorial structure on the 2-sphere

The odd sector

$$S = \int d^4x \sqrt{-g} \left[\frac{1}{2} M_1^2(r) R - \Lambda(r) - f(r) g^{rr} - \alpha(r) \bar{K}^\mu{}_\nu K^\nu{}_\mu \right] + S^{(2)} + S_{\text{pp}}$$

and

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Generalized RW equation

$$\frac{d^2 \Psi}{d\tilde{r}^2} + (\omega^2 + V(\tilde{r})) \Psi = S$$

$$\frac{d\tilde{r}}{dr} = 1 + (\dots) \frac{M}{r} + \mathcal{O} \left(\left(\frac{M}{r} \right)^2 \right) \quad V(\tilde{r}(r)) = -\frac{1}{r^2} \left(1 - \frac{2M}{r} \right) \left(l(l+1) - (\dots) \frac{M}{r} + \mathcal{O} \left(\left(\frac{M}{r} \right)^2 \right) \right)$$

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Related to h_0, h_1

Generalized RW equation

Point-particle source

$$\frac{d^2 \Psi}{d\tilde{r}^2} + (\omega^2 + V(\tilde{r})) \Psi = \tilde{S}$$

$$\frac{d\tilde{r}}{dr} = 1 + (\dots) \frac{M}{r} + \mathcal{O} \left(\left(\frac{M}{r} \right)^2 \right) \quad V(\tilde{r}(r)) = -\frac{1}{r^2} \left(1 - \frac{2M}{r} \right) \left(l(l+1) - (\dots) \frac{M}{r} + \mathcal{O} \left(\left(\frac{M}{r} \right)^2 \right) \right)$$

Related to the expansion of a, b

Solution of the RW equation

$$\frac{d^2\Psi}{d\tilde{r}^2} + (\omega^2 + V(\tilde{r}))\Psi = S$$

One can **perturbatively solve** this equation in the PN regime :

$$\Psi(r) = \Psi_0(r) + (M\omega)\Psi_1(r) + (M\omega)^2\Psi_2(r) + \dots$$

Poisson '93
Sasaki '94

This gives access to the **dissipated power** and the **GW phase**

$$P \propto \sum_{l,m} \left| \frac{d\Psi}{dt} \right|^2 \quad \Rightarrow \quad \tilde{h}(f) = A(f)e^{i\Phi(f)}$$

which reads

$$\frac{P}{P_N} = (\dots)v^2 + (\dots)v^4 + \dots$$

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which reads

« Newtonian » quadrupole \rightarrow

$$\frac{P}{P_N} = (\dots)v^2 + (\dots)v^4 + \dots$$

\uparrow Odd sector \nwarrow We go up to 3.5PN !

$$P_N = \frac{32}{5} \left(\frac{\mu_0}{M} \right)^2 v^{10}$$

Comparison with ppE

$$\tilde{h}(f) = A(f)e^{i\Phi(f)}$$

ppE framework :

$$\Phi = \Phi_0 + 2\pi f t_0 + \sum_k \phi_k v^{b_k}$$

Yunes et al. '09

GR :

$$b_k = (k - 5)/3$$
$$\phi_k = \phi_k^{\text{PN}}$$

Comparison with ppE

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ppE framework : $\Phi = \Phi_0 + 2\pi f t_0 + \sum_k \phi_k v^{b_k}$

Drawbacks : • Starting from a generic theory, it is quite a lot of work to obtain ϕ_k, b_k even at leading order

→ Give me your metric, I will give you your waveform !

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→ Give me your metric, I will give you your waveform !

- No **modeled search**, only a posteriori constraints

→ Modeled search with additional non-GR coefficients !
($\bar{\alpha}$ for GB)

Conclusions

- We extend the **EFT of Inflation/Dark Energy** to gravitational waves
- Our EFT provides **waveform templates** for **hairy black holes** in ST theories and in the **test-mass limit**
- Next steps :
 - ▶ Include the **even** sector
 - ▶ Extend to **spinning** solutions
 - ▶ Reconsider no-hair theorems