Three-body effects in waveforms

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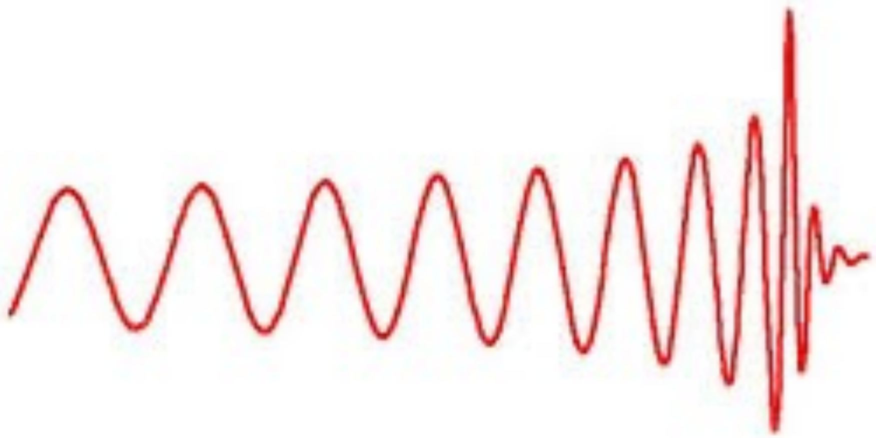
INTRODUCTION

Detection of GW beautifully corresponds to two-body systems in isolation



$$\Phi(f) = \phi_0 + 2\pi f t_0 + \sum_{k=0}^{7} \alpha_k f^{(k-5)/3}$$

m_1, m_2, χ_1, χ_2

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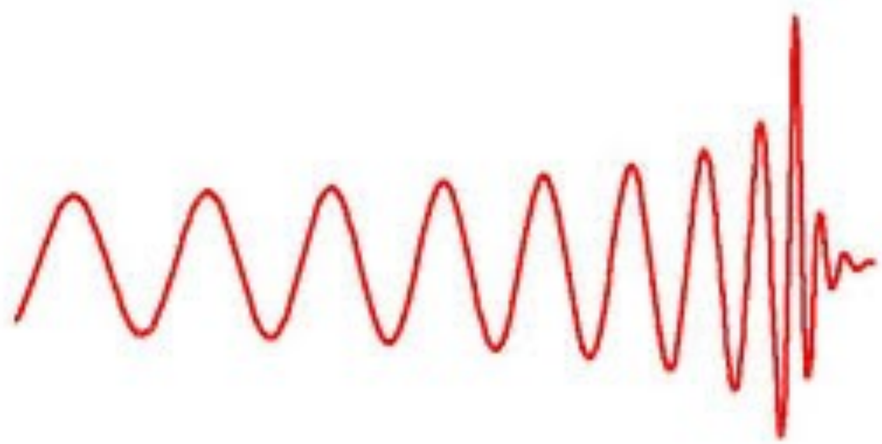

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Three-body systems are also quite common !

- 90% of low-mass binaries are expect to belong to a 'hierarchical' triple system

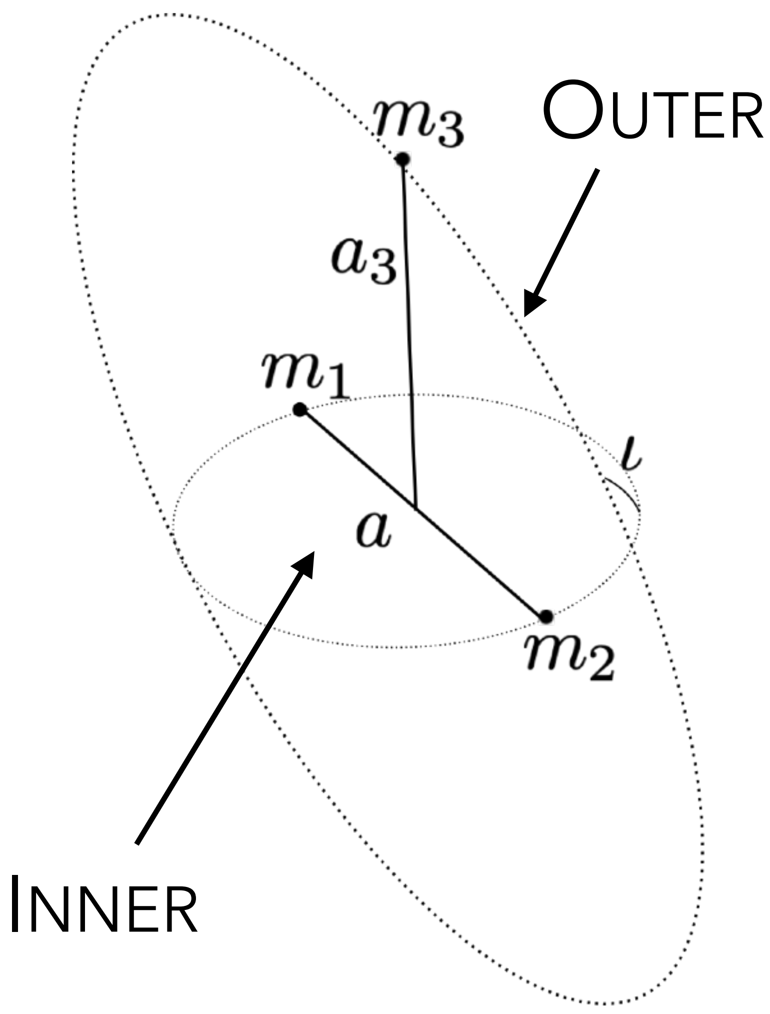
Tokovinin et al. 2006

- 'Migration traps' around SMBH at $R \sim 20 - 600 R_{\mathrm{sch}}$

Bellovary et al. 2015

Can we detect and measure parameters of the third body from waveform ?

HIERARCHICAL THREE-BODY SYSTEMS



$$m = m_1 + m_2$$

$$M = m_1 + m_2 + m_3$$

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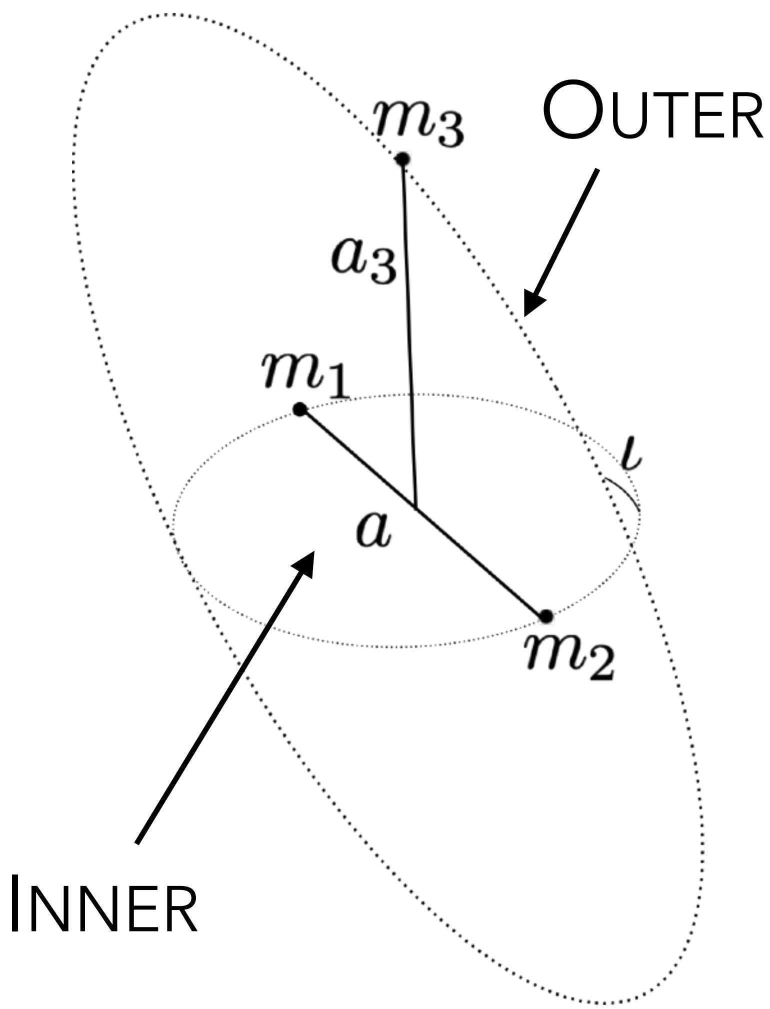

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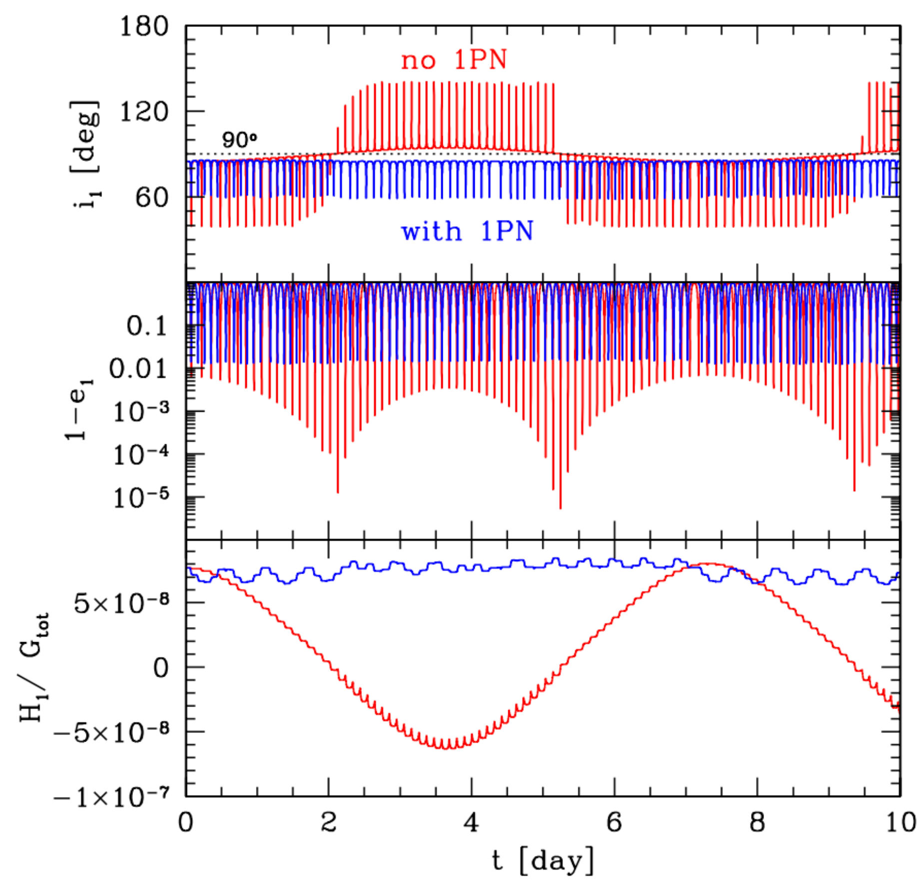
Timescales:

- $t_{\rm quad}$ Kozai-Lidov oscillations
- $t_{\rm PN}$ Perihelion precession

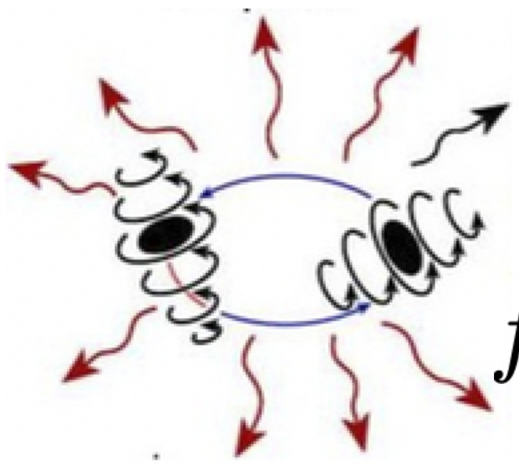


Naoz et al 2013

Outer object cannot influence the eccentricity of very relativistic inner binary

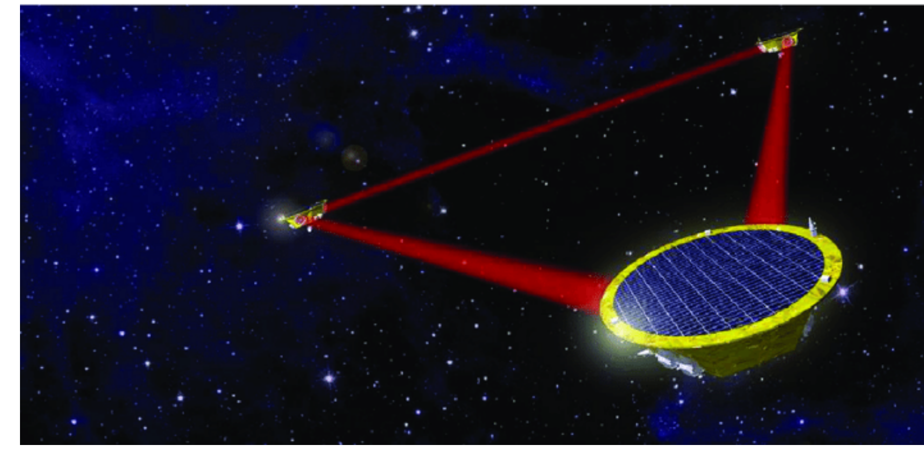
DOPPLER EFFECT

V



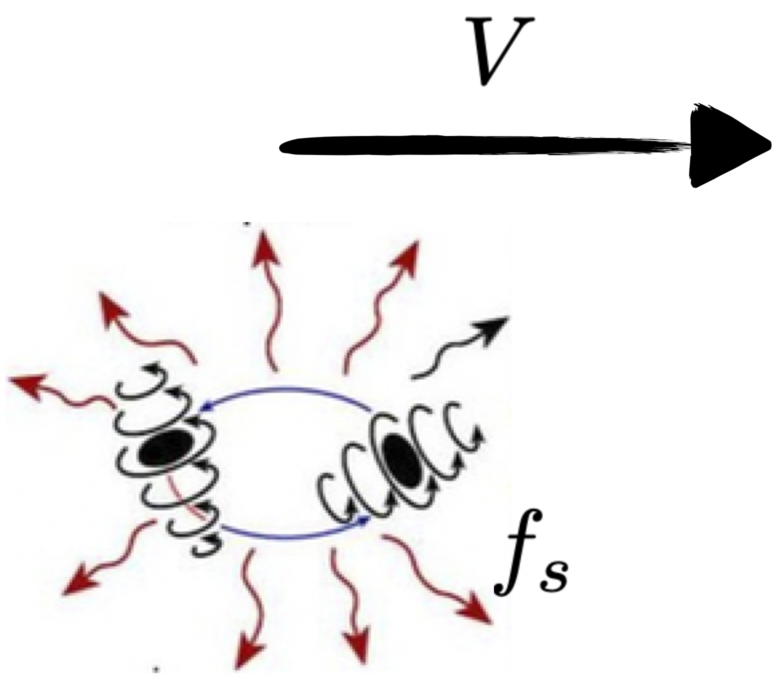
f_s

$f_s(1+V)$



If $V =$ Cst: similar to redshift $\begin{cases} m \to m(1+z) \\ f \to f/(1+z) \end{cases}$ Unobservable !

Doppler effect



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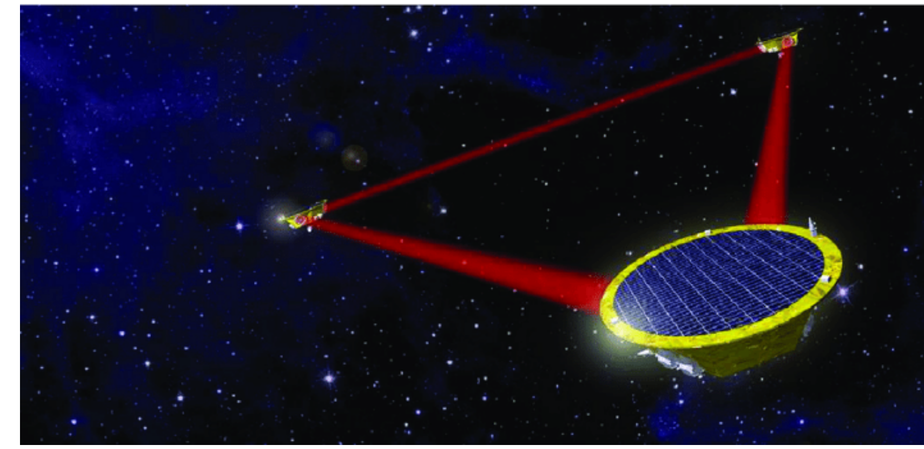

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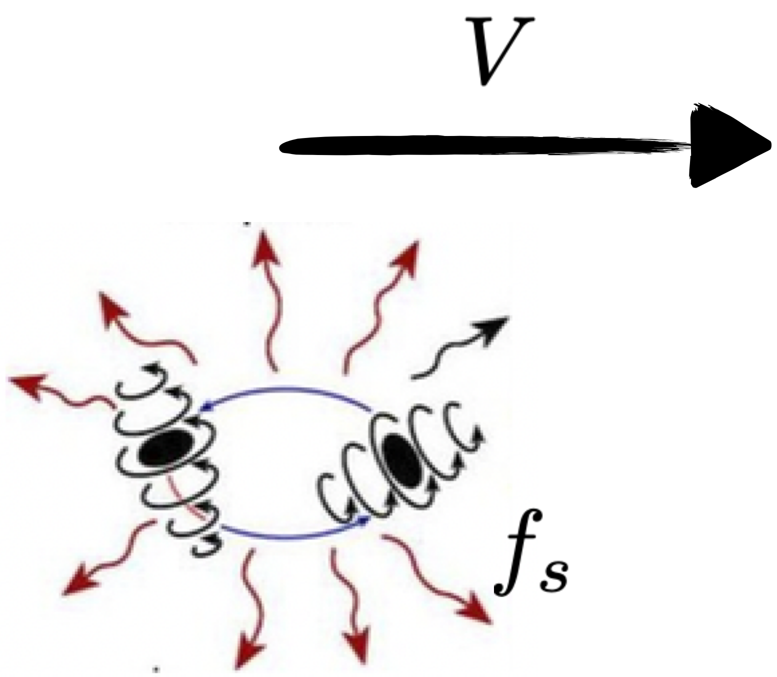
Time-dependence: $V \sim |V| \cos(2\pi t/P_{\rm out} + \psi) \sim |V|(1 + t/P_{\rm out})$

$T_{\rm obs} \ll P_{\rm out}$

Doppler effect



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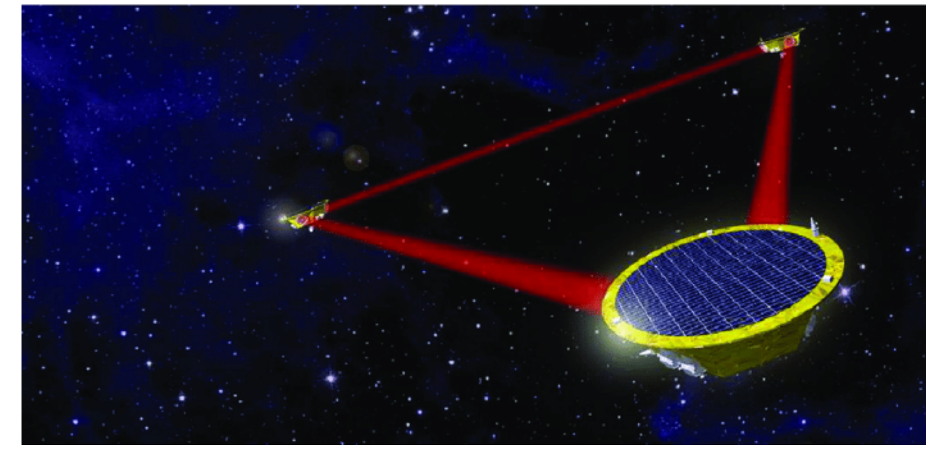

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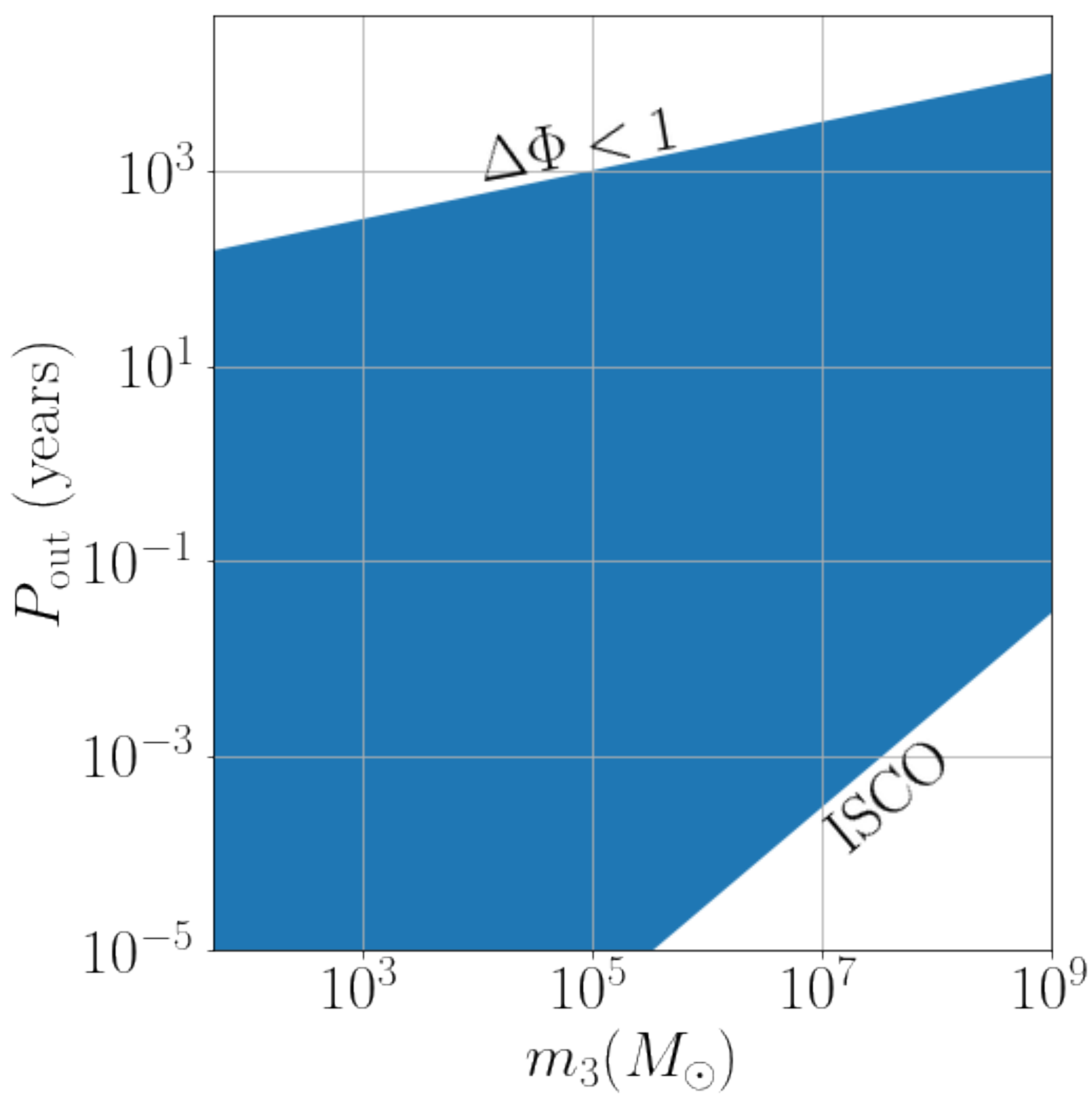
Phase shift: $\Phi = 2 \int \mathrm{d}t \; f_s (1+V) \sim 2 \int \mathrm{d}t \; f_s \left(1 + |V| t/P_{\rm out}\right)$

A « -4PN » effect since $t_{\rm coal} - t = \dfrac{5Gm}{256\nu} \left(\dfrac{Gm}{a}\right)^{-4}$

Doppler phase in LISA

$$\Delta\Phi \sim |V| \frac{T_{\rm obs}}{P_{\rm out}} f_s T_{\rm obs} \sim \left(\frac{GM}{P_{\rm out}^4}\right)^{1/3} f_s T_{\rm obs}^2$$



For $m = 50 M_\odot$ that will merge within $T_{\rm obs} = 4{\rm yr}$

Limits of Doppler

- If $P_{\rm out} \gg T_{\rm obs}$, we measure only one -4PN parameter while outer orbit depends on

$$m_3, P_{\rm out}, e_{\rm out}, \iota$$

  - If $P_{\rm out} \lesssim T_{\rm obs}$ one can break degeneracies Yu&Chen 2020
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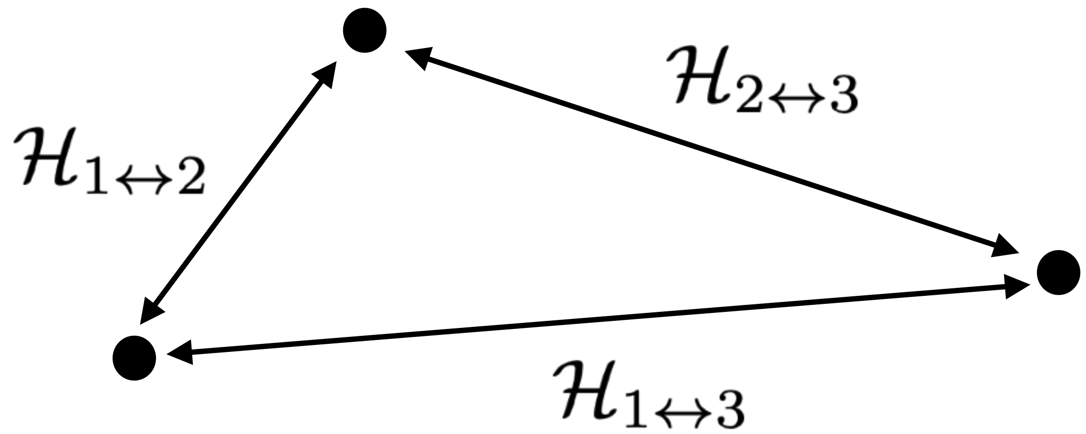
- If the perturber is close enough (e.g. $P_{\rm out} \lesssim T_{\rm obs}$), one has to take into account other effects for accurate waveform modelling ! (i.e. solving a relativistic three-body problem)

These two shortcomings may cure each other !

Relativistic three-body problem

A COMMON MISCONCEPTION

$$\mathcal{H}_{3-\mathrm{body}} = \mathcal{H}_{1\leftrightarrow 2} + \mathcal{H}_{1\leftrightarrow 3} + \mathcal{H}_{2\leftrightarrow 3}$$



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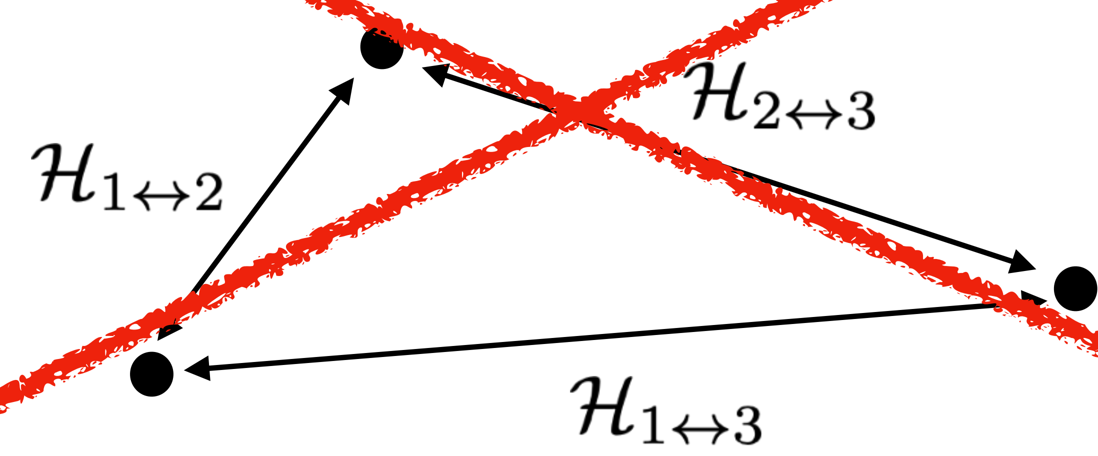


GR IS A NONLINEAR THEORY !

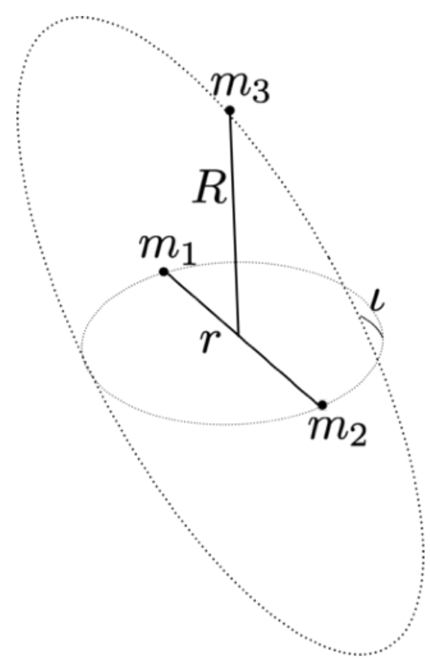
$$g_{\mu\nu} \neq g^{(1)}_{\mu\nu} + g^{(2)}_{\mu\nu} \quad \text{(would not solve } R_{\mu\nu} = 0\text{)}$$

GR CORRECTIONS AT 1PN

The ‘hardcore’ way: use EOM and expand in the CM frame to quadrupole order

Will (2014) Lim and Rodriguez (2020)

$$\boldsymbol{a}_a = -\sum_{b\neq a}\frac{Gm_b\boldsymbol{x}_{ab}}{r_{ab}^3} + \frac{1}{c^2}\sum_{b\neq a}\frac{Gm_b\boldsymbol{x}_{ab}}{r_{ab}^3}\left[4\frac{Gm_b}{r_{ab}} + 5\frac{Gm_a}{r_{ab}} + \sum_{c\neq a,b}\frac{Gm_c}{r_{bc}} + 4\sum_{c\neq a,b}\frac{Gm_c}{r_{ac}} - \frac{1}{2}\sum_{c\neq a,b}\frac{Gm_c}{r_{bc}^3}(\boldsymbol{x}_{ab}\cdot\boldsymbol{x}_{bc}) - v_a^2 + 4\boldsymbol{v}_a\cdot\boldsymbol{v}_b \right.$$
$$\left. - 2\boldsymbol{v}_b^2 + \frac{3}{2}(\boldsymbol{v}_b\cdot\boldsymbol{n}_{ab})^2\right] - \frac{7}{2c^2}\sum_{b\neq a}\frac{Gm_b}{r_{ab}}\sum_{c\neq a,b}\frac{Gm_c\boldsymbol{x}_{bc}}{r_{bc}^3} + \frac{1}{c^2}\sum_{b\neq a}\frac{Gm_b}{r_{ab}^3}\boldsymbol{x}_{ab}\cdot(4\boldsymbol{v}_a - 3\boldsymbol{v}_b)(\boldsymbol{v}_a - \boldsymbol{v}_b), \quad (3.1)$$



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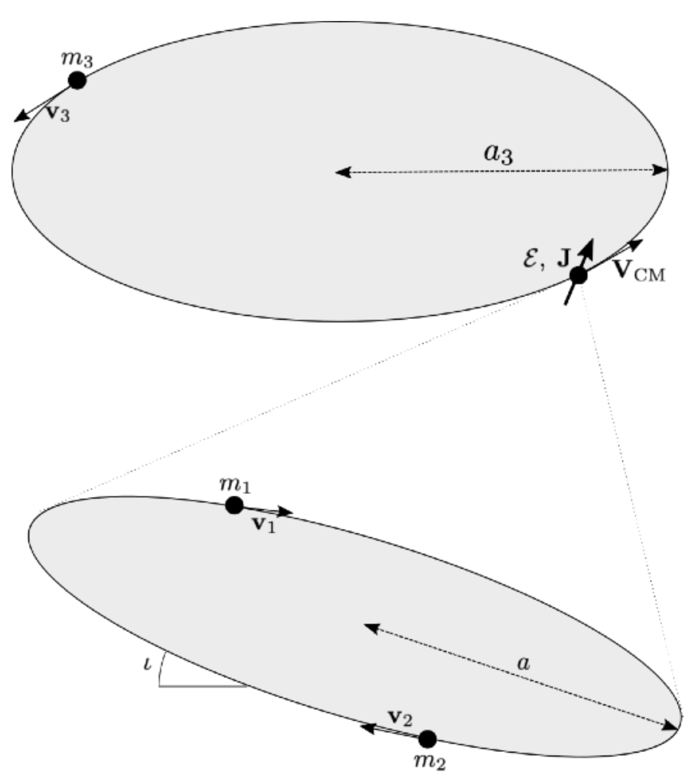
The 'intuitive' way: 'EFFECTIVE TWO-BODY' AK, F. Serra, E. Trincherini 2021



3-body motion = 2-body with spin !

The binary EFT

AK, F. Serra, E. Trincherini 2021

Proper time

$$\mathscr{L}_{\text{full}} = \sum_{A=1}^{3} - m_A \sqrt{-g_{\mu\nu} v_A^\mu v_A^\nu} \quad \Leftrightarrow \quad \mathscr{L}_{\text{EFT}} = - \mathscr{E} \sqrt{-g_{\mu\nu} V_{\text{CM}}^\mu V_{\text{CM}}^\nu} + \frac{1}{2} J_{\mu\nu} \Omega^{\mu\nu} - m_3 \sqrt{-g_{\mu\nu} v_3^\mu v_3^\nu}$$

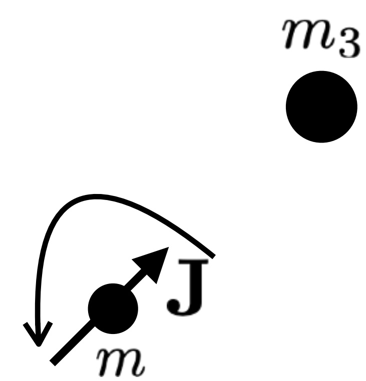
The equivalence principle fixes nearly everything!

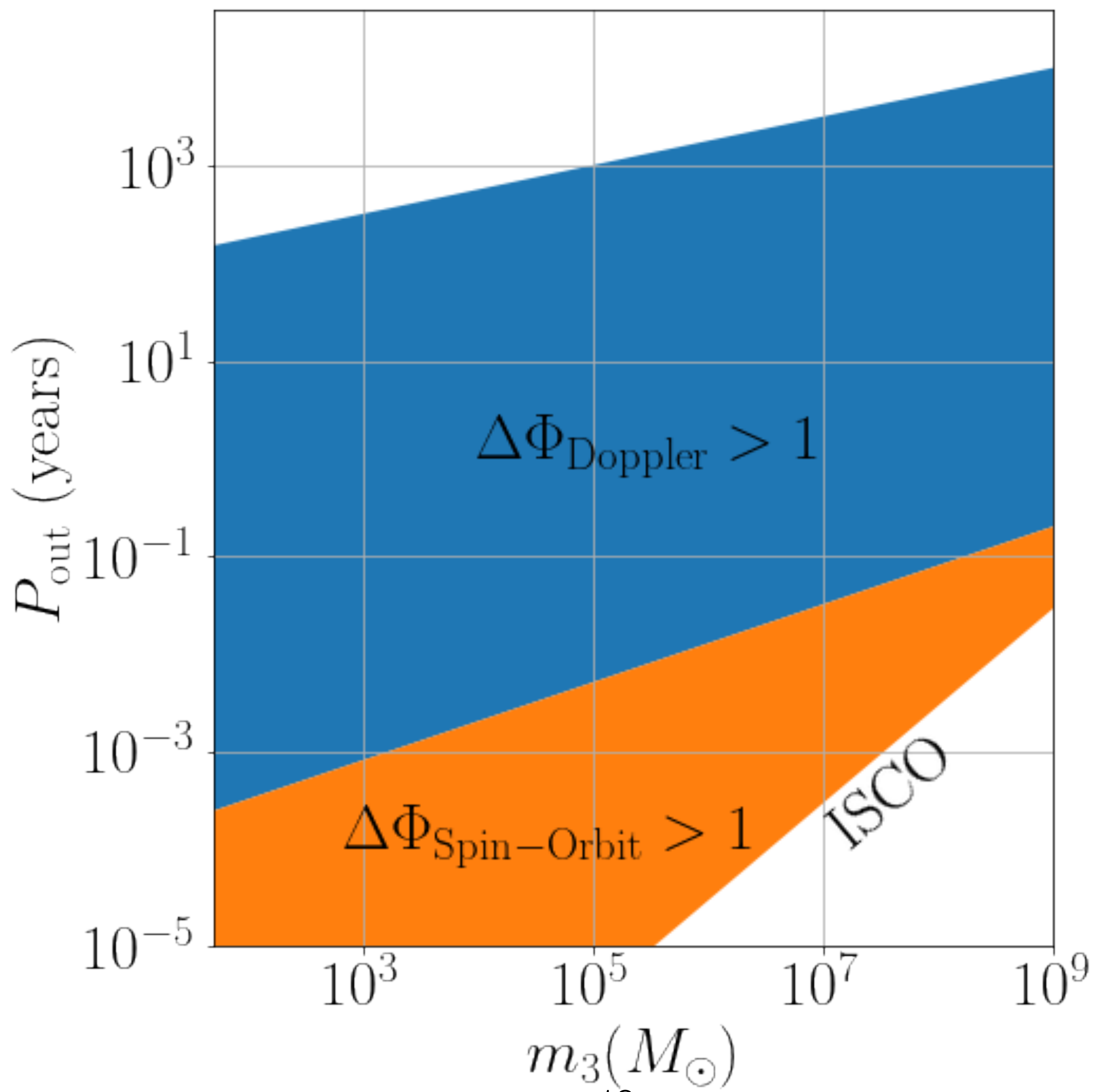
$$\mathscr{E} = m - \frac{G_N m \mu}{2a} , \qquad J_{ij} = \epsilon_{ijk} J^k , \quad \Omega_{ij} = \epsilon_{ijk} \Omega^k , \qquad \mathbf{J} = \sqrt{G_N m a (1 - e^2)}\, \hat{\mathbf{j}} , \qquad \boldsymbol{\Omega} = \hat{\mathbf{e}} \times \dot{\hat{\mathbf{e}}}$$

$\hat{\mathbf{e}} \equiv$ Unit Runge-Lenz vector

SPIN-ORBIT COUPLING

$$\mathcal{H}_{\rm SO} = \frac{4m + 3m_3}{2m} \frac{G}{a_3^3(1-e_3^2)^{3/2}} \mathbf{J} \cdot \mathbf{J}_3$$





Yu&Chen 2020

CONCLUSIONS

- LISA will see triple effects in waveforms
- Doppler effect can be probed for perturber at large distance but mass is degenerate with inclination
- Further relativistic three-body effects may allow to break this degeneracy