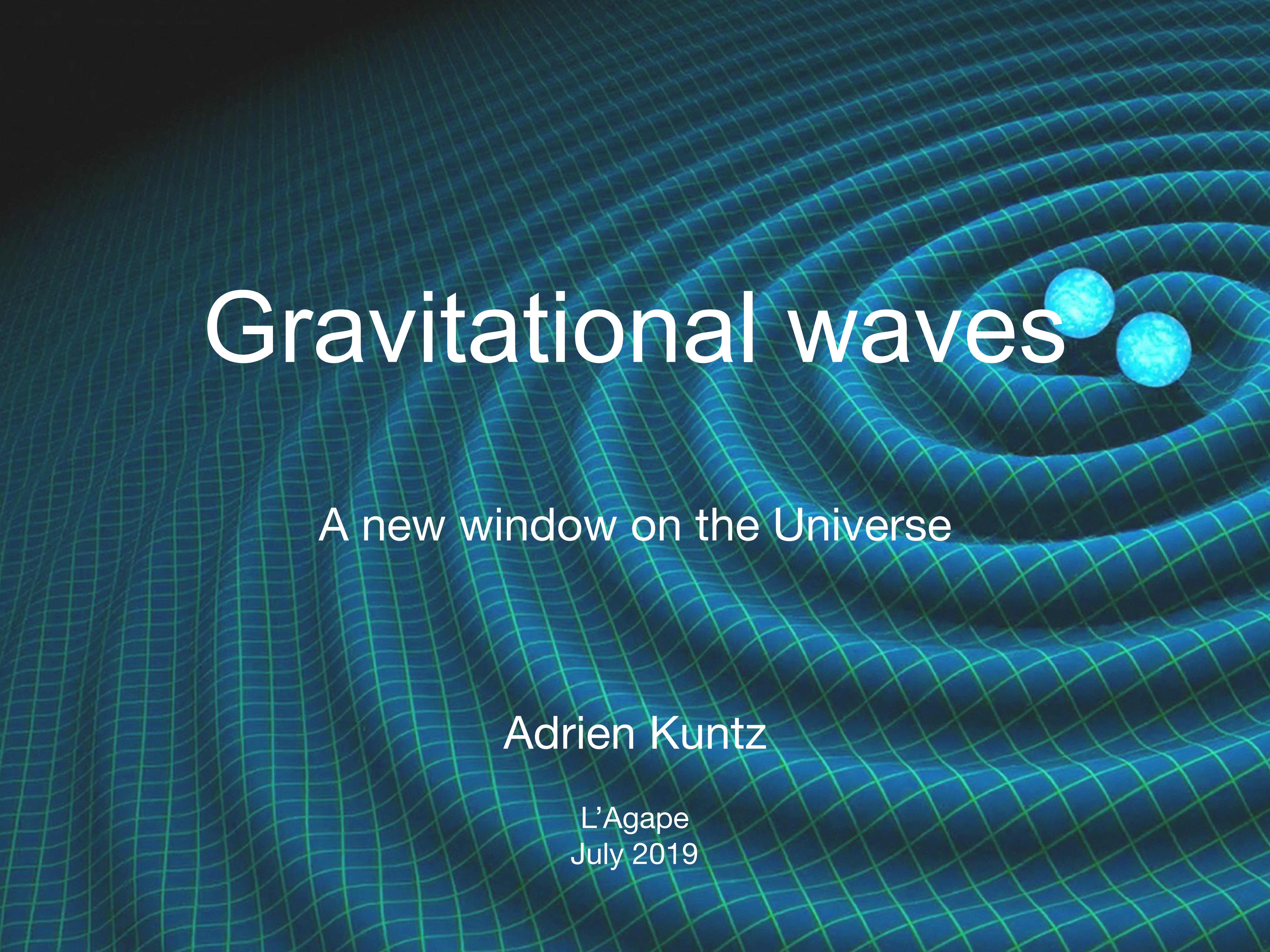

Gravitational waves

A new window on the Universe

Adrien Kuntz

L’Agape
July 2019

Part I : a short history

A bit of history

$$\nu^2 \nabla^2 \mathbf{e} = \frac{\partial^2 \mathbf{e}}{\partial t^2},$$

Quand nous parlerons donc de la position ou de la vitesse du corps attirant, il s'agira de cette position ou de cette vitesse à l'instant où l'*onde gravifique* est partie de ce corps; quand nous parlerons de la position ou de la vitesse du corps attiré, il s'agira de cette position ou de cette vitesse à l'instant où ce corps attiré a été atteint par l'onde gravifique émanée de l'autre corps; il est clair que le premier instant est antérieur au second.

A bit of history

$$\nu^{2}\nabla^{2}\mathbf{e} = \frac{\partial^{2}\mathbf{e}}{\partial t^{2}},$$

Mathematics is an experimental science, and definitions do not come first, but later on. They make themselves, when the nature of the subject has developed itself

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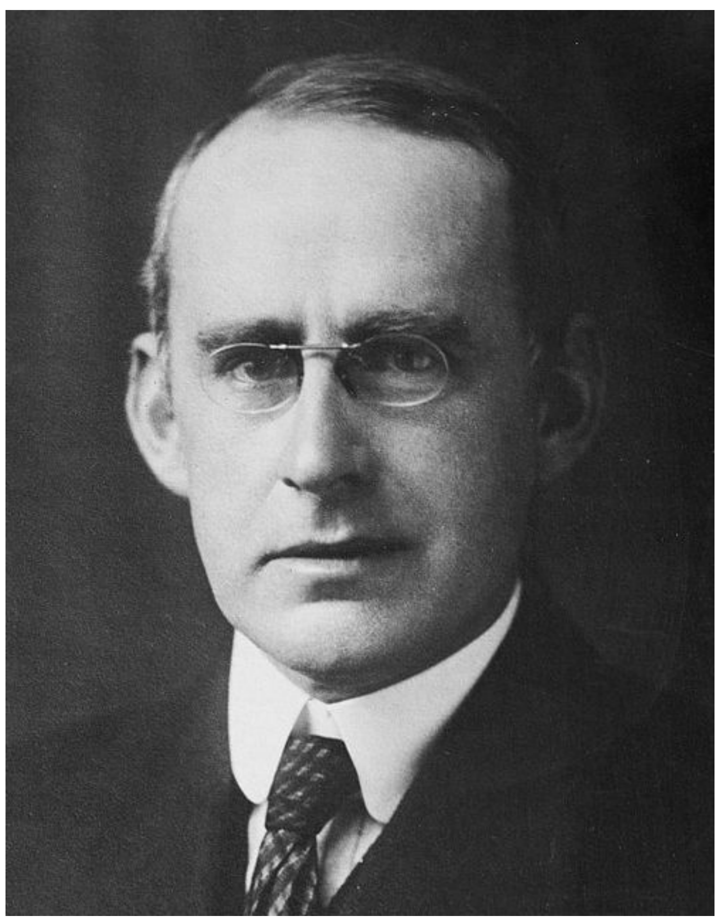
A bit of history

$$h = \frac{2G}{c^4}\frac{1}{r}\frac{\partial^2 Q}{\partial t^2}$$

Transverse-transverse	h_{22}, h_{33}, h_{23}.
Longitudinal-transverse	h_{12}, h_{13}, h_{24}, h_{34}.
Longitudinal-longitudinal	h_{11}, h_{14}, h_{44}.



They are not objective, and (like absolute velocity) are not detectable by any conceivable experiment. They are merely sinuosities in the co-ordinate-system, and the only speed of propagation relevant to them is " the speed of thought."

A bit of history

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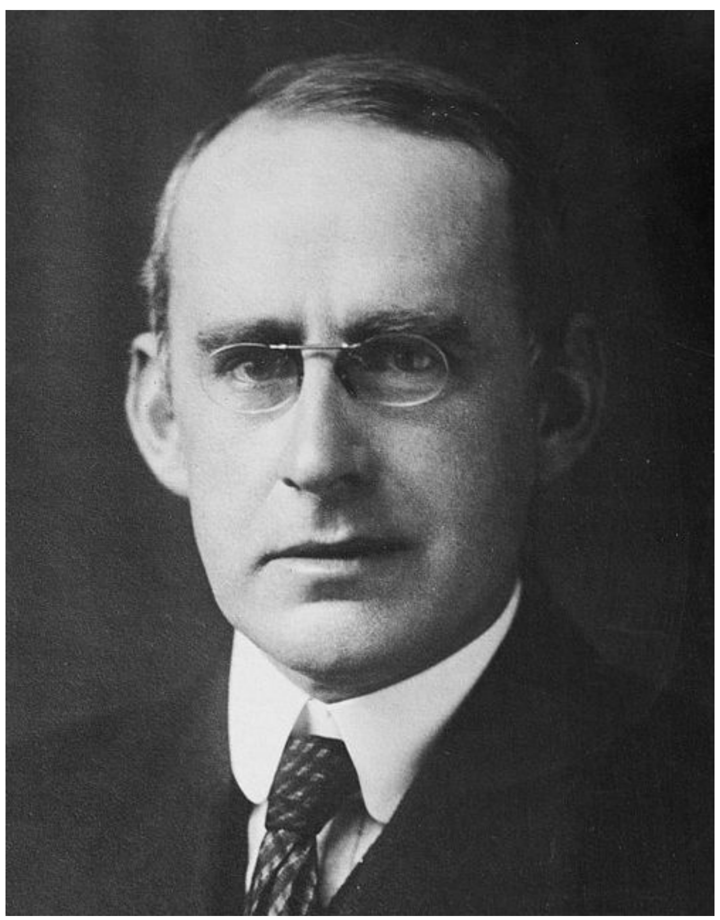
Together with a young collaborator, I arrived at the interesting result that gravitational waves do not exist, though they had been assumed a certainty to the first approximation.



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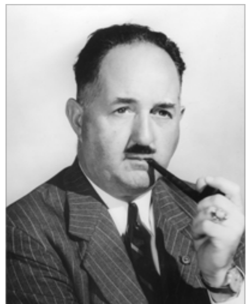
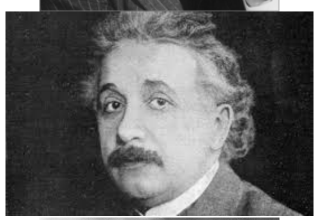
To cut a long story short…

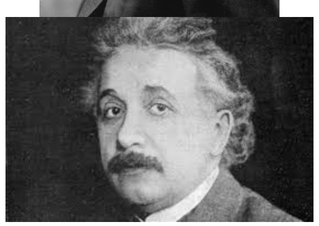


Robertson (referee) to Einstein and Rosen : your conclusion an artifact of your coordinate system



Einstein : I will never publish in Physical Review again

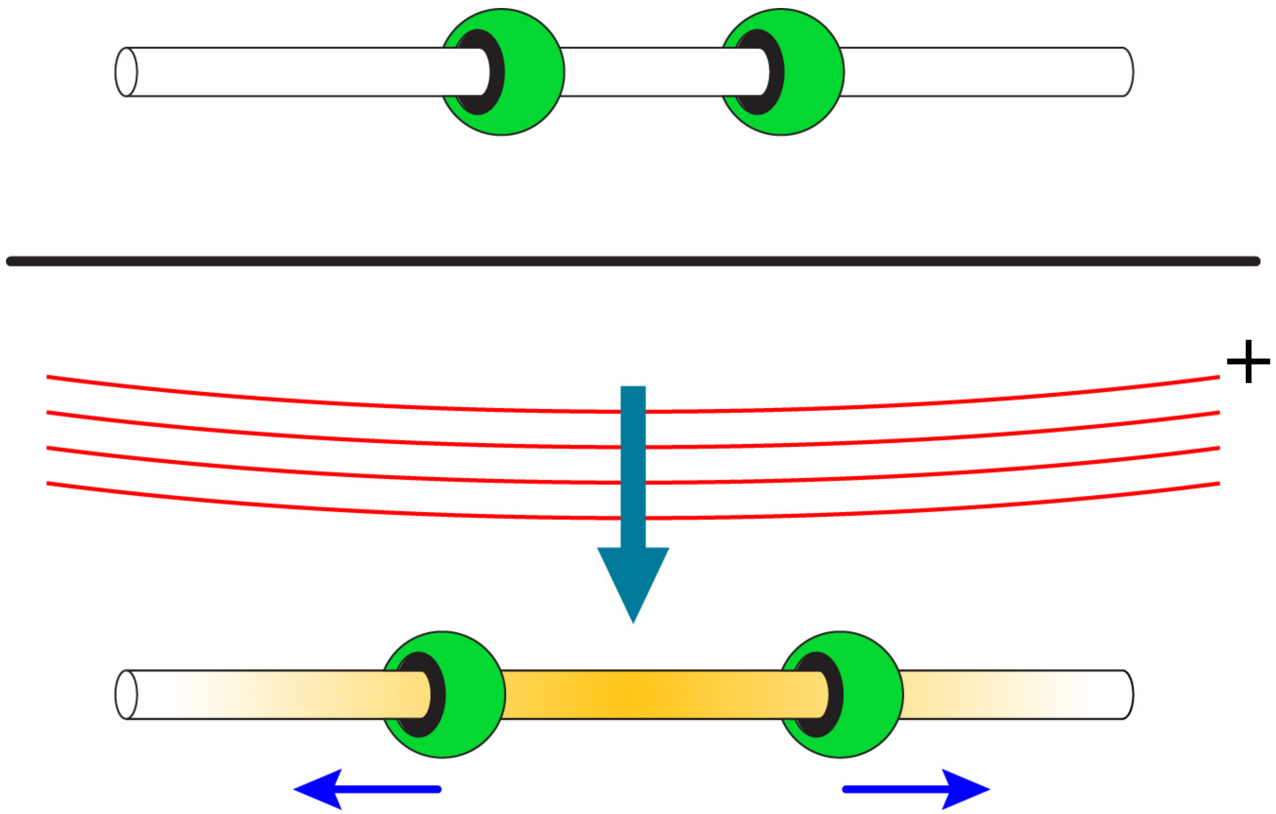
Infeld : Look, Einstein, Robertson was right



Einstein to Rosen (1937) : OK, so GW really exist. Let’s just change the conclusion of our article.

Rosen : I still don’t believe it and I prefer to publish my own version

Later on (1955)

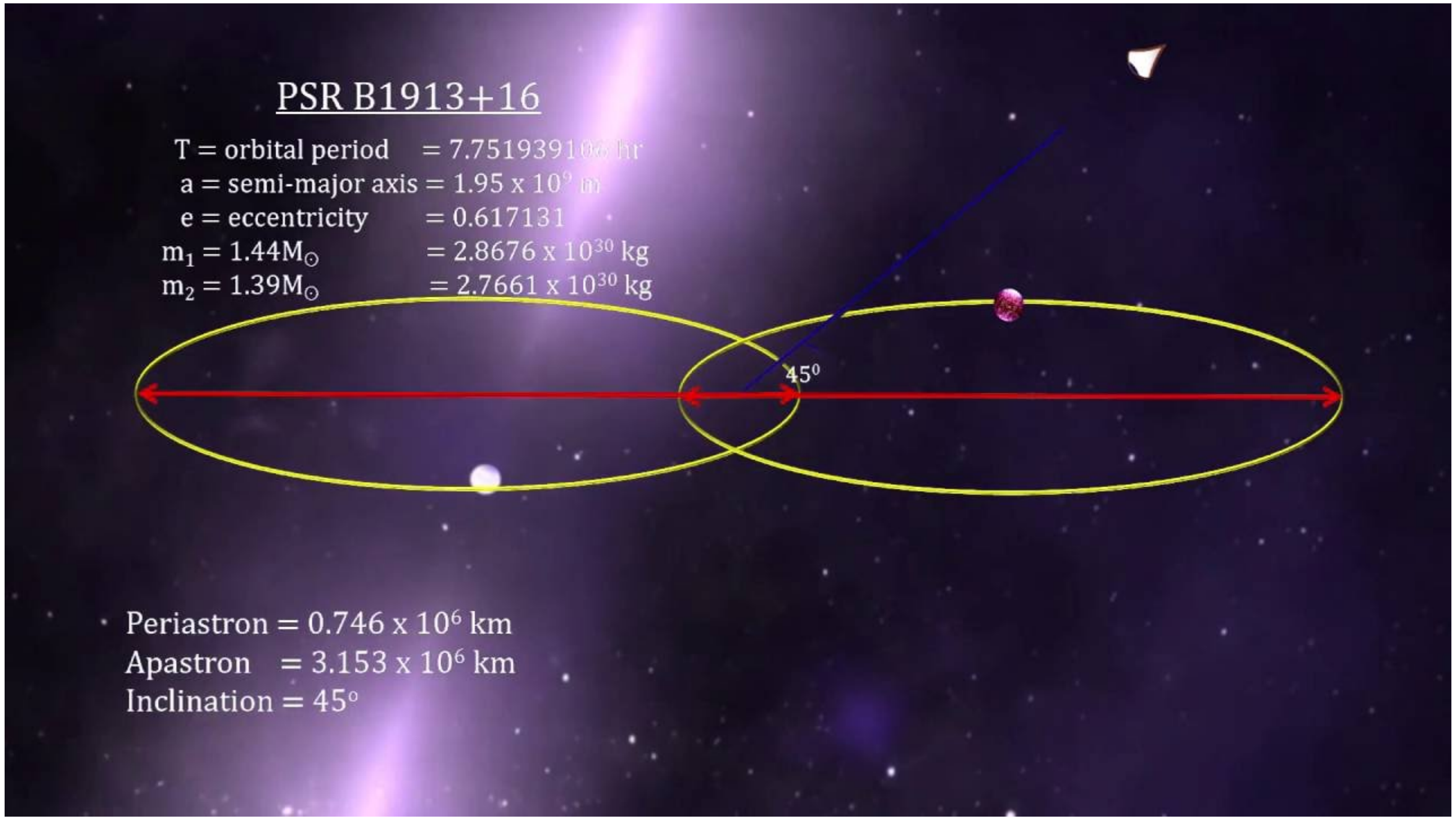


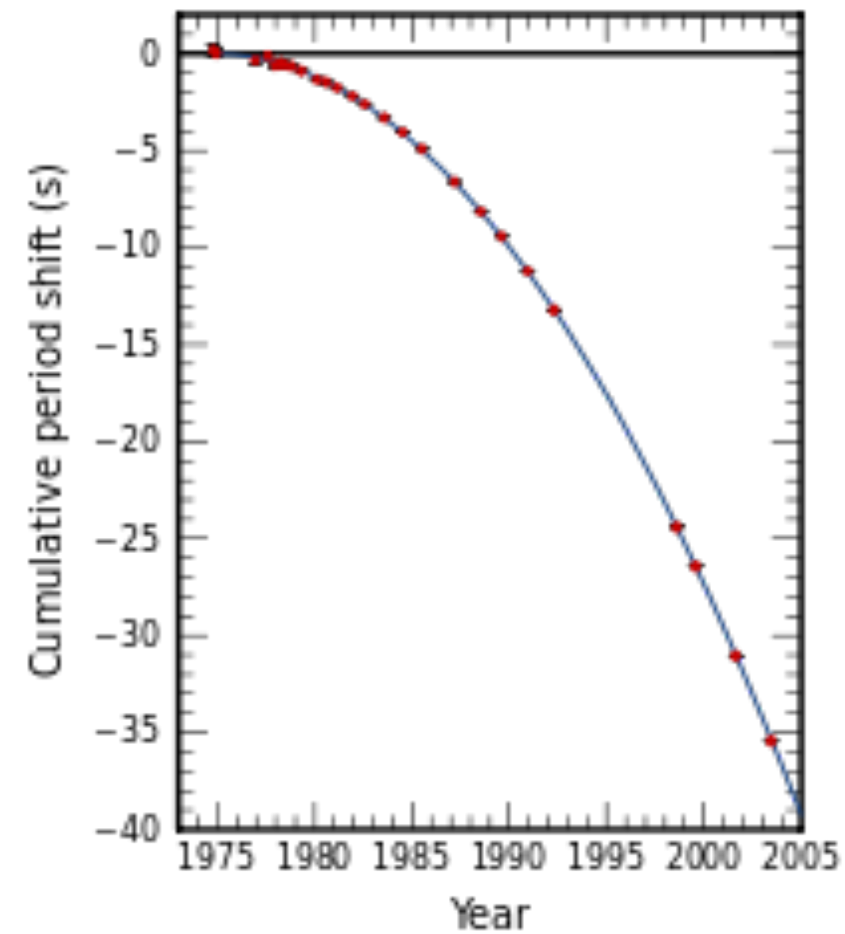
+ Pirani, Bondi, Wheeler..

Rosen still did not believe until the 70's…

(And even today I heard strange statements from a physics teacher in Marseille, whose name I will keep secret…)

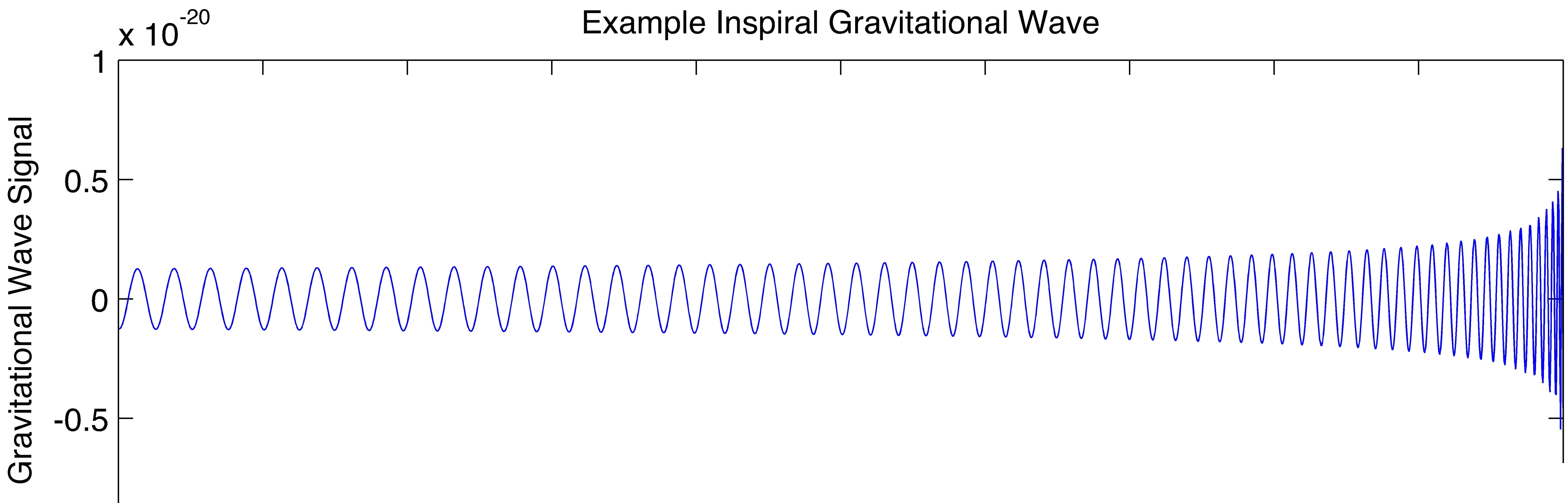
The Hulse-Taylor pulsar

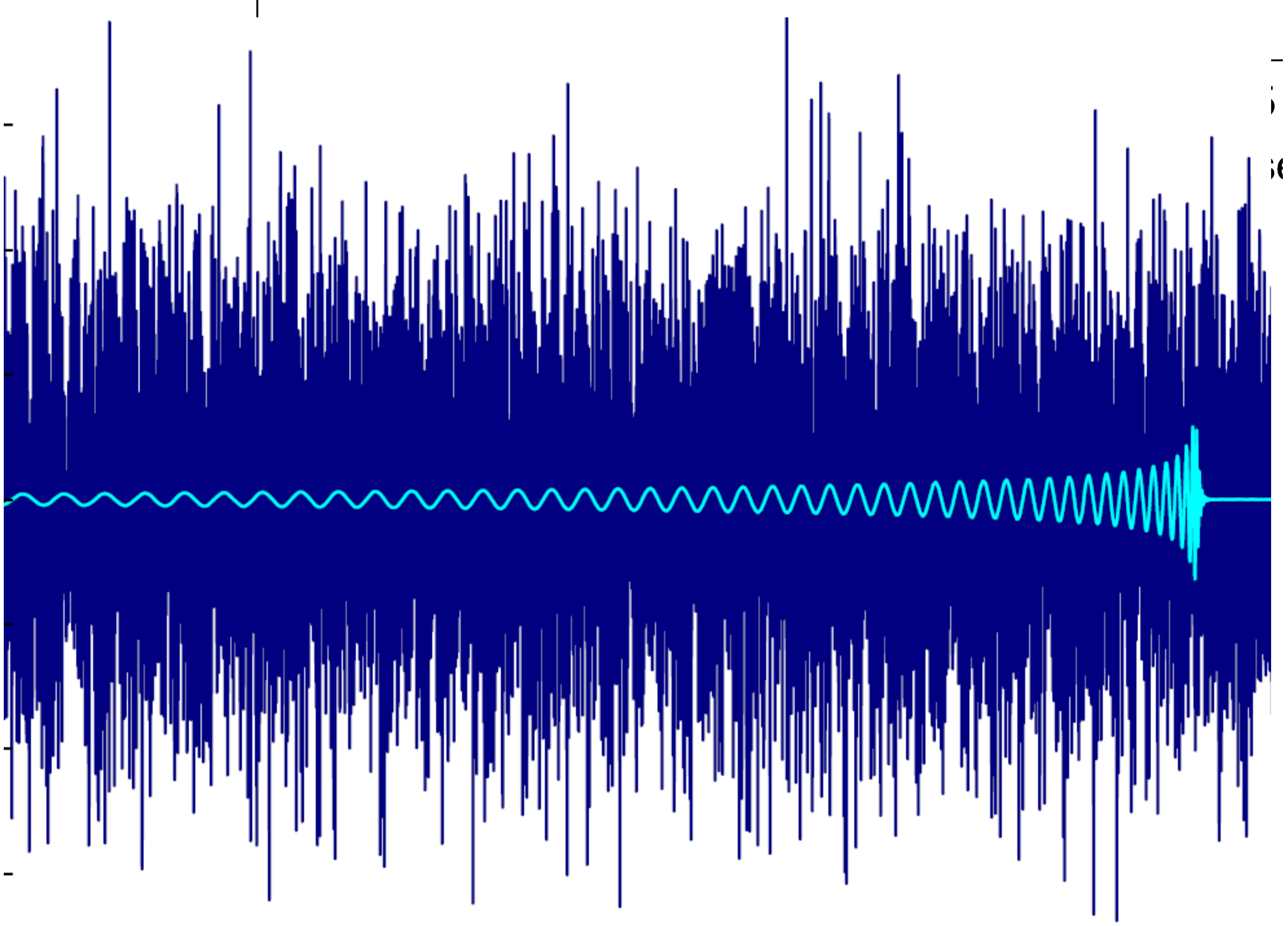


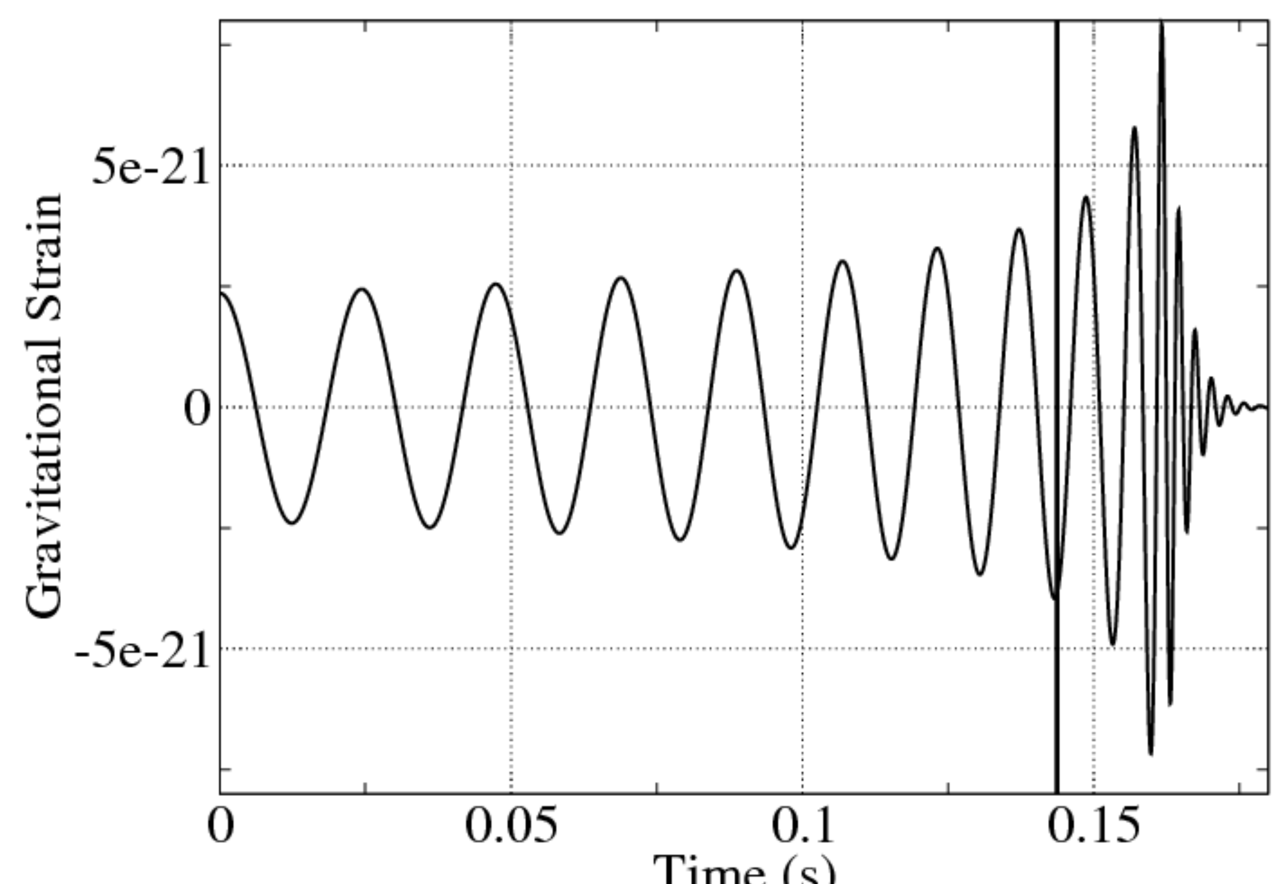


Consistent with GR at the 0.2% level !

Today


Example Inspiral Gravitational Wave
x 10^-20
1
0.5
0
-0.5
Gravitational Wave Signal




5e-21
0
-5e-21
Gravitational Strain
0
0.05
0.1
0.15
Time (s)

Part II : sources of GW

Landscape

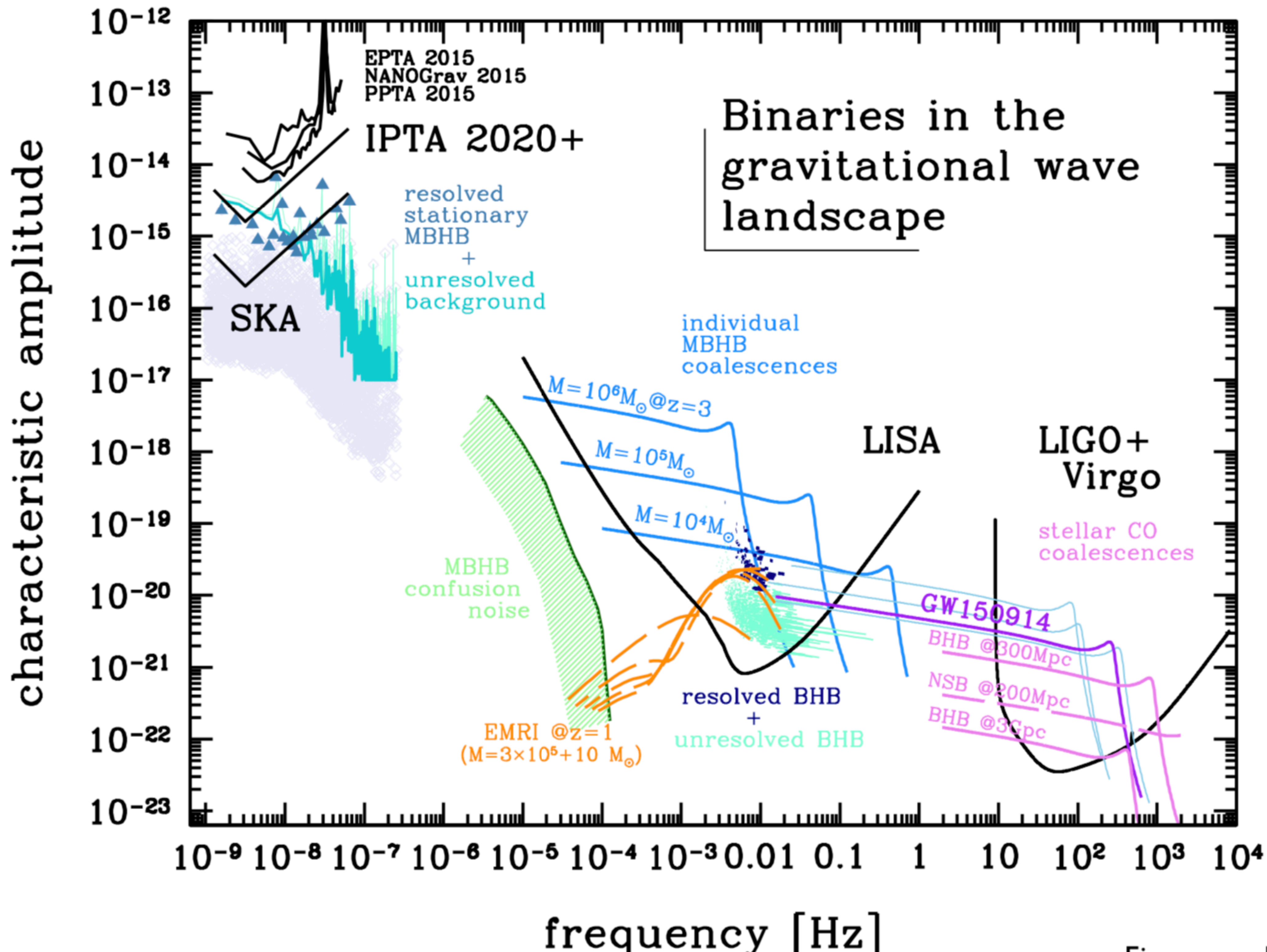
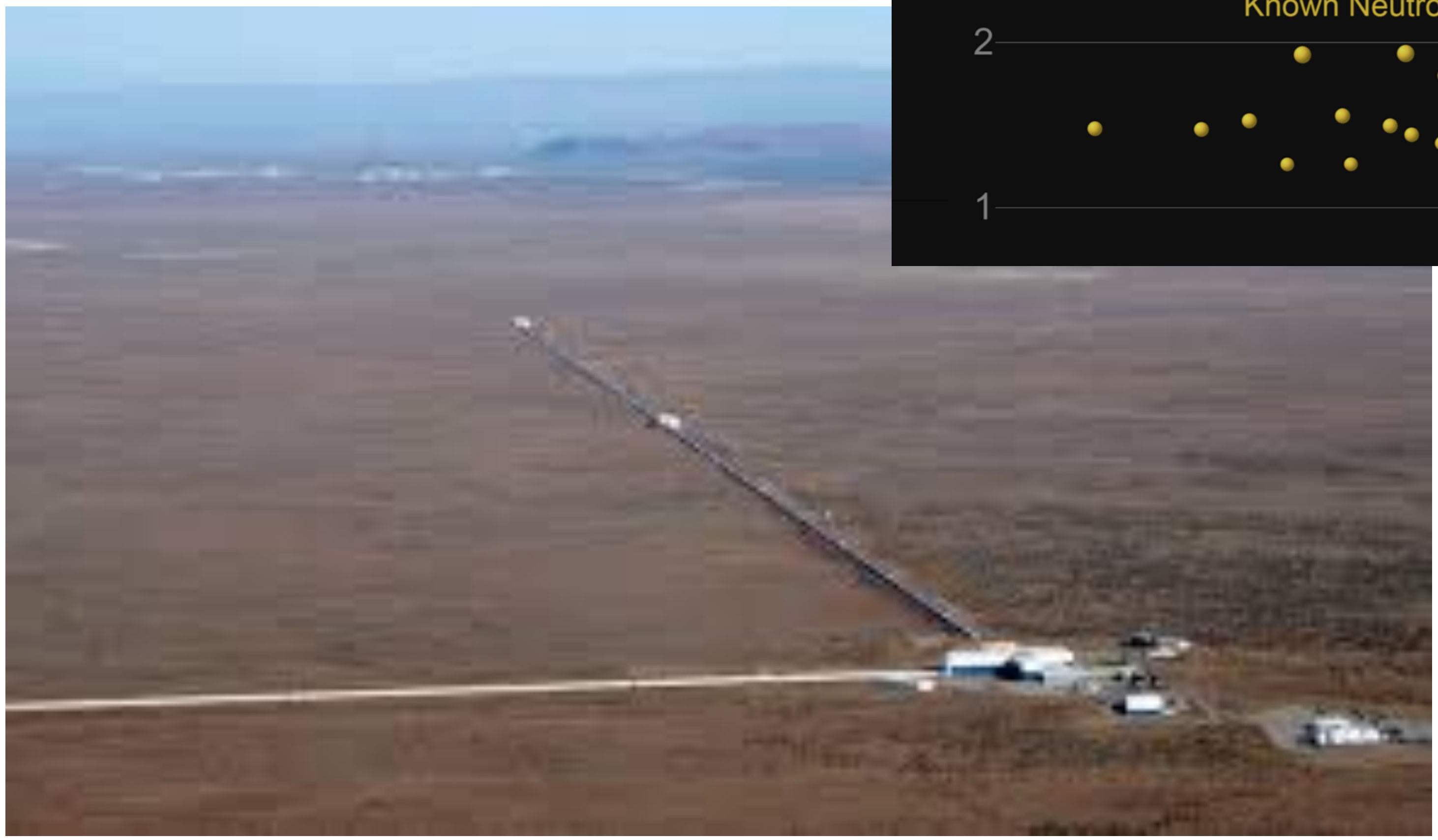
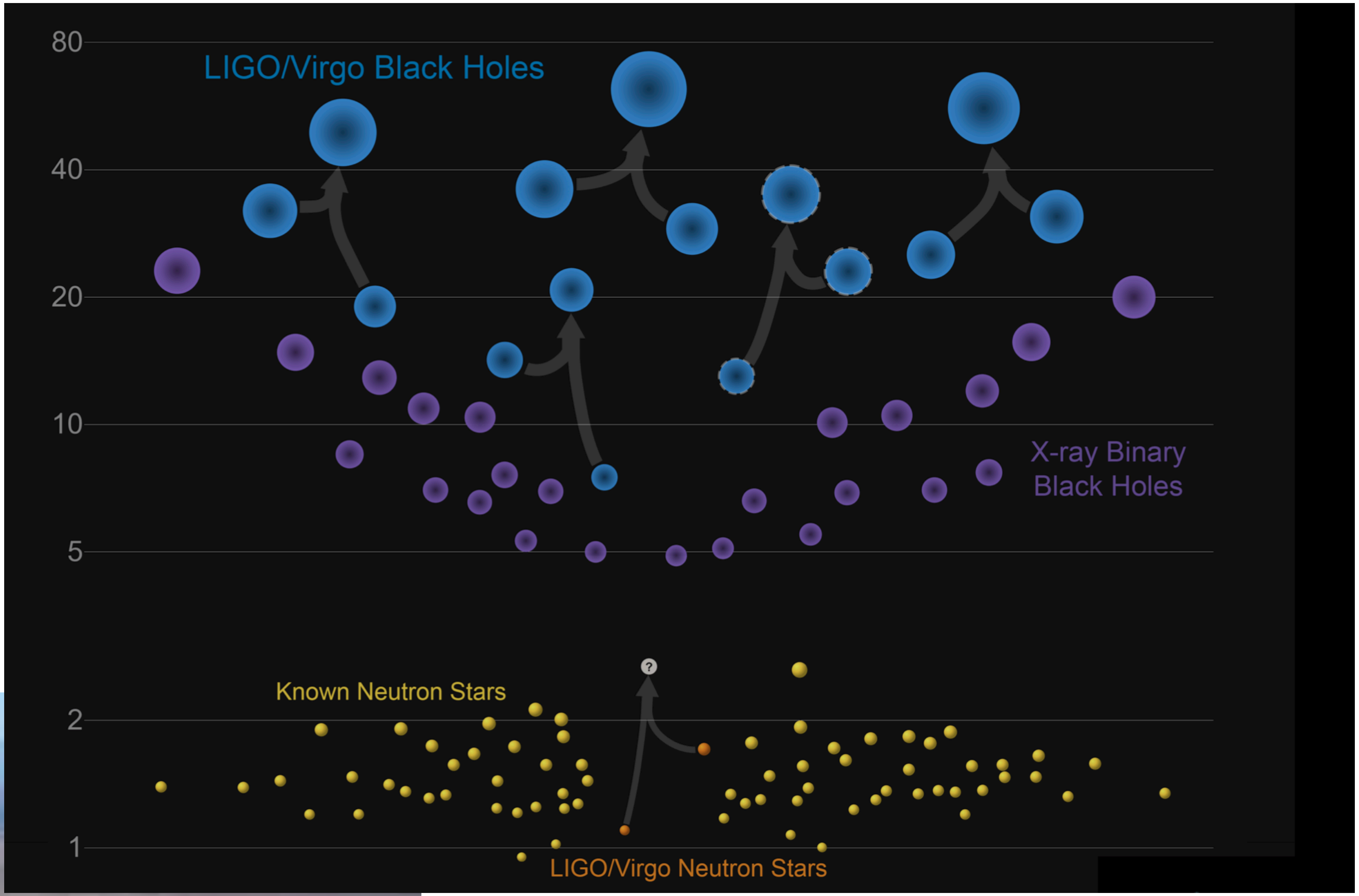


Figure credit: Alberto Sesana

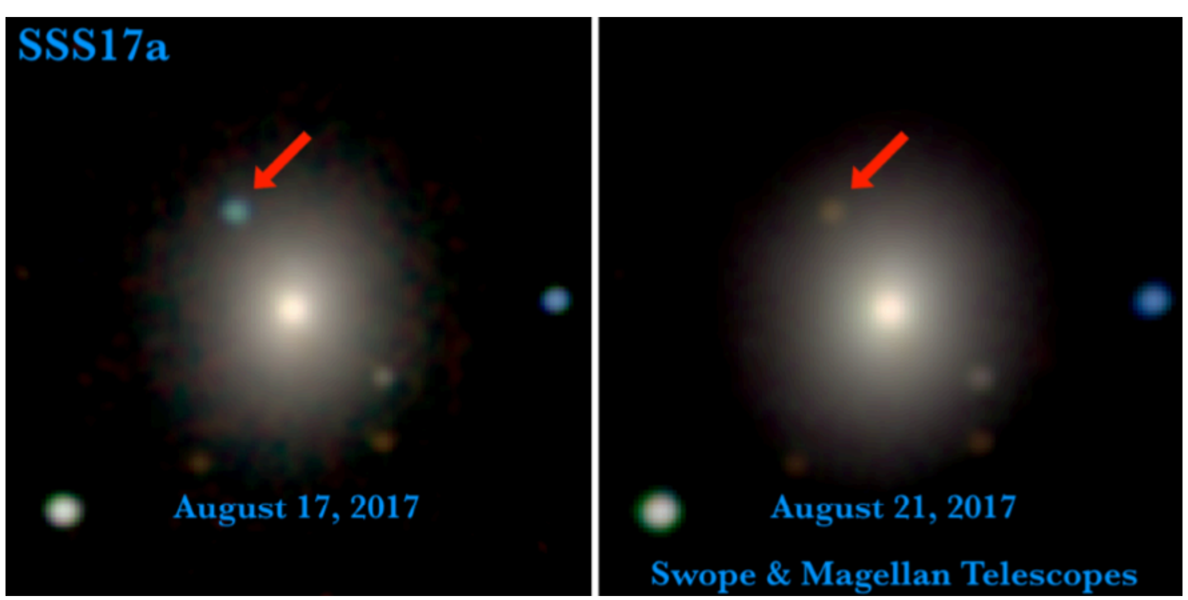
LIGO/VIRGO

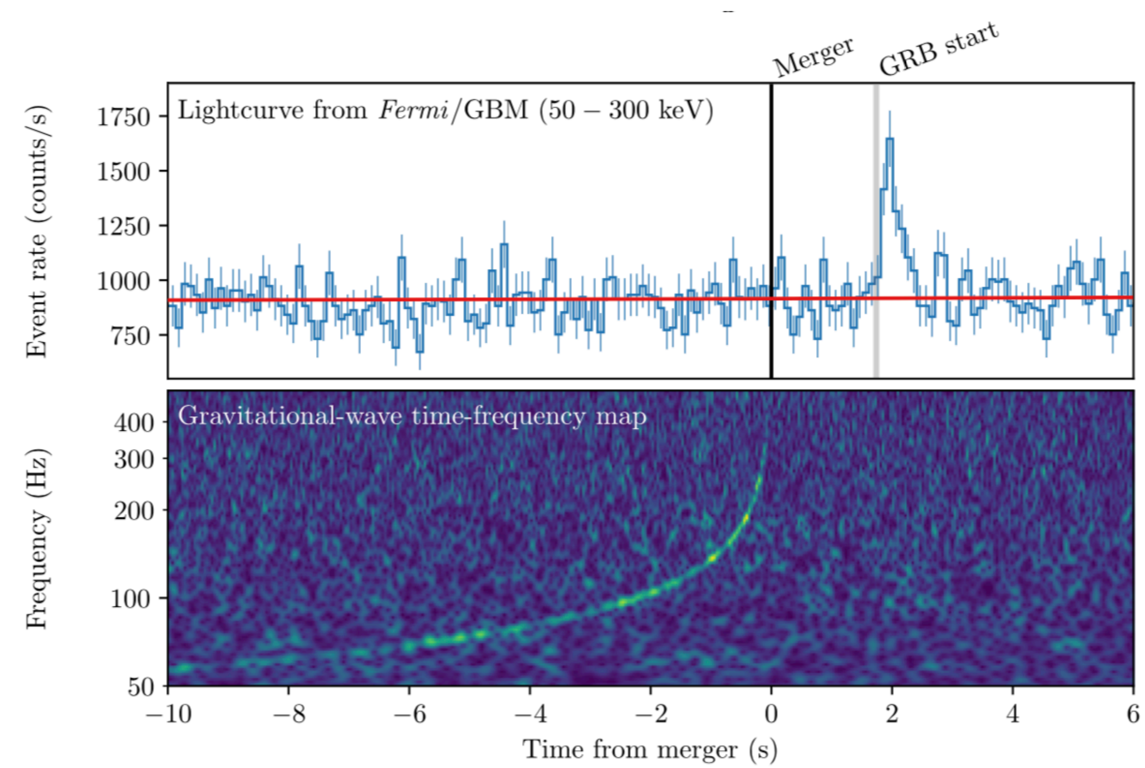



80
40
20
10
5
2
1
LIGO/Virgo Black Holes
X-ray Binary
Black Holes
Known Neutron Stars
LIGO/Virgo Neutron Stars

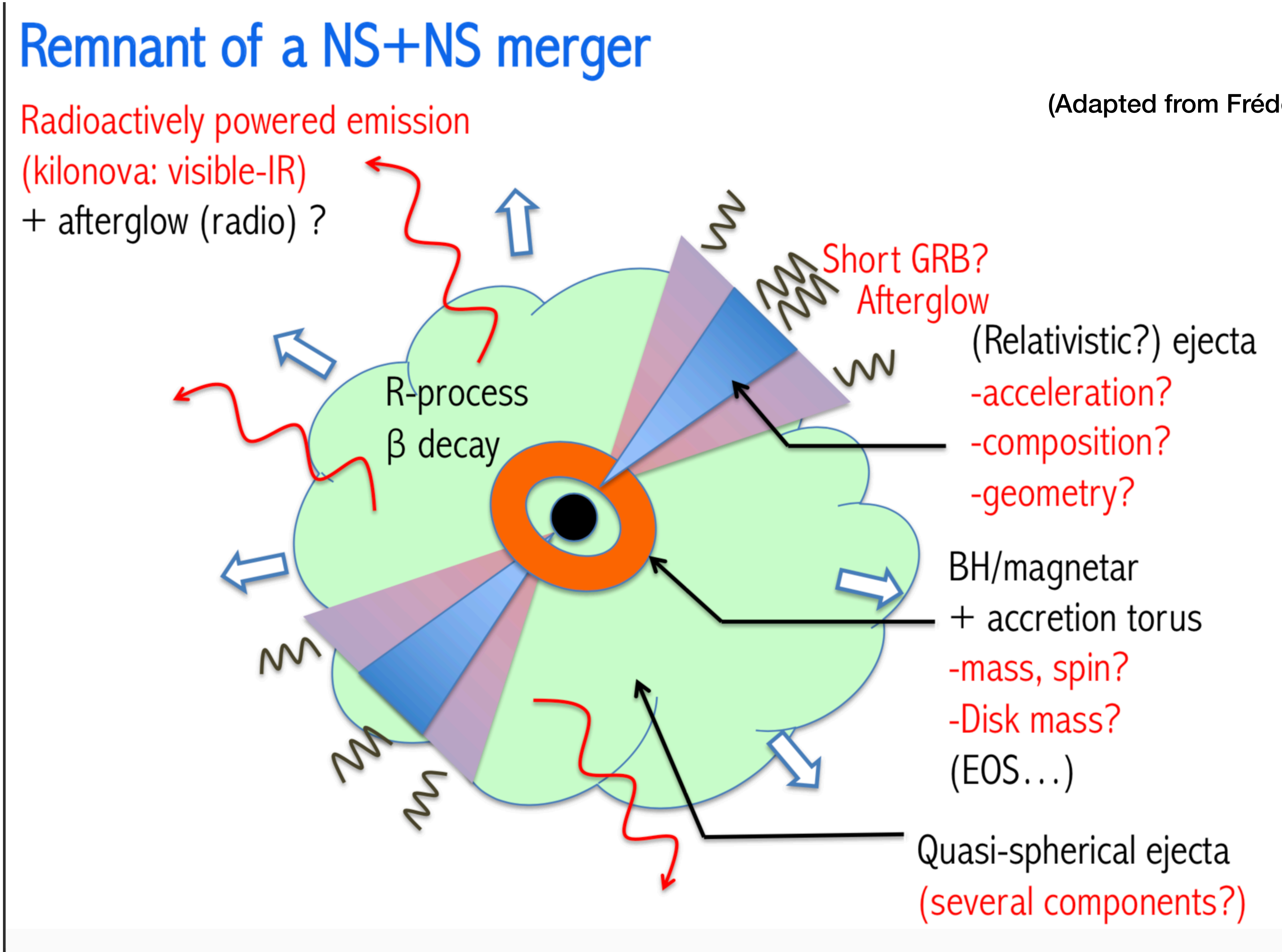
LIGO sources

- Most sources at z ~ 0.1 (less than 1 GPc) : close universe
- BH-BH : spin and mass distribution, formation (star or primordial ?), no EM counterpart ?
- BH-NS : expected but not found yet
- NS-NS : 1 detection sor far. EM counterpart : EOS of NS, speed of gravity, kilonova !



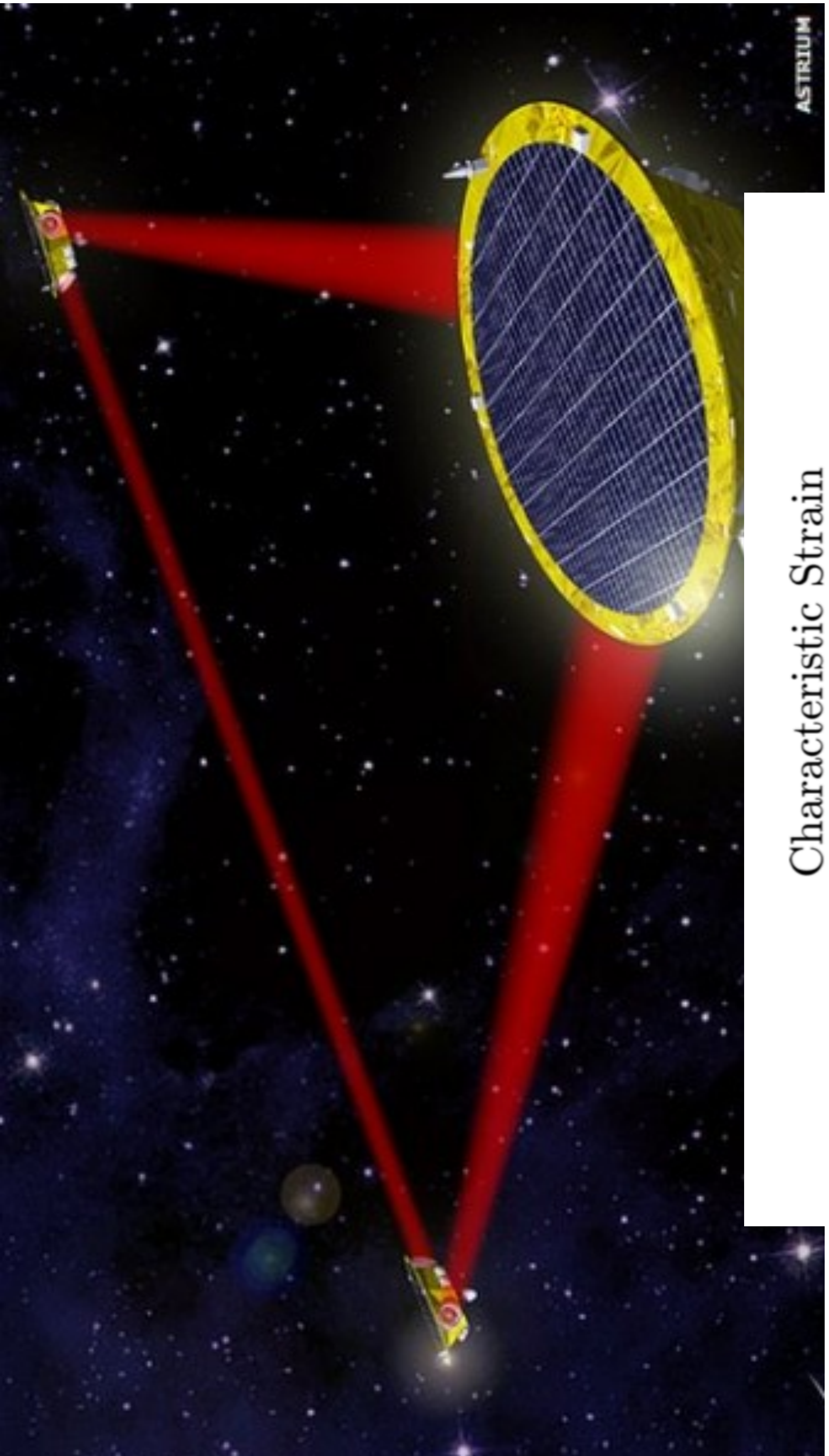


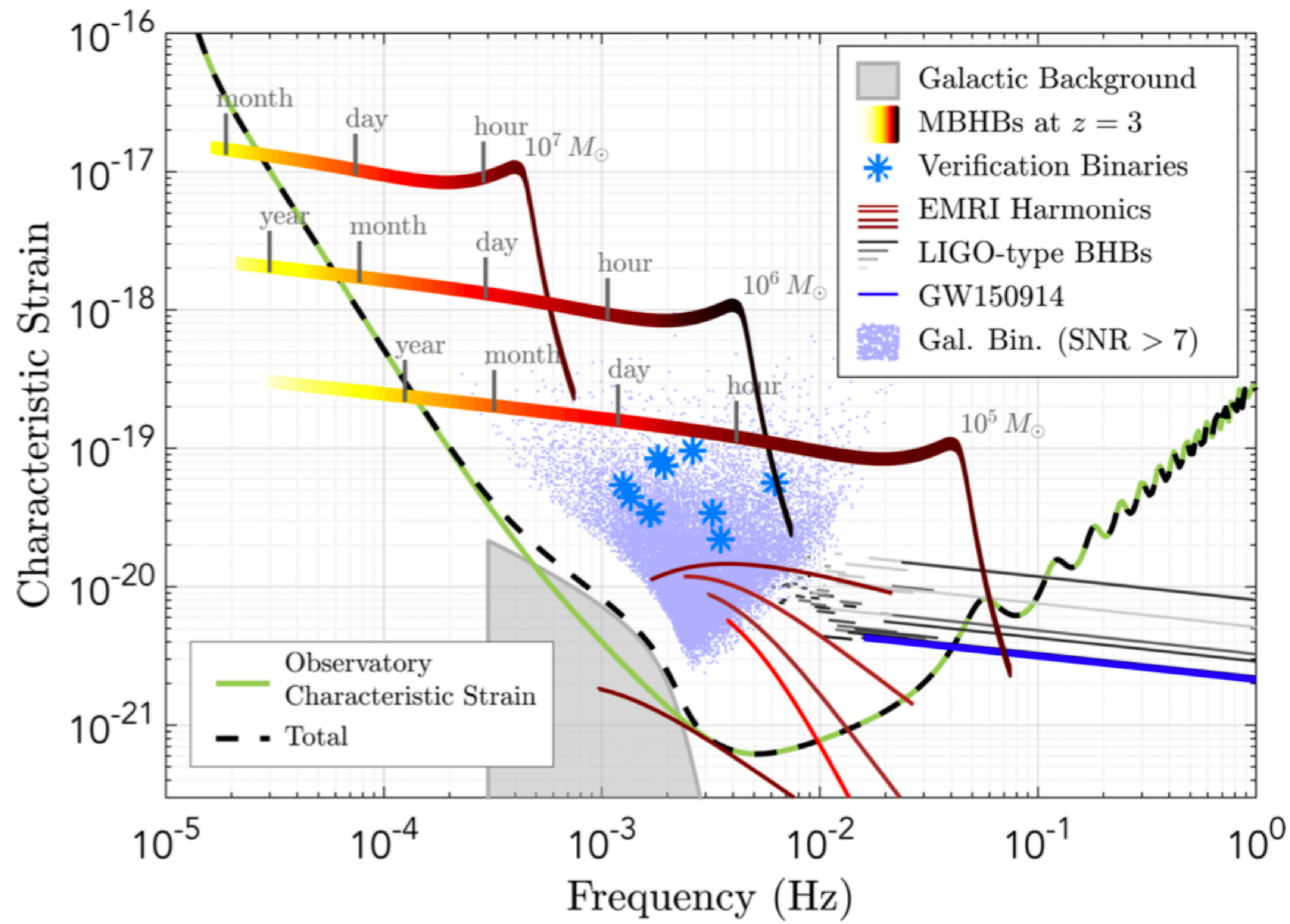
LIGO sources



(Adapted from Frédéric Daigne)

LISA





(Credit : Maria Volonteri)

LISA sources

- Binaries in our Galaxy (background noise + 25000 resolved), before they enter the LIGO band !
- Supermassive BH merger, up to z ~ 15 => population models, observation of the formation of a quasar in real time, H0 measurement and cosmology,… Few events per year
- Extreme mass ratio inspiral (60 $M_\odot$ VS $10^5 M_\odot$) up to z=4 => Very accurate measurement of the spin, eccentricity, inclination and test of gravity. Few events per year
- Stochastic GW background (inflation, cosmic strings…)
- Core collapse supernovaes
- Exotic and unmodeled sources !

Some questions LISA will try to answer

How many galaxies host MBHs

➔ when, where, how they form

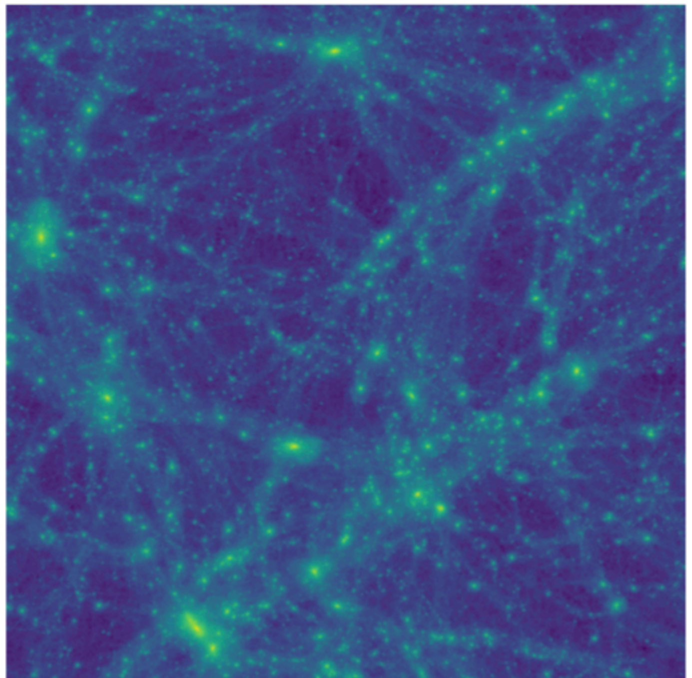
How long it takes for MBHs to merge in halo/galaxy merger

➔ dynamics of MBHs in mergers

How MBHs grow in mass over time

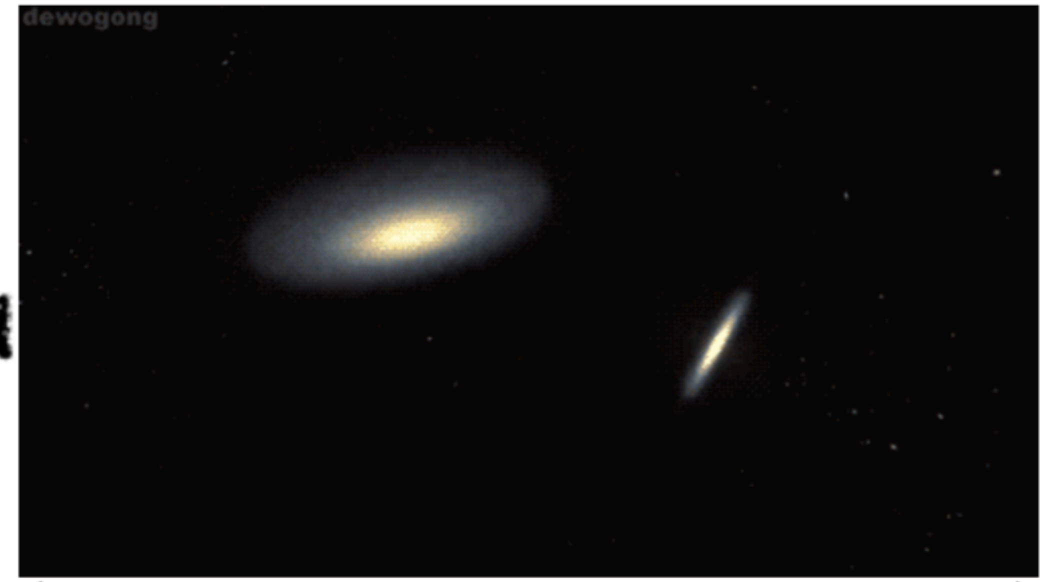
➔ accretion vs MBH-MBH mergers

Some questions LISA will try to answer



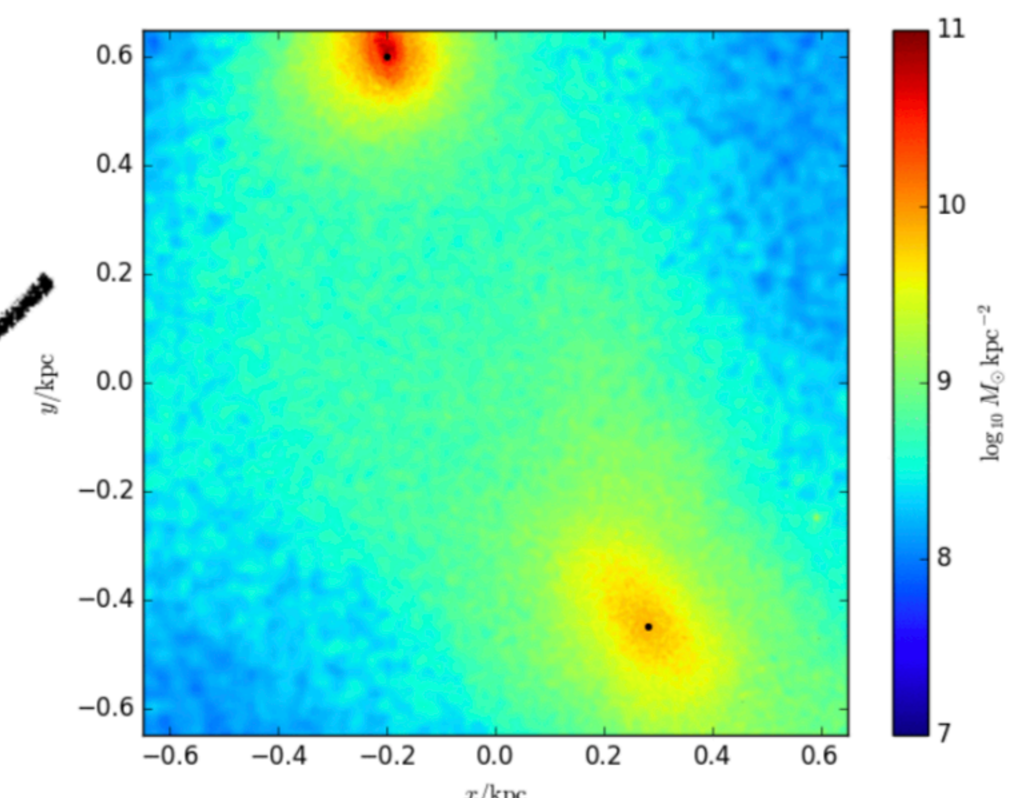
~100 Mpc (cosmology)

Gravity



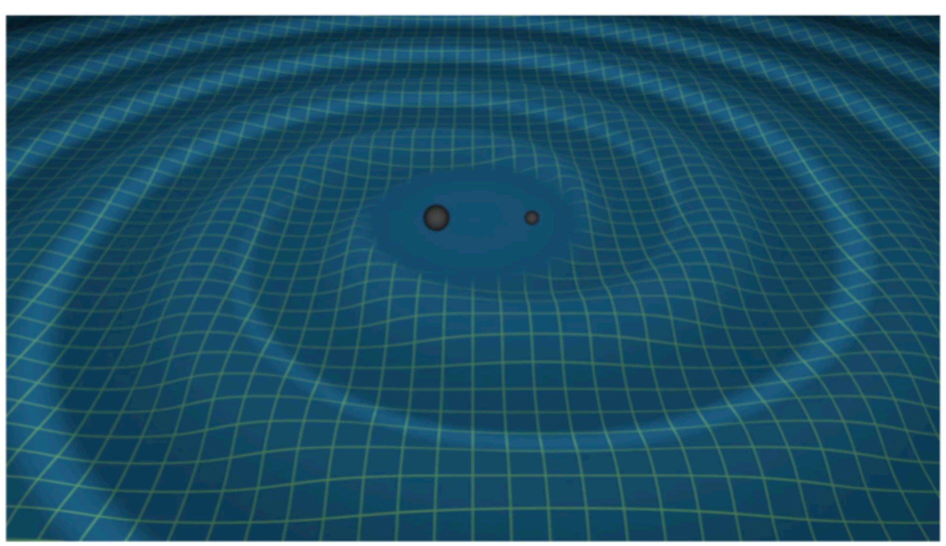
~100 kpc -1 kpc (galaxy mergers)

Dynamical Friction



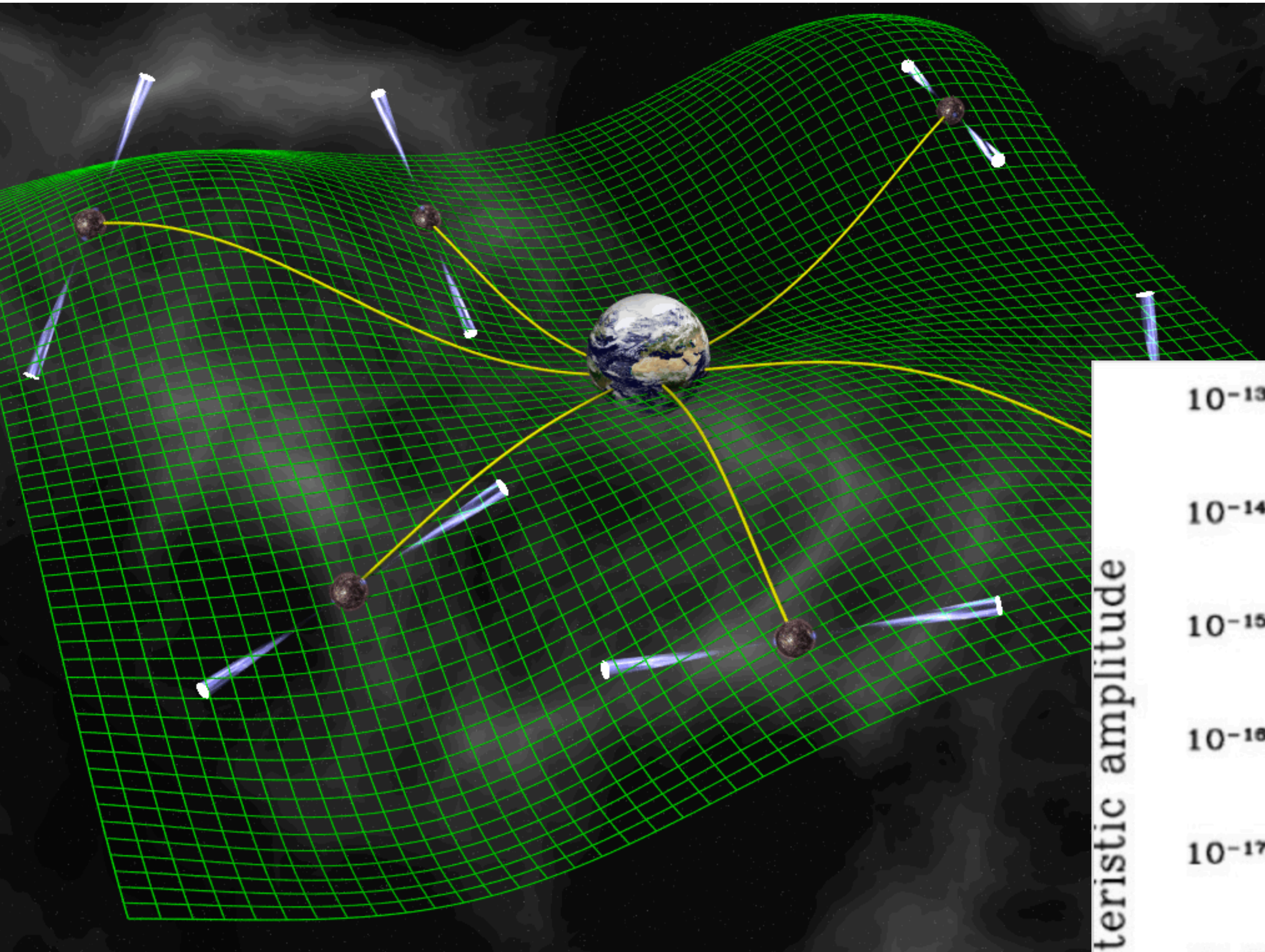
~1 kpc-1pc (binary formation)

Gas torques?
Stellar scattering?
Last pc problem

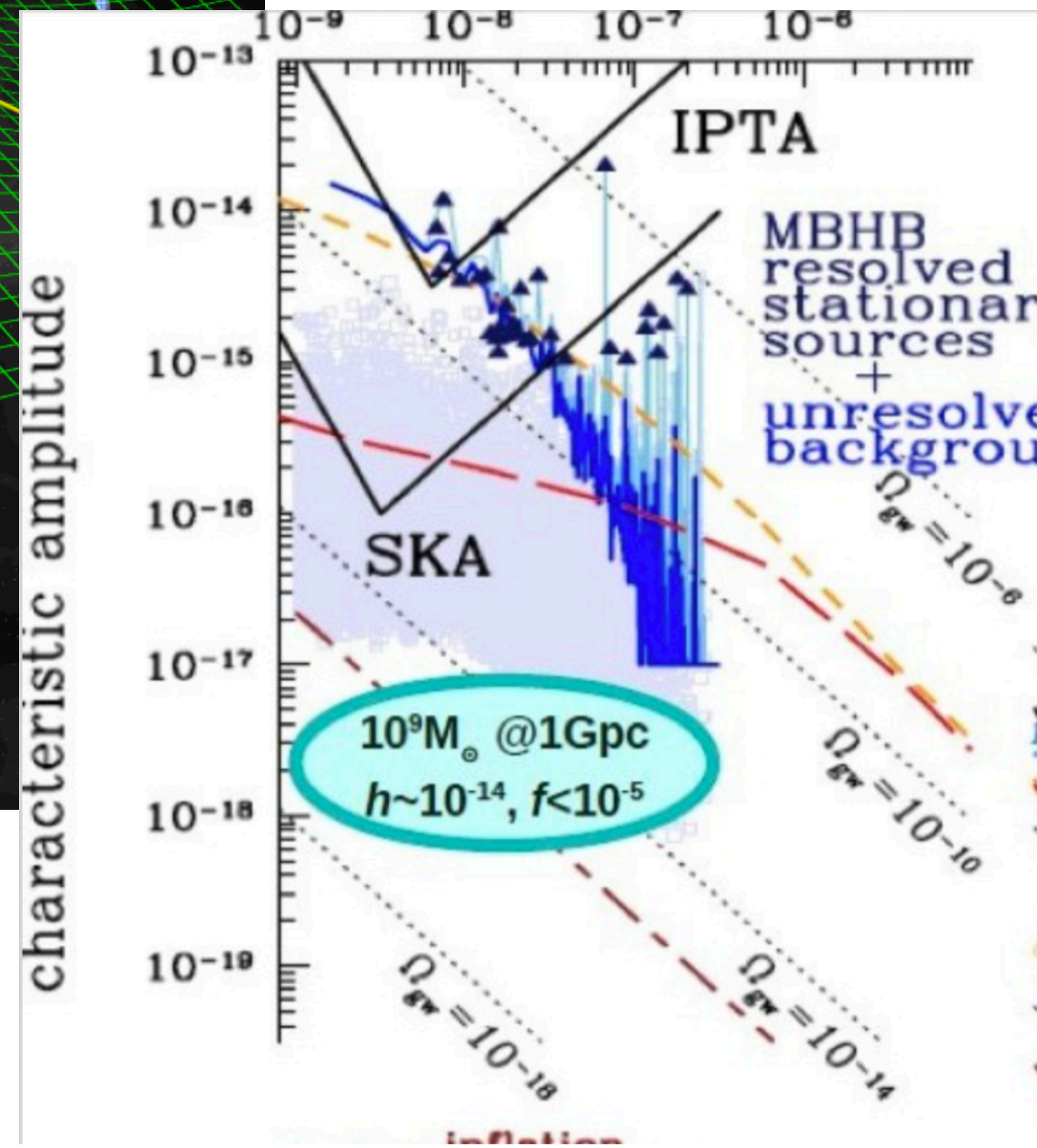


~1 millipc(BH merger)

Pulsar Timing Array

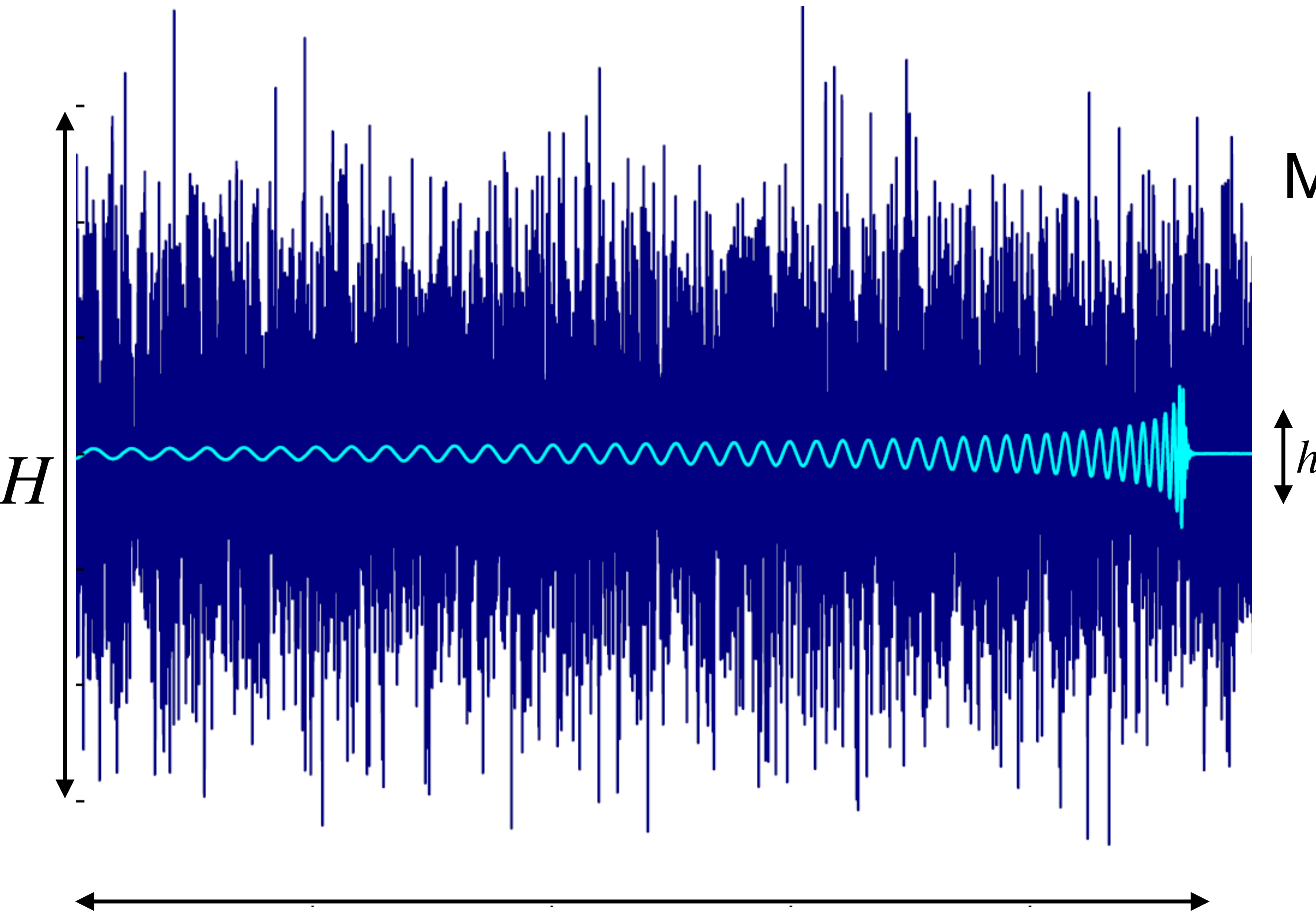


- Supermassive BH binaries
- Inflation
- Cosmic strings



Part III : modeling the waveform

The need for a high-precision template

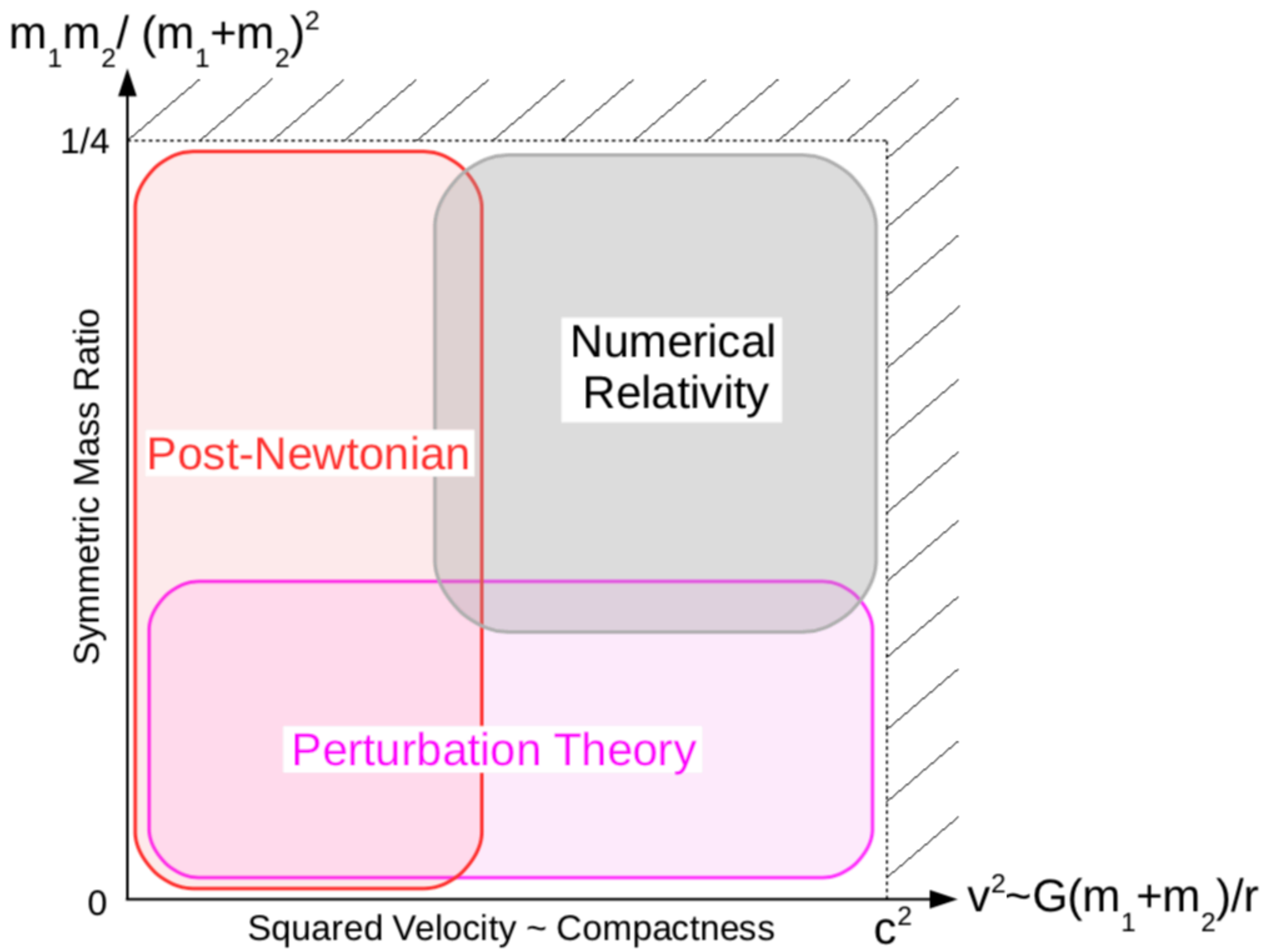


$T \simeq 10^4$oscillations

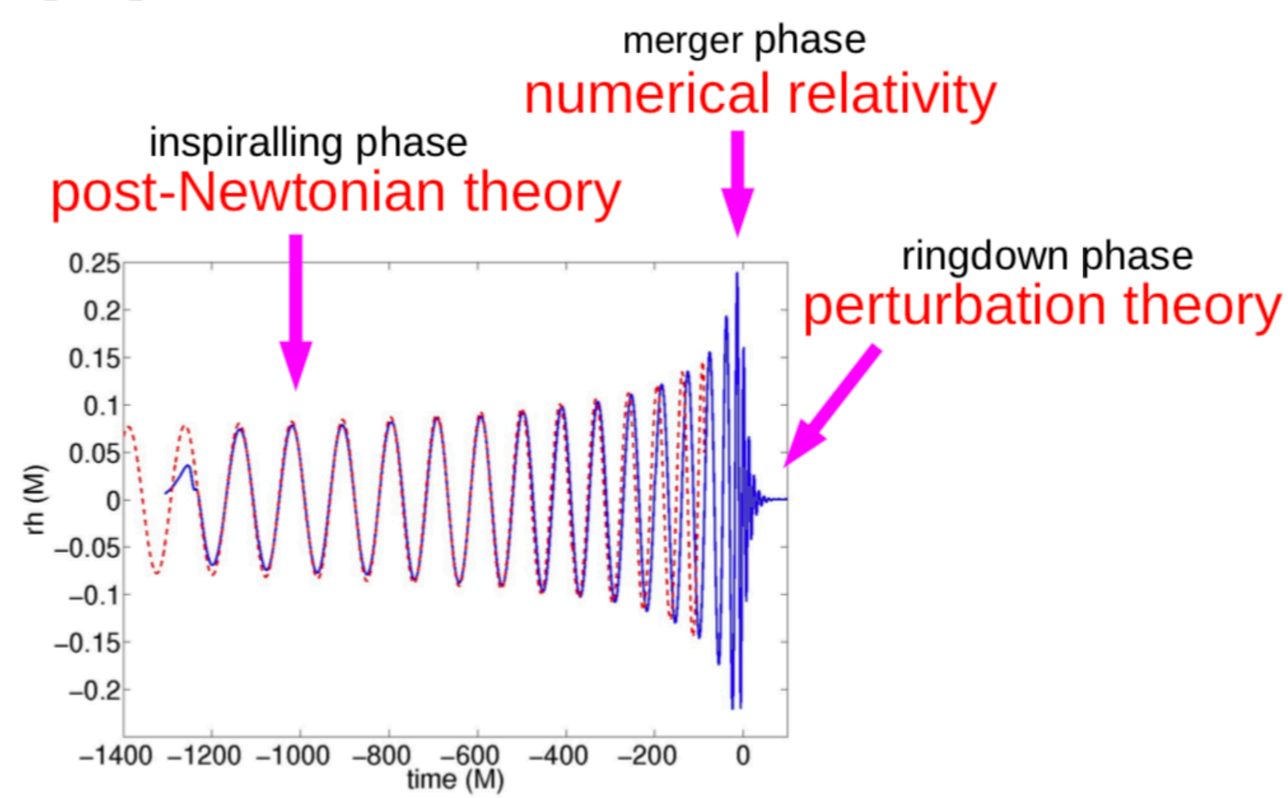
Matched filtering analysis

$$\frac{S}{N} \sim \frac{h}{H}\sqrt{T}$$

The different methods



(Credit : Luc Blanchet)



Post-Newtonian approximation

Perturbative solution of the EOM

$G_{\mu\nu} = 8\pi T_{\mu\nu}$ **and** $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, \quad h_{\mu\nu} \sim \frac{GM}{r} \sim v^2 \ll 1$

Matter is modeled by point particles (finite size effects arise only at $\mathcal{O}(v^8)$!) :

$$S_m = -m_1 \int d\tau_1 - m_2 \int d\tau_2$$

Then plug back $g_{\mu\nu}$ in the action and obtain the two-body relativistic Lagrangian :

$$L = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 + \frac{G m_1 m_2}{r} + \text{relativistic corrections}$$

A (small) part of the 3PN energy

$$
\begin{aligned}
E =\; & \frac{m_1 v_1^2}{2} - \frac{G m_1 m_2}{2 r_{12}} \\
& + \frac{1}{c^2}\left\{ \frac{G^2 m_1^2 m_2}{2 r_{12}^2} + \frac{3 m_1 v_1^4}{8} + \frac{G m_1 m_2}{r_{12}}\left(-\frac{1}{4}(n_{12}v_1)(n_{12}v_2) + \frac{3}{2}v_1^2 - \frac{7}{4}(v_1 v_2) \right) \right\} \\
& + \frac{1}{c^4}\left\{ -\frac{G^3 m_1^3 m_2}{2 r_{12}^3} - \frac{19 G^3 m_1^2 m_2^2}{8 r_{12}^3} + \frac{5 m_1 v_1^6}{16} \right. \\
& \qquad + \frac{G m_1 m_2}{r_{12}}\left(\frac{3}{8}(n_{12}v_1)^3(n_{12}v_2) + \frac{3}{16}(n_{12}v_1)^2(n_{12}v_2)^2 - \frac{9}{8}(n_{12}v_1)(n_{12}v_2)v_1^2 \right. \\
& \qquad\qquad - \frac{13}{8}(n_{12}v_2)^2 v_1^2 + \frac{21}{8}v_1^4 + \frac{13}{8}(n_{12}v_1)^2(v_1 v_2) + \frac{3}{4}(n_{12}v_1)(n_{12}v_2)(v_1 v_2) \\
& \qquad\qquad \left. - \frac{55}{8}v_1^2(v_1 v_2) + \frac{17}{8}(v_1 v_2)^2 + \frac{31}{16}v_1^2 v_2^2 \right) \\
& \qquad \left. + \frac{G^2 m_1^2 m_2}{r_{12}^2}\left(\frac{29}{4}(n_{12}v_1)^2 - \frac{13}{4}(n_{12}v_1)(n_{12}v_2) + \frac{1}{2}(n_{12}v_2)^2 - \frac{3}{2}v_1^2 + \frac{7}{4}v_2^2 \right) \right\} \\
& + \frac{1}{c^6}\left\{ \frac{35 m_1 v_1^8}{128} \right. \\
& \qquad + \frac{G m_1 m_2}{r_{12}}\left(-\frac{5}{16}(n_{12}v_1)^5(n_{12}v_2) - \frac{5}{16}(n_{12}v_1)^4(n_{12}v_2)^2 - \frac{5}{32}(n_{12}v_1)^3(n_{12}v_2)^3 \right. \\
& \qquad\qquad + \frac{19}{16}(n_{12}v_1)^3(n_{12}v_2)v_1^2 + \frac{15}{16}(n_{12}v_1)^2(n_{12}v_2)^2 v_1^2 + \frac{3}{4}(n_{12}v_1)(n_{12}v_2)^3 v_1^2 \\
& \qquad\qquad + \frac{19}{16}(n_{12}v_2)^4 v_1^2 - \frac{21}{16}(n_{12}v_1)(n_{12}v_2)v_1^4 - 2(n_{12}v_2)^2 v_1^4 \\
& \qquad\qquad + \frac{55}{16}v_1^6 - \frac{19}{16}(n_{12}v_1)^4(v_1 v_2) - (n_{12}v_1)^3(n_{12}v_2)(v_1 v_2) \\
& \qquad\qquad - \frac{15}{32}(n_{12}v_1)^2(n_{12}v_2)^2(v_1 v_2) + \frac{45}{16}(n_{12}v_1)^2 v_1^2(v_1 v_2) \\
& \qquad\qquad + \frac{5}{4}(n_{12}v_1)(n_{12}v_2)v_1^2(v_1 v_2) + \frac{11}{4}(n_{12}v_2)^2 v_1^2(v_1 v_2) - \frac{139}{16}v_1^4(v_1 v_2)
\end{aligned}
$$

(Source : Luc Blanchet)

Dissipative dynamics

Similarly complicated calculations lead to the final result (for circular orbits) :

$$\phi = -\frac{x^{-5/2}}{32\nu}\Big\{1 + \left(\frac{3715}{1008} + \frac{55}{12}\nu\right)x - 10\pi x^{3/2} + \left(\frac{15293365}{1016064} + \frac{27145}{1008}\nu + \frac{3085}{144}\nu^2\right)x^2$$
$$+ \left(\frac{38645}{1344} - \frac{65}{16}\nu\right)\pi x^{5/2}\ln\left(\frac{x}{x_0}\right)$$
$$+ \left[\frac{12348611926451}{18776862720} - \frac{160}{3}\pi^2 - \frac{1712}{21}C - \frac{856}{21}\ln(16\,x)\right.$$
$$\left. + \left(-\frac{15737765635}{12192768} + \frac{2255}{48}\pi^2\right)\nu + \frac{76055}{6912}\nu^2 - \frac{127825}{5184}\nu^3\right]x^3$$
$$+ \left(\frac{77096675}{2032128} + \frac{378515}{12096}\nu - \frac{74045}{6048}\nu^2\right)\pi x^{7/2} + \mathcal{O}\left(\frac{1}{c^8}\right)\Big\},$$

$$x = (GM\omega)^{2/3} \sim v^2$$

$$\nu = \frac{m_1 m_2}{(m_1+m_2)^2}$$

	$2\times 1.4\,M_\odot$	$10\,M_\odot + 1.4\,M_\odot$	$2\times 10\,M_\odot$
Newtonian order	16031	3576	602
1PN	441	213	59
1.5PN (dominant tail)	−211	−181	−51
2PN	9.9	9.8	4.1
2.5PN	−11.7	−20.0	−7.1
3PN	2.6	2.3	2.2
3.5PN	−0.9	−1.8	−0.8

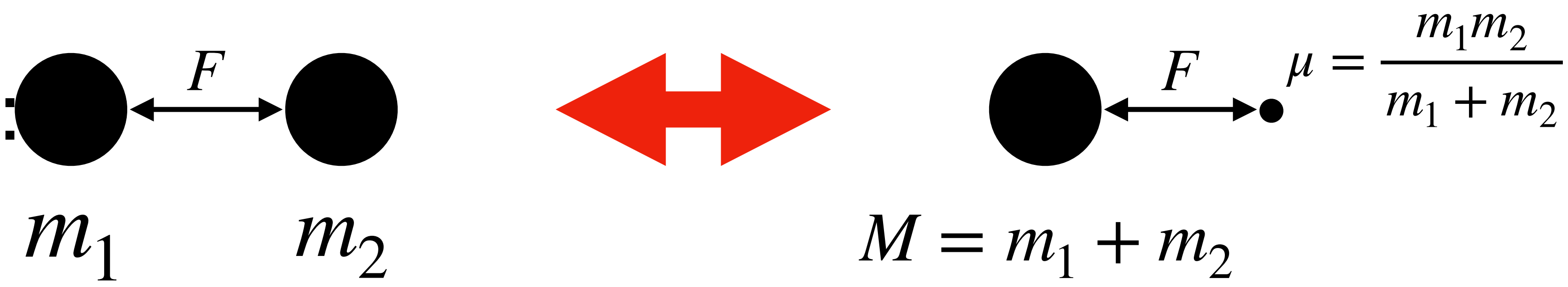
(source : Luc Blanchet)

Note the slow convergence !

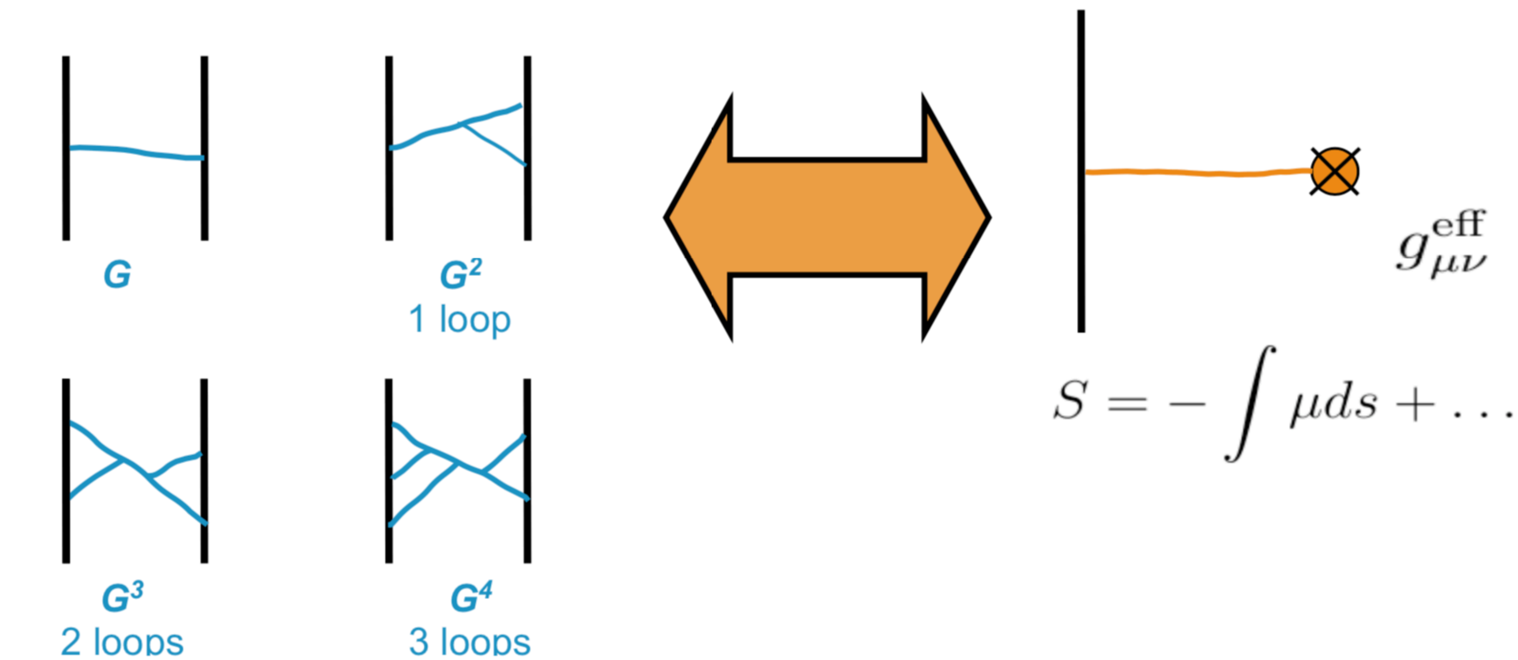
Effective One-Body (Buonanno-Damour 98)

We want to resum as much as possible the (badly convergent) PN expansion

Basic idea :



Real dynamics | **Effective dynamics**



$$H = H_0 + \left(G H_1 + \frac{G^2}{c^2} H_2 + \frac{G^3}{c^4} H_3 + \frac{G^4}{c^6} H_4 \right) \left(1 + \frac{1}{c^2} + \ldots \right)$$

$$A(R) = 1 - \frac{2GM}{c^2\,R} + 2\nu \left(\frac{GM}{c^2\,R} \right)^3 + \cdots ,$$

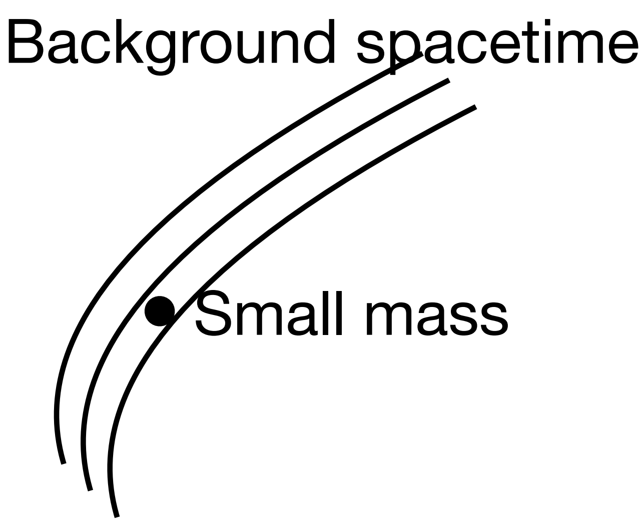
$$B(R) = 1 + \frac{2GM}{c^2\,R} + (4 - 6\nu) \left(\frac{GM}{c^2\,R} \right)^2 + \cdots$$

Effective metric

$$ds^2_{\text{eff}} = -A(r)dt^2 + B(r)dr^2 + r^2 \left(d\theta^2 + \sin^2\theta d\varphi^2 \right)$$

Gravitational self-force

EMRI : find the waveform as an expansion in $\epsilon = m/M$. This is far from being achieved.



The tensorial form of the equations of motion is

$$\frac{Du^\mu}{d\tau} = -\frac{1}{2}\left(g^{\mu\nu} + u^\mu u^\nu\right)\left(2h^{\text{tail}}_{\nu\lambda\rho} - h^{\text{tail}}_{\lambda\rho\nu}\right)u^\lambda u^\rho$$

Deviation from geodesics

where

$$h^{\text{tail}}_{\mu\nu\lambda} = 4m\int_{-\infty}^{\tau-\epsilon} \nabla_\lambda\left(G_{\mu\nu\mu'\nu'} - \frac{1}{2}g_{\mu\nu}G^{\rho}{}_{\rho\mu'\nu'}\right)\left(z(\tau), z(\tau')\right)u^{\mu'}u^{\nu'}\,d\tau'$$

(MiSaTaQuWa equations)

Mathematically challenging to go to higher orders, but $\mathcal{O}(\epsilon^3)$ is needed !

Conclusions

- We will now be able for the first time to observe the universe with another channel than photons !
- We begin to observe the close universe, and we will observe it to cosmological distances in few decades
- Modeling of the signal is analytically and numerically challenging

Il n’y a pas de problèmes résolus, seulement des problèmes plus ou moins résolus