Testing gravity with the two-body problem

ADRIEN KUNTZ

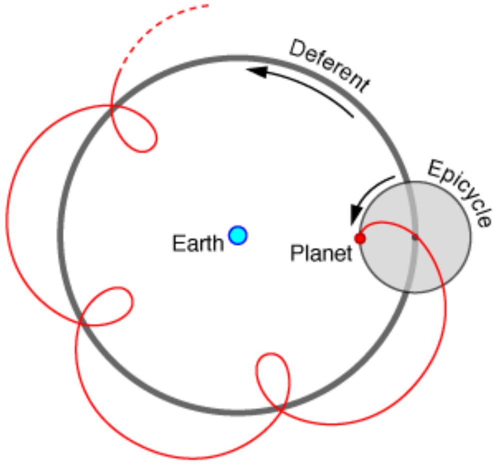
17/09/2020

Aix*Marseille université
Initiative d'excellence

CPT

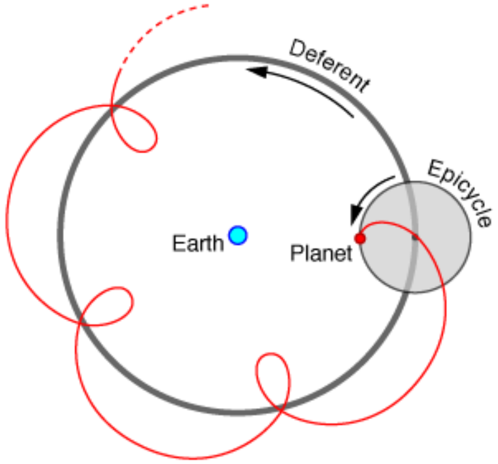
A LONG AND RICH HISTORY…

THE GREEKS…

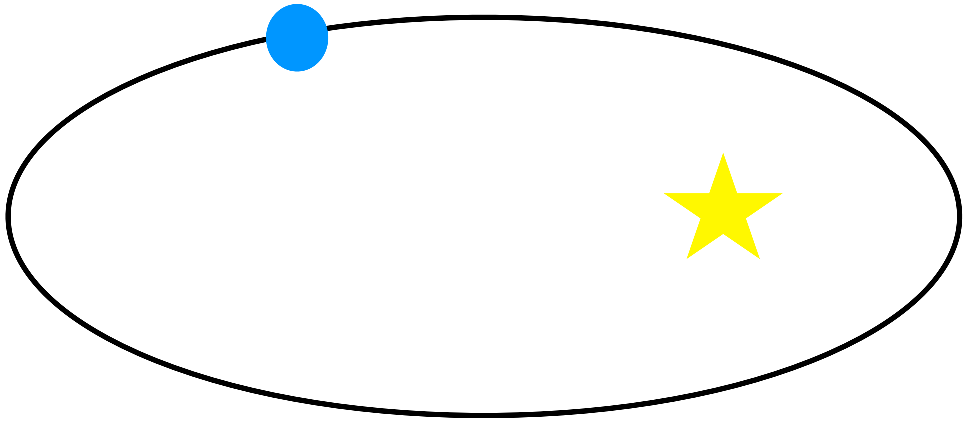


A LONG AND RICH HISTORY…

THE GREEKS…

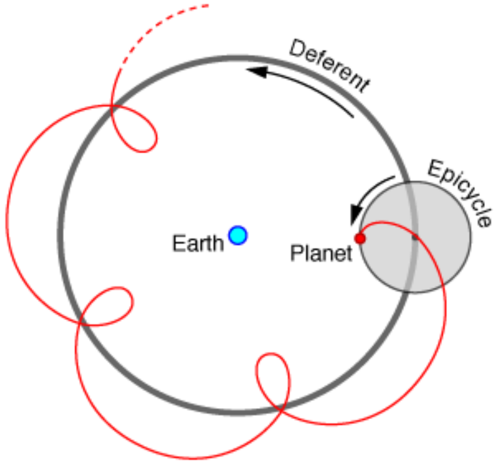


KEPLER…

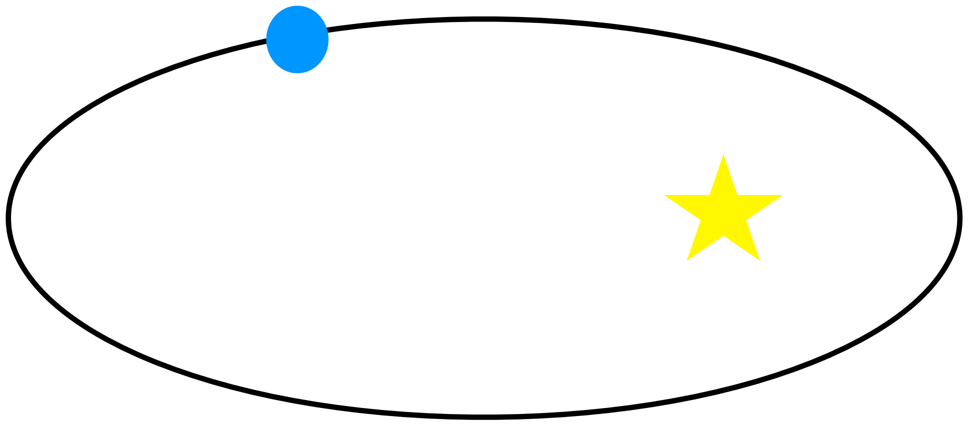


A LONG AND RICH HISTORY…

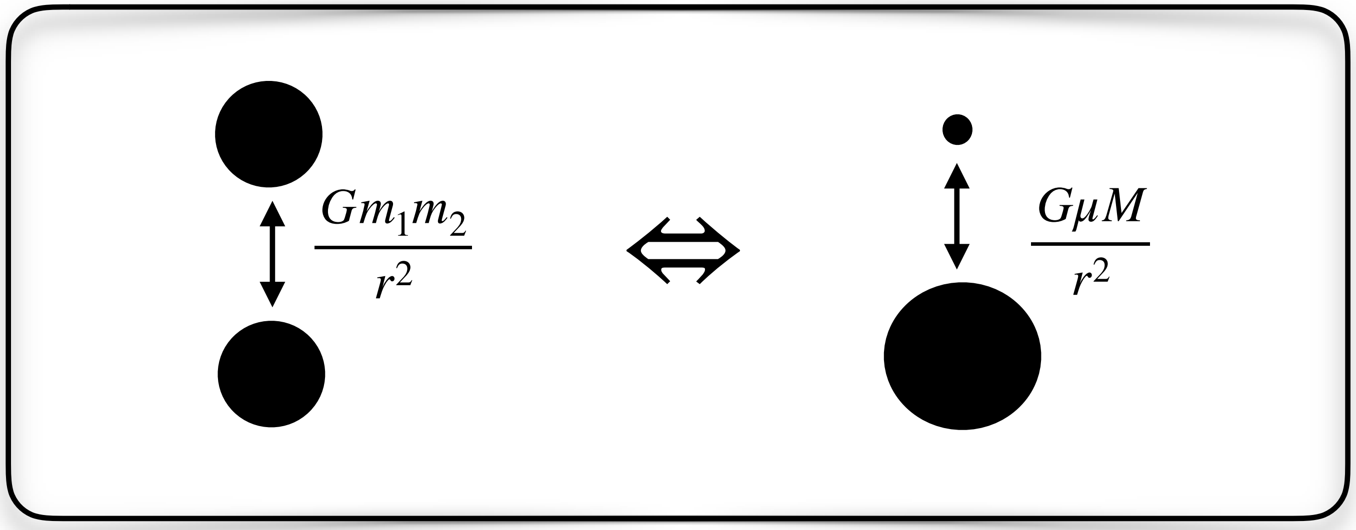
THE GREEKS…



KEPLER…



NEWTON…

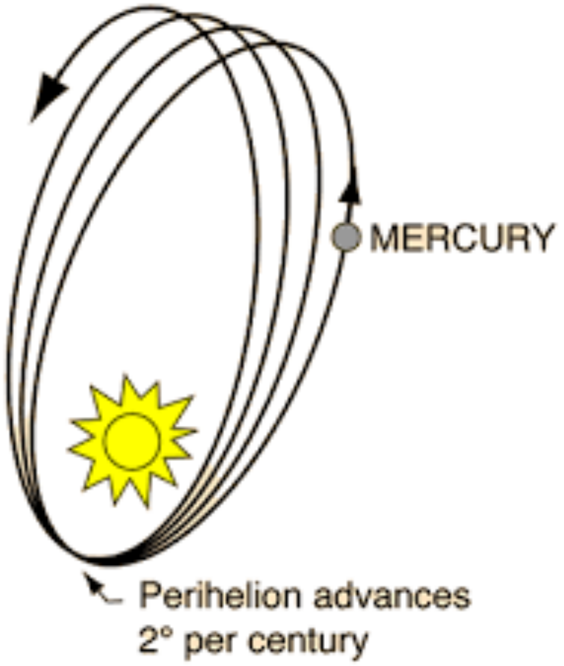


A LONG AND RICH HISTORY…

EINSTEIN'S GENERAL RELATIVITY (GR)

Post-Newtonian force

$$\mathbf{F} = -\frac{Gm_1m_2}{r^3}\mathbf{r}\left[1 + \mathcal{O}\left(\frac{v^2}{c^2}\right)\right]$$

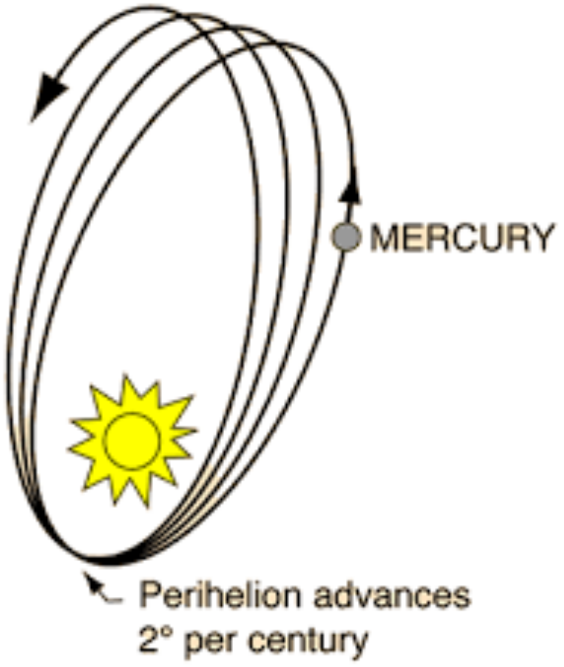


A LONG AND RICH HISTORY…

EINSTEIN'S GENERAL RELATIVITY (GR)

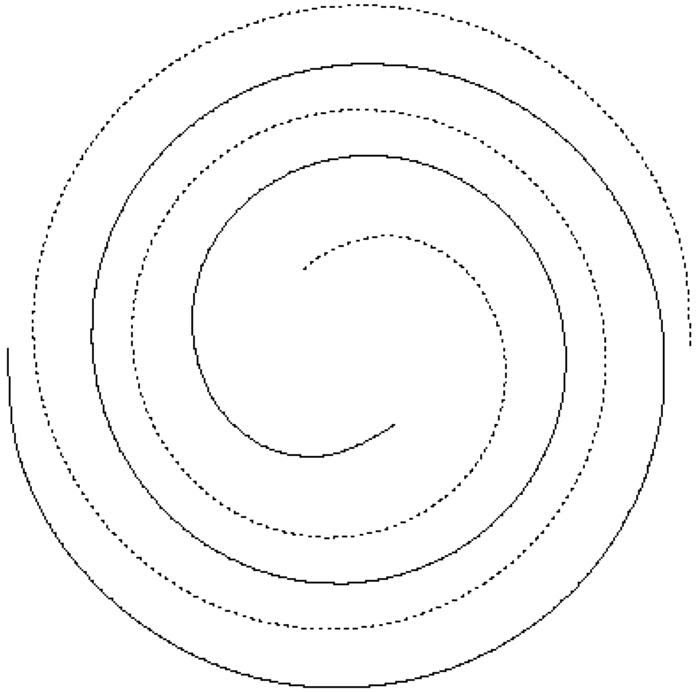
Post-Newtonian force

$$\mathbf{F} = -\frac{Gm_1m_2}{r^3}\mathbf{r}\left[1 + \mathcal{O}\left(\frac{v^2}{c^2}\right)\right]$$



Gravitational Waves (GW)

$$P = \frac{G}{5}\left\langle \dddot{Q}^{kl}\dddot{Q}_{kl} \right\rangle$$



A LONG AND RICH HISTORY…

THE SOUND OF GRAVITATIONAL WAVES

PROBLEMATIC

What can we learn from gravity using two-body trajectories ? Where should we look for new physics, and which parameters control possible deviations from GR ?

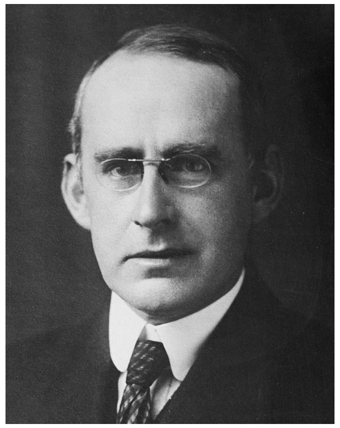
$\Rightarrow$ EFFECTIVE FIELD THEORY (EFT) ideas are crucial

PROBLEMATIC

What can we learn from gravity using two-body trajectories ? Where should we look for new physics, and which parameters control possible deviations from GR ?

$\Rightarrow$ EFFECTIVE FIELD THEORY (EFT) ideas are crucial

A simple example : Eddington parameters



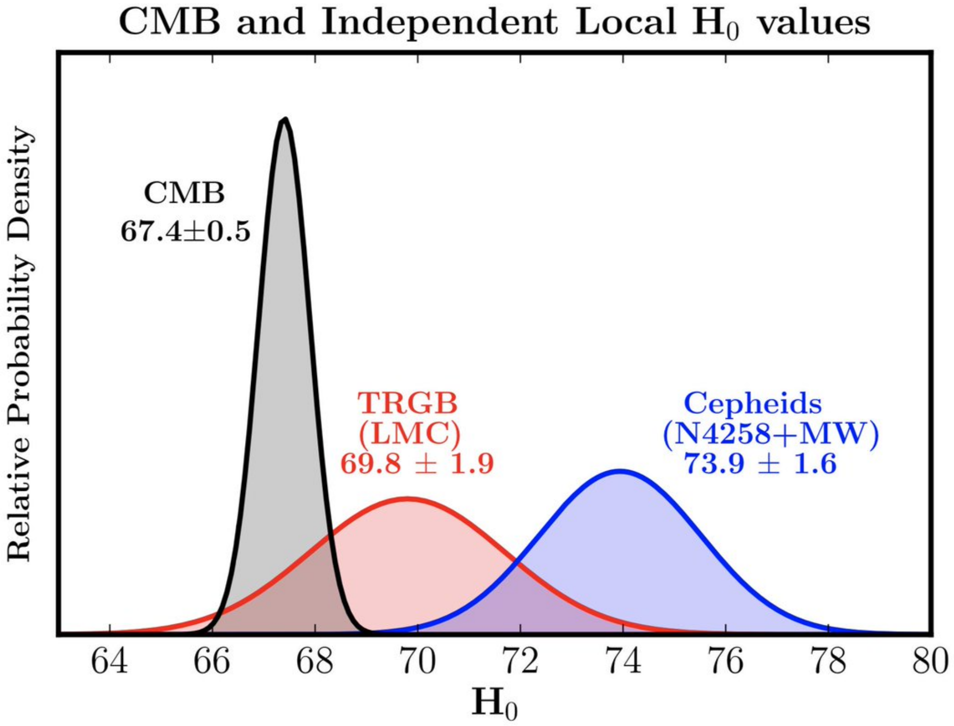
$$g_{\mu\nu}\mathrm{d}x^{\mu}\mathrm{d}x^{\nu} \simeq -\left(1-\frac{2GM}{r}+\beta\frac{2G^2M^2}{r^2}+\dots\right)\mathrm{d}t^2+\left(1+\gamma\frac{2GM}{r}+\dots\right)(\mathrm{d}x^2+\mathrm{d}y^2+\mathrm{d}z^2)\ .$$

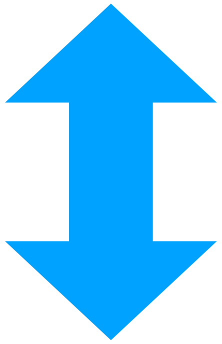
Today's constraints : $|\gamma-1| \lesssim 2\times 10^{-5}$ $|\beta-1| \lesssim 8\times 10^{-5}$

WHY MODIFY GRAVITY ?

COSMOLOGICAL CONSTANT PROBLEM

HUBBLE TENSION

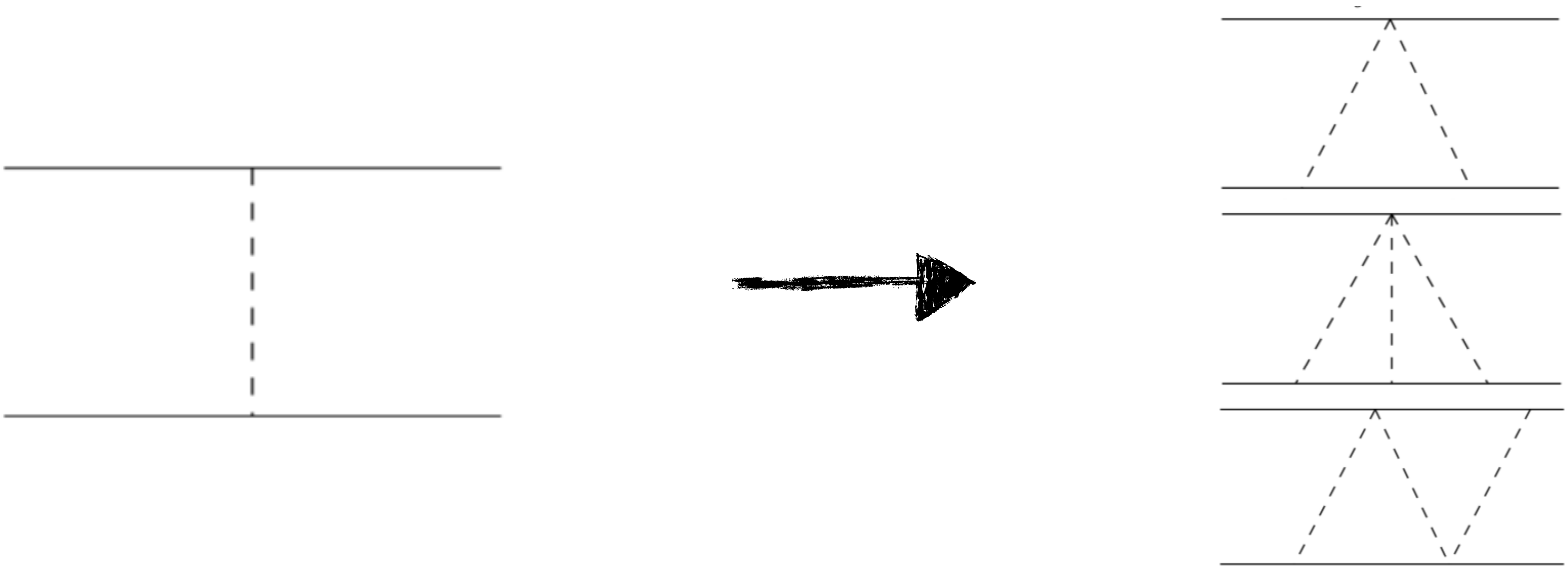




SCALAR-TENSOR THEORIES: $g_{\mu\nu} \quad + \quad \varphi$

PLAN

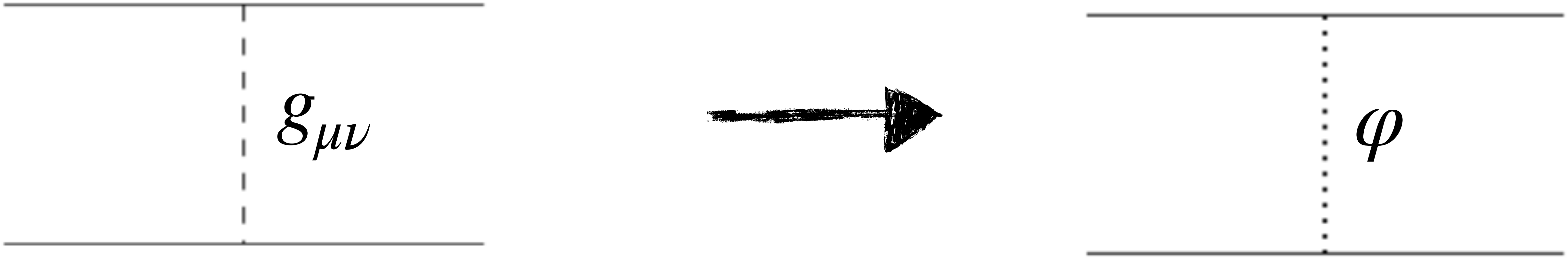
1. THE TWO-BODY PROBLEM IN GR : AN EFT APPROACH



PLAN

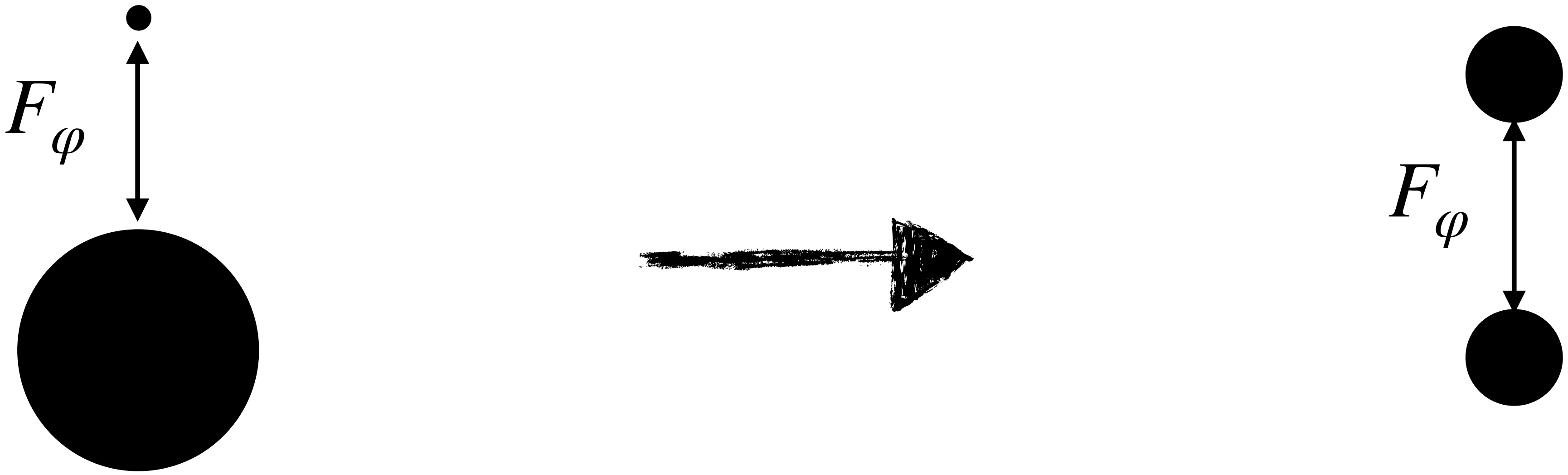
1. THE TWO-BODY PROBLEM IN GR : AN EFT APPROACH

2. THE TWO-BODY PROBLEM IN SCALAR-TENSOR THEORIES



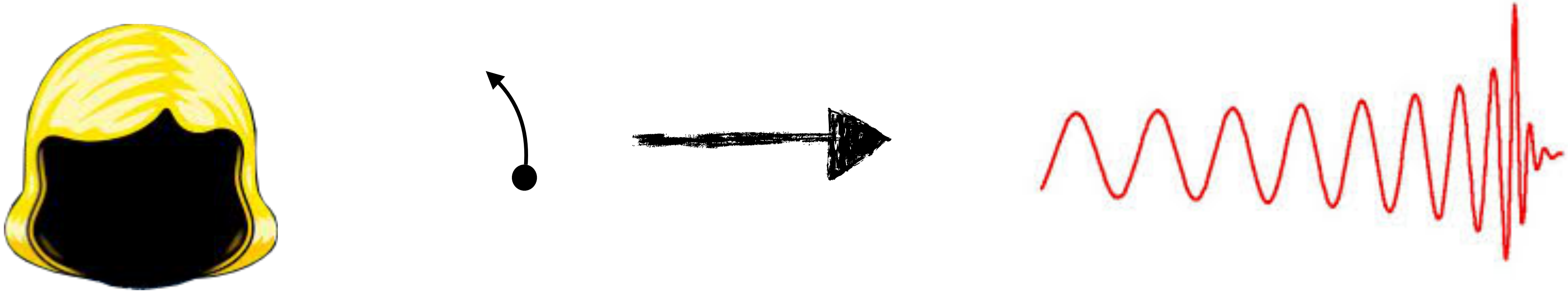
PLAN

1. THE TWO-BODY PROBLEM IN GR : AN EFT APPROACH
2. THE TWO-BODY PROBLEM IN SCALAR-TENSOR THEORIES
3. TWO-BODY PROBLEM AND SCREENING MECHANISMS



PLAN

1. THE TWO-BODY PROBLEM IN GR : AN EFT APPROACH
2. THE TWO-BODY PROBLEM IN SCALAR-TENSOR THEORIES
3. TWO-BODY PROBLEM AND SCREENING MECHANISMS
4. EXTREME MASS RATIO INSPIRALS AND SCALAR HAIR



PLAN

1. THE TWO-BODY PROBLEM IN GR : AN EFT APPROACH

   A. Kuntz (PRD) 20

2. THE TWO-BODY PROBLEM IN SCALAR-TENSOR THEORIES

   A. Kuntz, F. Piazza, F. Vernizzi (JCAP) 19

   P. Brax, AC. Davis, A. Kuntz (PRD) 19

3. TWO-BODY PROBLEM AND SCREENING MECHANISMS

   A. Kuntz (PRD) 19

   P. Brax, L. Heisenberg, A. Kuntz (JCAP) 20

4. EXTREME MASS RATIO INSPIRALS AND SCALAR HAIR

   A. Kuntz, R. Penco, F. Piazza (JCAP) 20

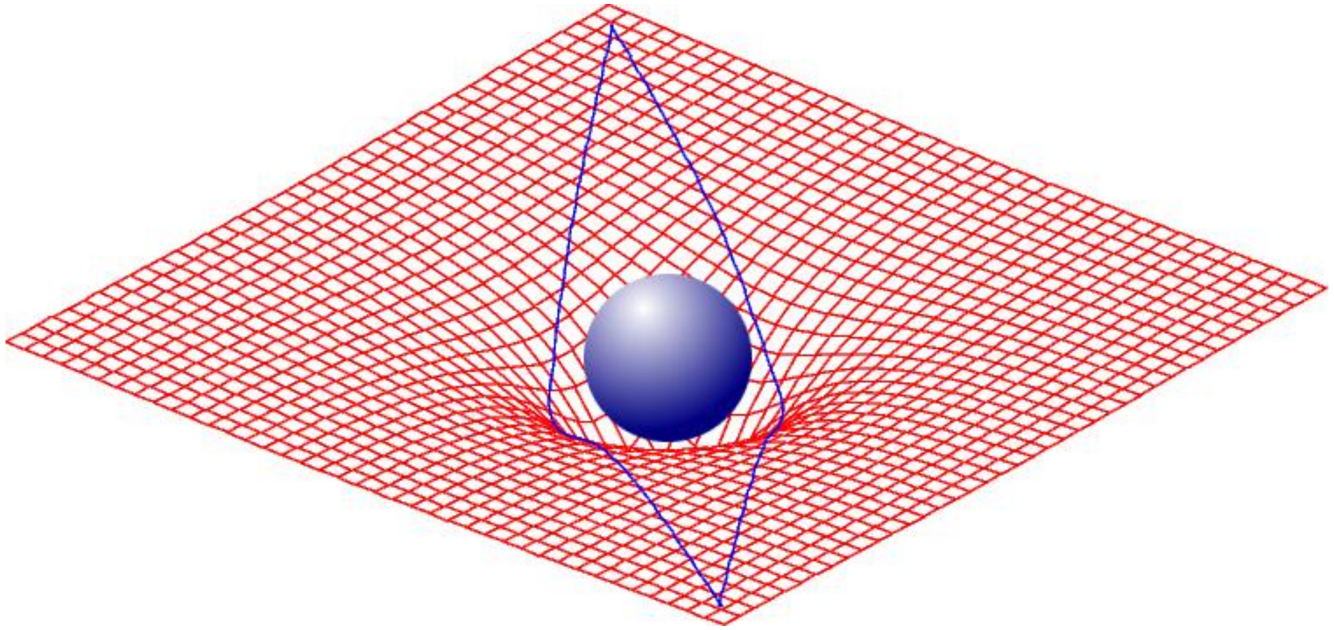
THE TWO-BODY PROBLEM IN GR

Basic ingredient of GR : the METRIC $g_{\mu\nu}$

Action principle (in vacuum) : $S_{\text{EH}} = \frac{M_P^2}{2}\int d^4x\sqrt{-g}R \quad \Rightarrow \quad G_{\mu\nu} = 0$

A POINT-PARTICLE in GR : $S_{\text{pp,A}} = -\, m_A\int d\tau_A = -\, m_A\int dt\sqrt{-g_{\mu\nu}\frac{dx_A^\mu}{dt}\frac{dx_A^\nu}{dt}}$

$$\Downarrow$$



$$\frac{d^2x_A^\mu}{d\tau^2} + \Gamma^\mu_{\nu\rho}\frac{dx_A^\nu}{d\tau}\frac{dx_A^\rho}{d\tau} = 0$$

THE TWO-BODY PROBLEM IN GR

EFT approach : use field theory tools

Goldberger and Rothstein 06
Porto 06
+ many others…

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

$$\mathcal{O}(v^2) = \mathcal{O}\left(\frac{GM}{r}\right) \ll 1$$

THE TWO-BODY PROBLEM IN GR

EFT approach : use field theory tools

Goldberger and Rothstein 06
Porto 06
+ many others…

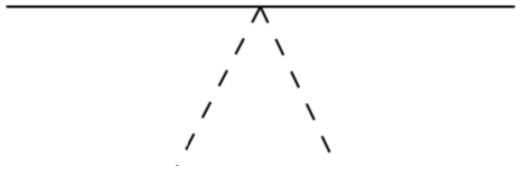
$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \quad \Rightarrow \quad S = S^{(2)} + S_{\rm int}$$

GREEN FUNCTION or PROPAGATOR:

$$S^{(2)} = -\frac{1}{8}\int d^4x \left[-\frac{1}{2}(\partial_\mu h^\alpha_\alpha)^2 + (\partial_\mu h_{\nu\rho})^2\right]$$

INTERACTION VERTEX:

$$S_{\rm int} \supset m\int {\rm d}t\; h_{00}\;, \qquad m\int {\rm d}t\; h_{00}^2\;, \qquad \int {\rm d}^4x\; \partial^2 h^3$$



THE TWO-BODY PROBLEM IN GR

The two-body dynamics is encoded in the EFFECTIVE ACTION :

$$e^{iS_{\mathrm{eff}}[\mathbf{x}_1(t),\mathbf{x}_2(t)]} = \int \mathscr{D}h_{\mu\nu} e^{iS[\mathbf{x}_1(t),\mathbf{x}_2(t),h_{\mu\nu}]}$$

REAL PART: CONSERVATIVE

IMAGINARY PART: DISSIPATIVE

THE TWO-BODY PROBLEM IN GR

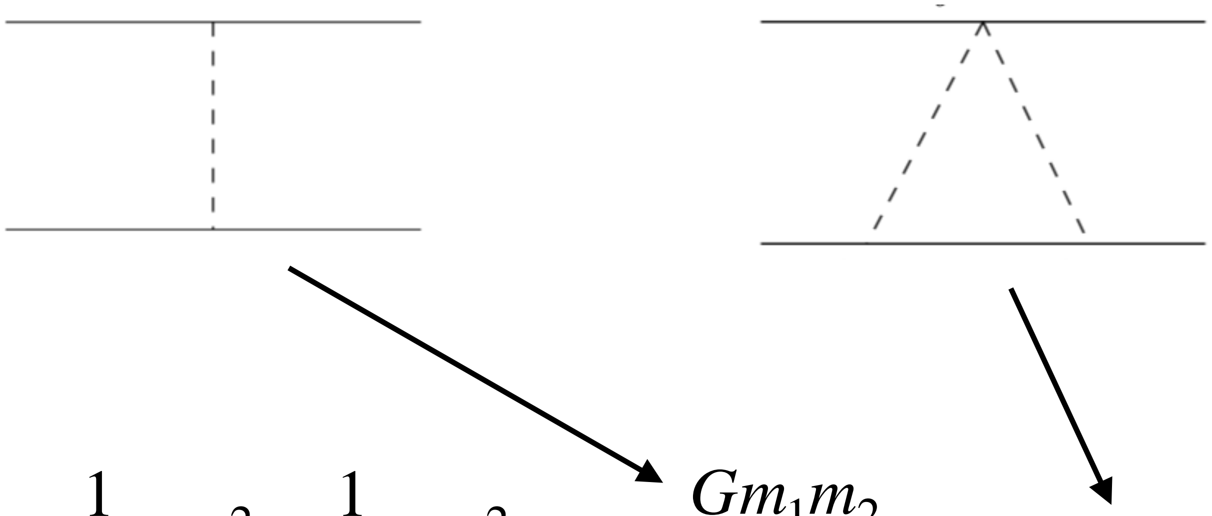
The two-body dynamics is encoded in the EFFECTIVE ACTION :

$$e^{iS_{\text{eff}}[\mathbf{x}_1(t),\mathbf{x}_2(t)]} = \int \mathscr{D}h_{\mu\nu} e^{iS[\mathbf{x}_1(t),\mathbf{x}_2(t),h_{\mu\nu}]}$$

REAL PART: CONSERVATIVE

IMAGINARY PART: DISSIPATIVE

$$\mathfrak{R}(S_{\text{eff}}) = \int dt L[\mathbf{x}_A, \mathbf{v}_A]$$



$$L \quad = \frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2 + \frac{G m_1 m_2}{|\,\mathbf{x_1}(t) - \mathbf{x_2}(t)\,|} + L_{1\text{PN}} + \dots$$

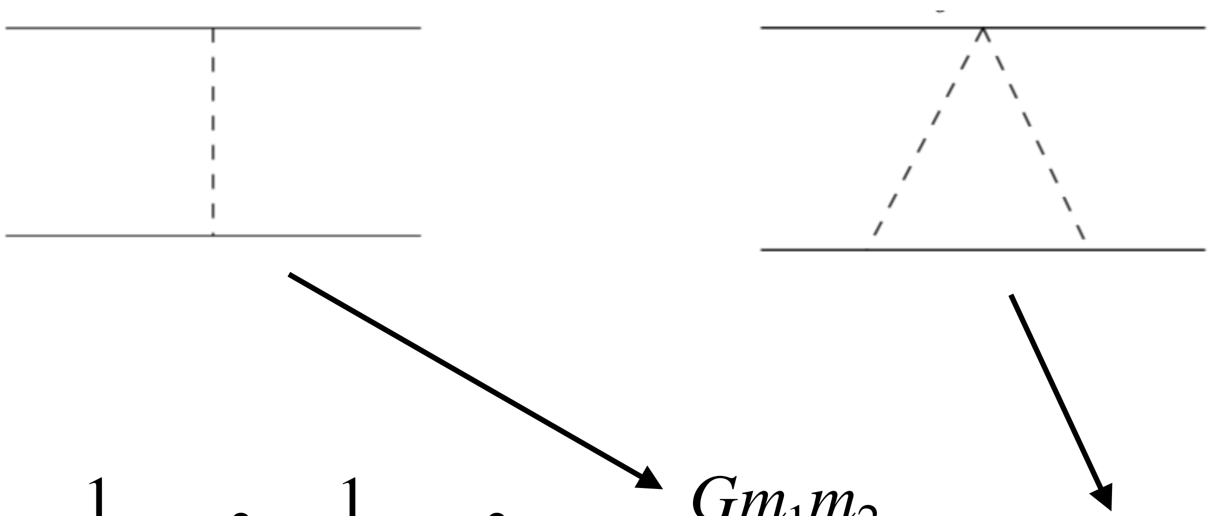
THE TWO-BODY PROBLEM IN GR

The two-body dynamics is encoded in the EFFECTIVE ACTION :

$$e^{iS_{\text{eff}}[\mathbf{x}_1(t),\mathbf{x}_2(t)]} = \int \mathcal{D}h_{\mu\nu} e^{iS[\mathbf{x}_1(t),\mathbf{x}_2(t),h_{\mu\nu}]}$$

REAL PART: CONSERVATIVE

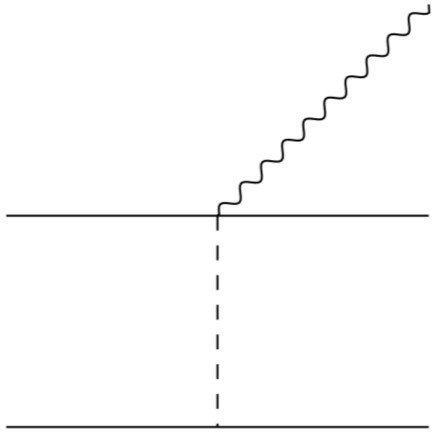
$$\mathfrak{R}(S_{\text{eff}}) = \int dt L[\mathbf{x}_A, \mathbf{v}_A]$$



$$L \quad = \frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2 + \frac{G m_1 m_2}{|\mathbf{x_1}(t) - \mathbf{x_2}(t)|} + L_{1\text{PN}} + \ldots$$

IMAGINARY PART: DISSIPATIVE

$$\mathfrak{I}(S_{eff}) = \frac{T}{2}\int dE d\Omega \frac{d^2\Gamma}{dE d\Omega}$$



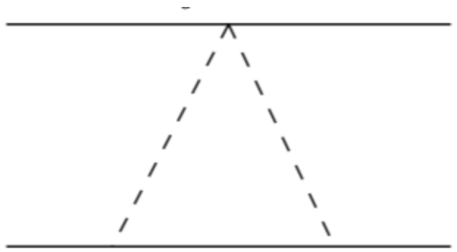
$$P = \frac{G}{5}\left\langle \dddot{Q}^{kl}\dddot{Q}_{kl}\right\rangle + \ldots$$

A RESUMMATION TECHNIQUE

A. Kuntz (PRD) 20

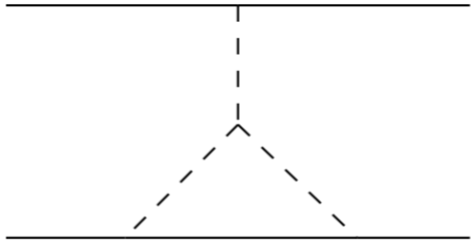
In the 1PN potential enter two types of vertex

$$S_{\rm pp} \supset \int \mathrm{d}t\, h_{00}^2$$



$$= \frac{G^2 m_1 m_2^2}{2r^2}$$

$$S_{\rm EH} \supset \int \mathrm{d}^4x\, \partial^2 h^3$$



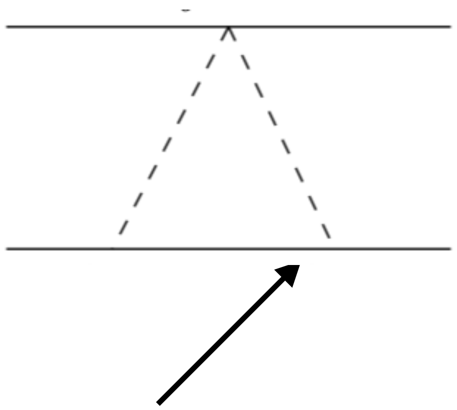
$$= -\frac{G^2 m_1 m_2^2}{r^2}$$

A RESUMMATION TECHNIQUE

A. Kuntz (PRD) 20

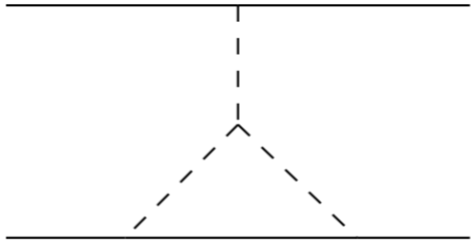
In the 1PN potential enter two types of vertex

$$S_{\rm pp} \supset \int \mathrm{d}t \; h_{00}^2$$



$$= \frac{G^2 m_1 m_2^2}{2r^2}$$

$$S_{\rm EH} \supset \int \mathrm{d}^4x \; \partial^2 h^3$$



$$= -\frac{G^2 m_1 m_2^2}{r^2}$$

The first one can be resummed exactly !

$$S_{\rm pp,A} = -\, m_A \int dt \sqrt{-g_{\mu\nu} v_A^\mu v_A^\nu} \quad \Leftrightarrow \quad S_{\rm pp,A} = -\frac{m_A}{2} \int \mathrm{d}t \left[e_A - \frac{g_{\mu\nu} v_A^\mu v_A^\nu}{e_A} \right]$$

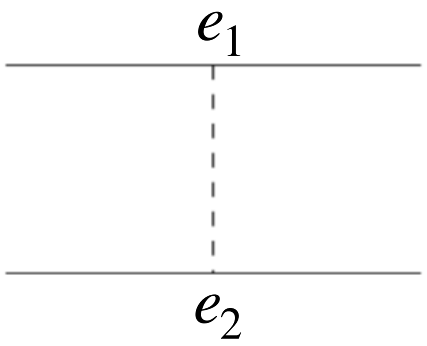
with $e_A = \sqrt{-g_{\mu\nu} v_A^\mu v_A^\nu}$

. . . The worldline couplings are now LINEAR

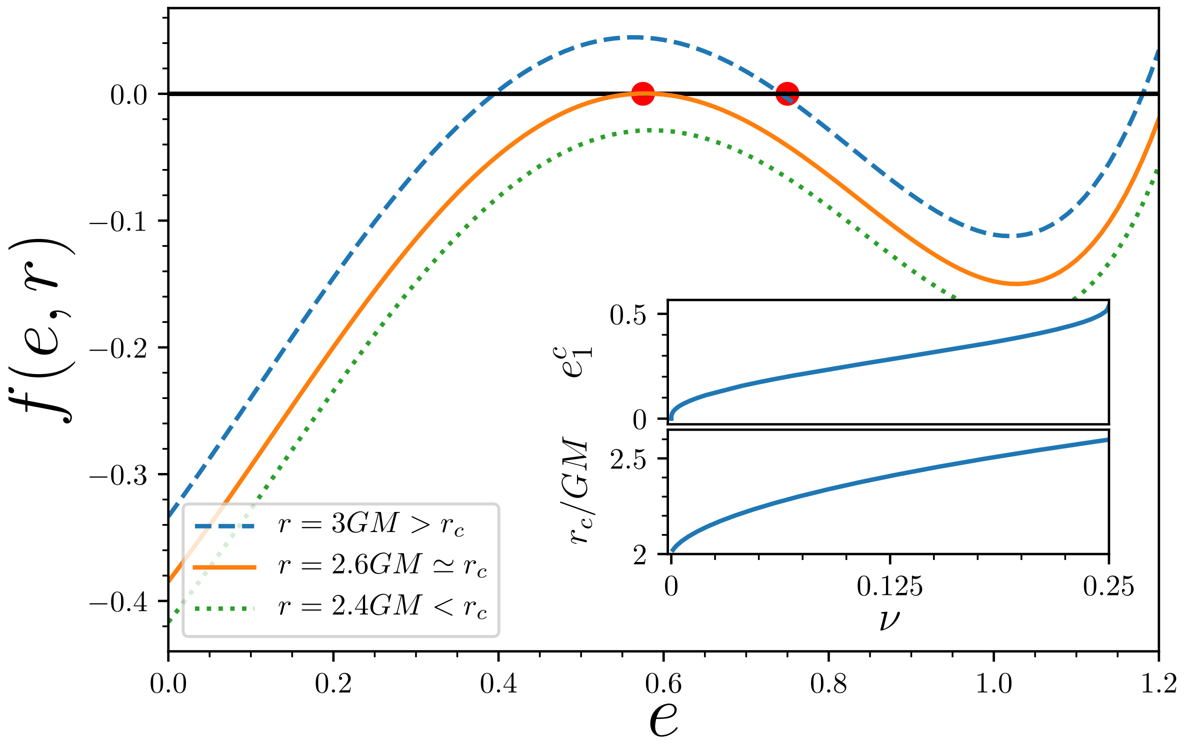
WORLDLINE PARAMETERS

A. Kuntz (PRD) 20

e_1, e_2 obey an interesting quintic equation. In the static case : $e_A = \sqrt{-g_{00}}$



$$\longrightarrow \qquad f_1(e_1, r) \equiv (e_1^2 - 1)^2\left(e_1 - \frac{2Gm_1}{r}\right) - \frac{4G^2m_2^2e_1}{r^2} = 0$$



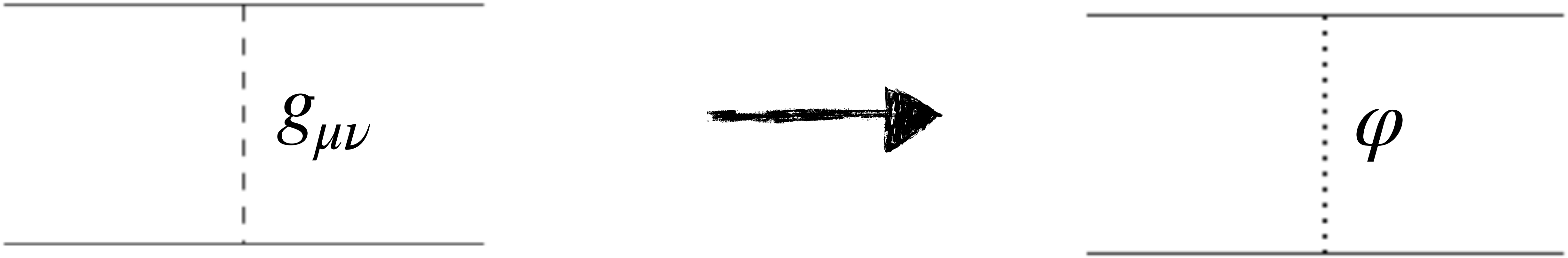
They define an ‘effective two-body horizon’ !

This can be generalised to gauge-invariant quantities for circular orbits

PLAN

1. THE TWO-BODY PROBLEM IN GR : AN EFT APPROACH

2. THE TWO-BODY PROBLEM IN SCALAR-TENSOR THEORIES



MODIFYING GR : SCALAR-TENSOR THEORIES

GR action: $S = \frac{M_P^2}{2}\int d^4x\sqrt{-g}R + S_m[g_{\mu\nu},\psi_i]$

A simple alternative to GR: $g_{\mu\nu} \quad + \quad \varphi$

$$S_\varphi = -\frac{1}{2}\int d^4x\sqrt{-g}\,g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi$$

Coupling of φ with matter, compatible with causality and equivalence principle:

$$S_m[\tilde{g}_{\mu\nu},\psi_i] \quad \text{with} \quad \tilde{g}_{\mu\nu} = \underset{\text{Conformal}}{\underset{\uparrow}{A(\varphi,X)}}g_{\mu\nu} + \underset{\text{Disformal}}{\underset{\nwarrow}{B(\varphi,X)}}\partial_\mu\varphi\partial_\nu\varphi \qquad X = g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi$$

Bekenstein 92

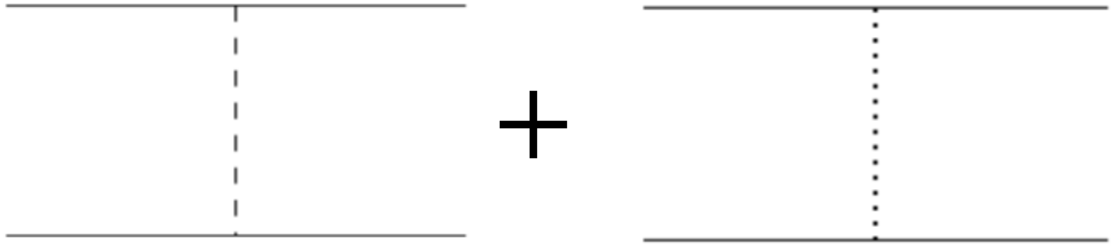
CONFORMAL COUPLING

Focus first on $\tilde{g}_{\mu\nu} = A(\varphi) g_{\mu\nu}$

A. Kuntz, F. Piazza, F. Vernizzi (JCAP) 19

$$S_{\rm pp} = -\int d\tau_A \, m_A(\varphi) = m_A \int d\tau \left(-1 + \alpha_A \frac{\varphi}{M_P} + \delta_A \left(\frac{\varphi}{M_P} \right)^2 + \ldots \right)$$

CONSERVATIVE



$$\tilde{G}_N = G_N \left(1 + 2\alpha_1 \alpha_2 \right)$$

DISSIPATIVE

Scalar-tensor theories

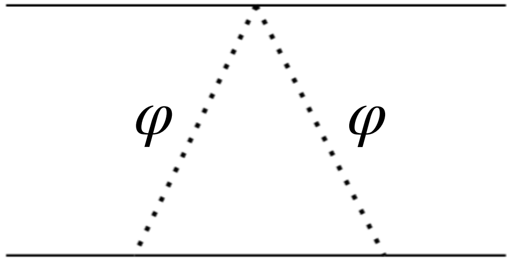
CONFORMAL COUPLING

Focus first on $\tilde{g}_{\mu\nu} = A(\varphi) g_{\mu\nu}$

A. Kuntz, F. Piazza, F. Vernizzi (JCAP) 19

$$S_{\rm pp} = -\int d\tau_A \, m_A(\varphi) = m_A \int d\tau \left(-1 + \alpha_A \frac{\varphi}{M_P} + \delta_A \left(\frac{\varphi}{M_P} \right)^2 + \ldots \right)$$

CONSERVATIVE

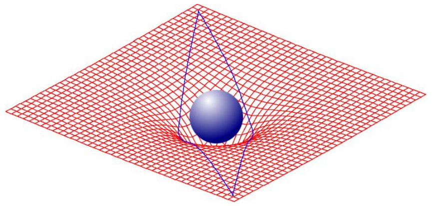


$+ \ldots$

PPN parameters :

$$\gamma_{AB} = 1 - 4 \frac{\alpha_A \alpha_B}{1 + 2\alpha_A \alpha_B}$$

$$\beta_{AB} = 1 - 2 \frac{\alpha_A^2 \alpha_B^2 + f_{AB}}{(1 + 2\alpha_A \alpha_B)^2}$$



DISSIPATIVE

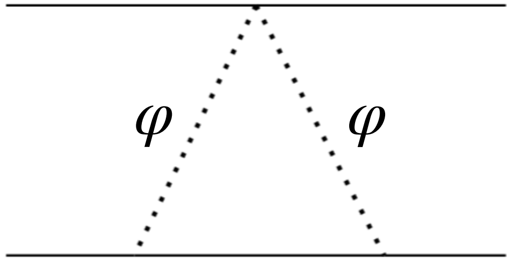
CONFORMAL COUPLING

Focus first on $\tilde{g}_{\mu\nu} = A(\varphi) g_{\mu\nu}$

A. Kuntz, F. Piazza, F. Vernizzi (JCAP) 19

$$S_{\rm pp} = -\int d\tau_A \, m_A(\varphi) = m_A \int d\tau \left(-1 + \alpha_A \frac{\varphi}{M_P} + \delta_A \left(\frac{\varphi}{M_P} \right)^2 + \dots \right)$$

CONSERVATIVE

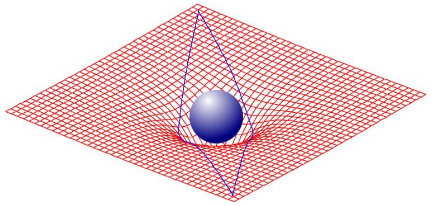


$+\dots$

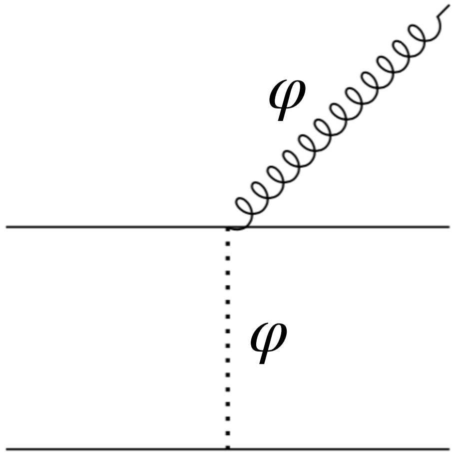
PPN parameters :

$$\gamma_{AB} = 1 - 4 \frac{\alpha_A \alpha_B}{1 + 2\alpha_A \alpha_B}$$

$$\beta_{AB} = 1 - 2 \frac{\alpha_A^2 \alpha_B^2 + f_{AB}}{(1 + 2\alpha_A \alpha_B)^2}$$



DISSIPATIVE



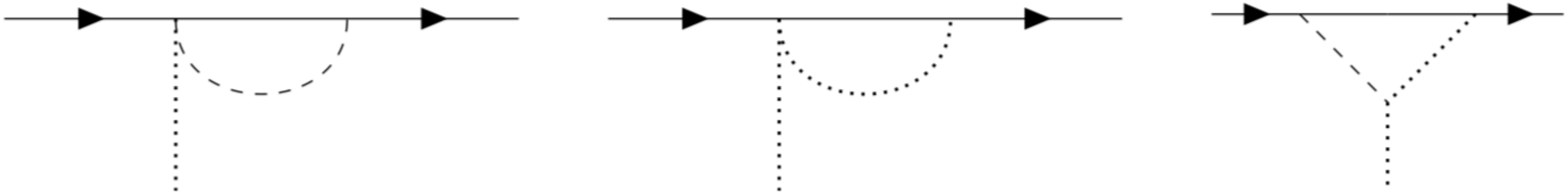
$$P_\phi = 2G_N \left(\left\langle \dot{I}_\phi{}^2 \right\rangle + \frac{1}{3} \left\langle \ddot{I}^i_\phi{}^2 \right\rangle + \frac{1}{30} \left\langle \dddot{I}^{ij}_\phi{}^2 \right\rangle + \dots \right)$$

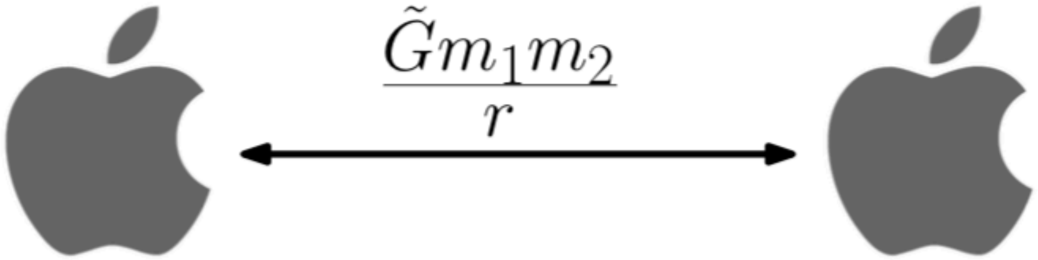
Monopole Dipole Quadrupole

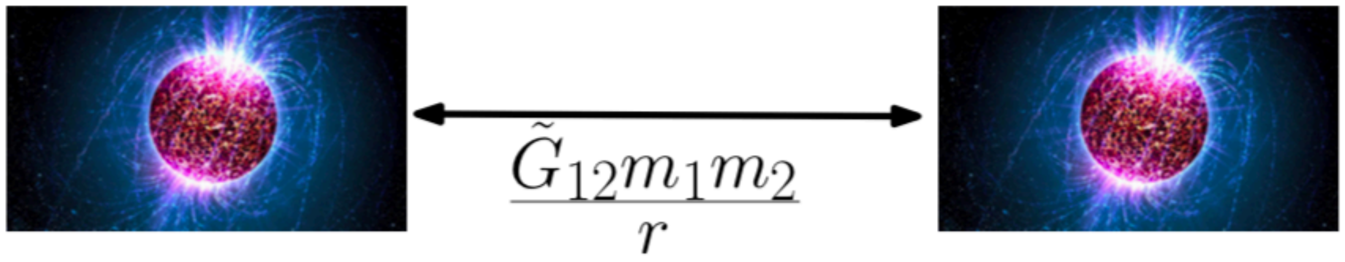
CHARGE RENORMALISATION

A. Kuntz, F. Piazza, F. Vernizzi (JCAP) 19

$$S_{\rm int} = -\int d\tau_A \, m_A(\varphi) = m_A \int d\tau \left(-1 + \alpha \frac{\varphi}{M_P} + \delta \left(\frac{\varphi}{M_P} \right)^2 + \ldots \right)$$







$$\tilde{G}_{12} = G_N \left(1 + 2\alpha_1 \alpha_2 \right)$$

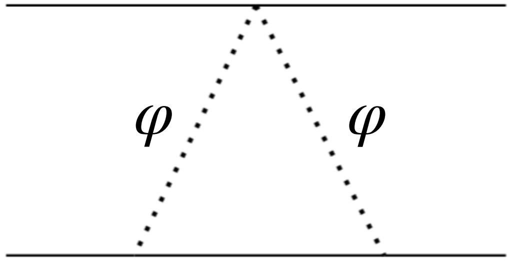
DISFORMAL COUPLING

$$\tilde{g}_{\mu\nu} = A(\varphi)g_{\mu\nu} + B(\varphi)\partial_\mu\varphi\partial_\nu\varphi$$

P. Brax, AC. Davis, A. Kuntz (PRD) 19

$$\Rightarrow \quad S_{\rm pp} = m_A \int d\tau \left(-1 + \alpha_A \frac{\varphi}{M_P} + \delta_A \left(\frac{\varphi}{M_P}\right)^2 + \ldots\right)\left(1 + \frac{1}{M^2 M_P^2}(\partial_\mu \varphi v_A^\mu)^2 + \ldots\right)$$

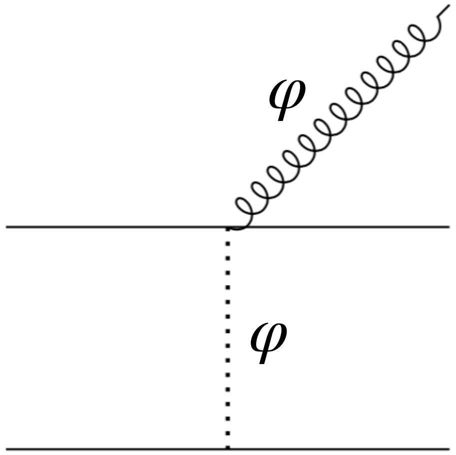
CONSERVATIVE



$+\ldots$

$$L_{\rm dis} = 4\alpha^2 b \frac{G^2 m_1 m_2 (m_1 + m_2)}{M^2}\left(\frac{d}{dt}\frac{1}{r}\right)^2$$

DISSIPATIVE



Monopole $$I_{\rm dis} = 8\alpha b \frac{G m_1 m_2}{M^2}\frac{d^2}{dt^2}\frac{1}{r}$$

$$r = |\mathbf{x_1}(t) - \mathbf{x_2}(t)|$$

DISFORMAL COUPLING

CIRCULAR TRAJECTORY

P. Brax, AC. Davis, A. Kuntz (PRD) 19

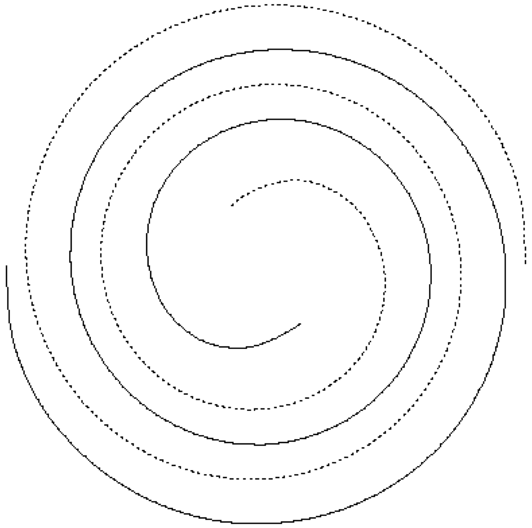
$$L_{\text{dis}} = 4\alpha^2 b \frac{G^2 m_1 m_2 (m_1 + m_2)}{M^2} \left(\frac{d}{dt} \frac{1}{r} \right)^2 \qquad I_{\text{dis}} = 8\alpha b \frac{G m_1 m_2}{M^2} \frac{d^2}{dt^2} \frac{1}{r}$$

For circular orbits : $\dot{r} = 0\,!$

No contribution of the disformal coupling. This is intuitive because :

$$\int d\tau (\partial_\mu \phi v_A^\mu)^2 = \int d\tau \left(\frac{d\phi}{d\tau} \right)^2$$

In this case I showed that only radiation reaction effects contribute



$$\Rightarrow \quad L_{\text{dis}} = \mathcal{O}(v^{14}), \quad I_{\text{dis}} = \mathcal{O}(v^{12})$$

DISFORMAL COUPLING

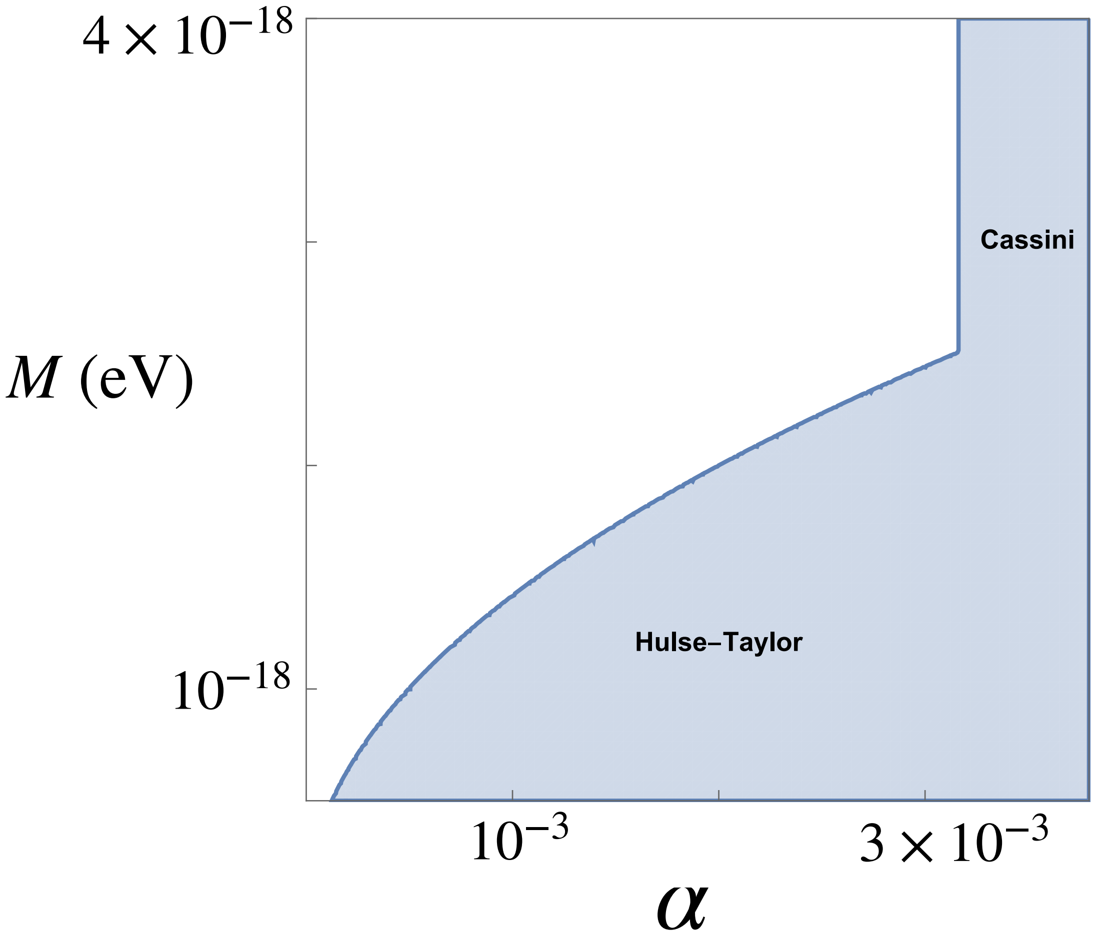
ELLIPTIC TRAJECTORY

P. Brax, AC. Davis, A. Kuntz (PRD) 19

Monopole $$I_{\rm dis} = 8\alpha b \frac{G m_1 m_2}{M^2} \frac{d^2}{dt^2} \frac{1}{r}$$

$$\Rightarrow \quad P_\phi^{\rm mono} \simeq \frac{64}{9G} \alpha^2 (G M_c \omega)^{10/3} \left[f_2(e) - 12 y f_3(e) + 36 y^2 f_4(e) \right]$$

$$y \simeq \left(\frac{\omega_{\rm Hulse-Taylor}}{M} \right)^2$$

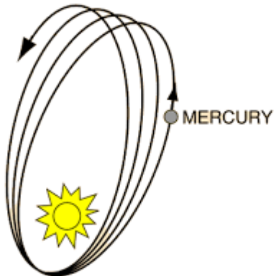


$$\omega_{\rm Hulse-Taylor} \sim 10^{-18} \text{ eV}$$

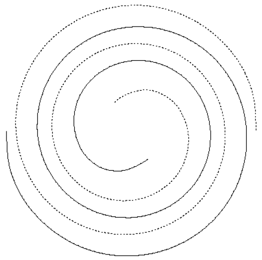
CONCLUSION PART 2

MAIN ASPECTS OF SCALAR-TENSOR THEORIES, WITH RESPECT TO GR:

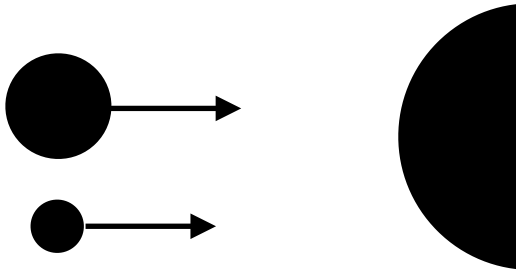
- Bending of light and perihelion is different



- Dipolar radiation



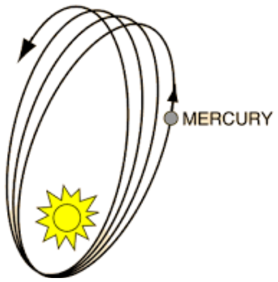
- Violations of the strong equivalence principle



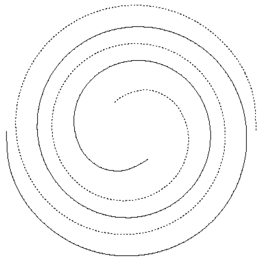
CONCLUSION PART 2

MAIN ASPECTS OF SCALAR-TENSOR THEORIES, WITH RESPECT TO GR:

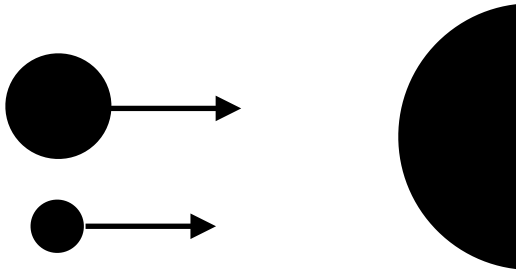
- Bending of light and perihelion is different



- Dipolar radiation



- Violations of the strong equivalence principle



Experimental tests are very stringent: the scalar coupling is small

$$\alpha \lesssim 10^{-2}$$

A screening mechanism could explain such a small value

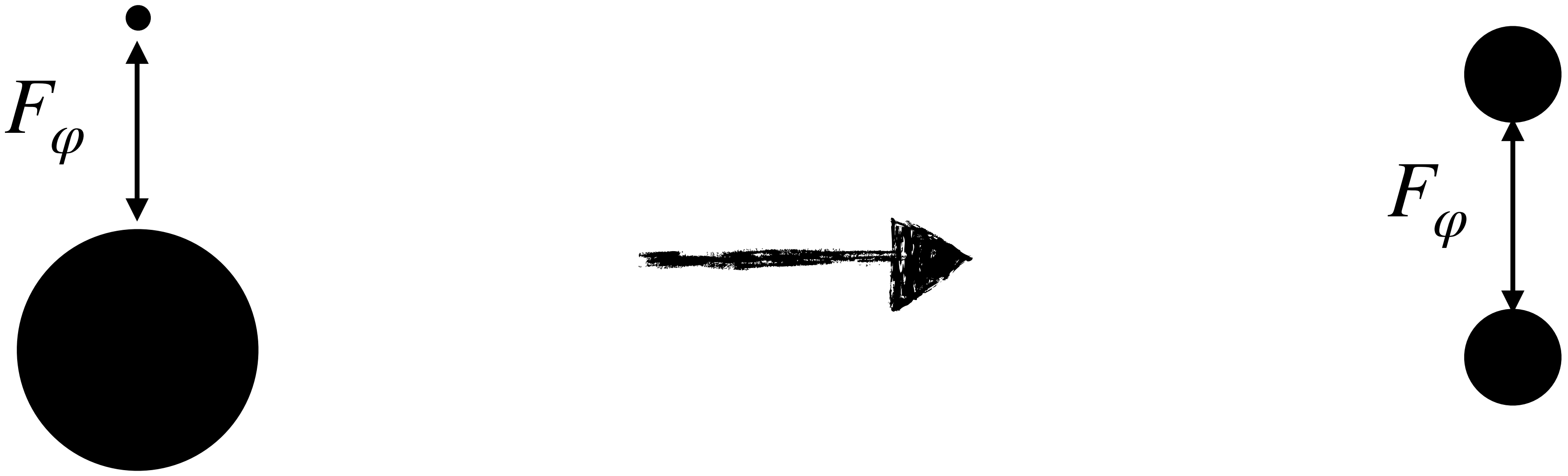
HOW TO FORMULATE THE TWO-BODY PROBLEM WITH A SCREENING MECHANISM?

PLAN

1. THE TWO-BODY PROBLEM IN GR : AN EFT APPROACH

2. THE TWO-BODY PROBLEM IN SCALAR-TENSOR THEORIES

3. TWO-BODY PROBLEM AND SCREENING MECHANISMS



K-MOUFLAGE SCREENING

$$S = \int d^4x \left[-\frac{(\partial\varphi)^2}{2} - \frac{1}{4\Lambda^4}(\partial\varphi)^4 + \frac{\varphi T}{M_P} \right]$$

For cosmological applications

$$\Lambda^2 \sim HM_P$$

Equation of motion around a static source:

$$\varphi_0' + \frac{\left(\varphi_0'\right)^3}{\Lambda^4} = \frac{M}{4\pi M_P r^2}$$

K-MOUFLAGE SCREENING

$$S = \int d^4x \left[-\frac{(\partial\varphi)^2}{2} - \frac{1}{4\Lambda^4}(\partial\varphi)^4 + \frac{\varphi T}{M_P} \right]$$

For cosmological applications

$$\Lambda^2 \sim H M_P$$

Equation of motion around a static source:

$$\varphi_0' + \frac{\left(\varphi_0'\right)^3}{\Lambda^4} = \frac{M}{4\pi M_P r^2}$$

$$\varphi_0'(r) \simeq \left(\frac{\Lambda^4 M}{4\pi M_P r^2} \right)^{1/3}$$

$$\varphi_0'(r) \simeq \frac{M}{4\pi M_P r^2}$$

$$\frac{\varphi_0}{\varphi_{\rm Newt}} \simeq \left(\frac{r}{r_*} \right)^{4/3}$$

$$\frac{\varphi_0}{\varphi_{\rm Newt}} \simeq 1$$

$$r_* = \left(\frac{M}{4\pi M_P \Lambda^2} \right)^{1/3}$$

(0.1 parsecs for the Sun)

K-MOUFLAGE SCREENING

$$S = \int d^4x \left[-\frac{(\partial\varphi)^2}{2} - \frac{1}{4\Lambda^4}(\partial\varphi)^4 + \frac{\varphi T}{M_P} \right]$$

For cosmological applications

$$\Lambda^2 \sim H M_P$$

Equation of motion around a static source:

$$\varphi_0' + \frac{\left(\varphi_0'\right)^3}{\Lambda^4} = \frac{M}{4\pi M_P r^2}$$

EFFECT ON THE PERIHELION

$$\frac{\varphi_0}{\varphi_{\rm Newt}} \sim \left(\frac{r}{r_*}\right)^{4/3} \leq 10^{-11}$$

L. Iorio 12

TWO-BODY PROBLEM

PERTURBATIVE EXPANSION BREAKS DOWN…

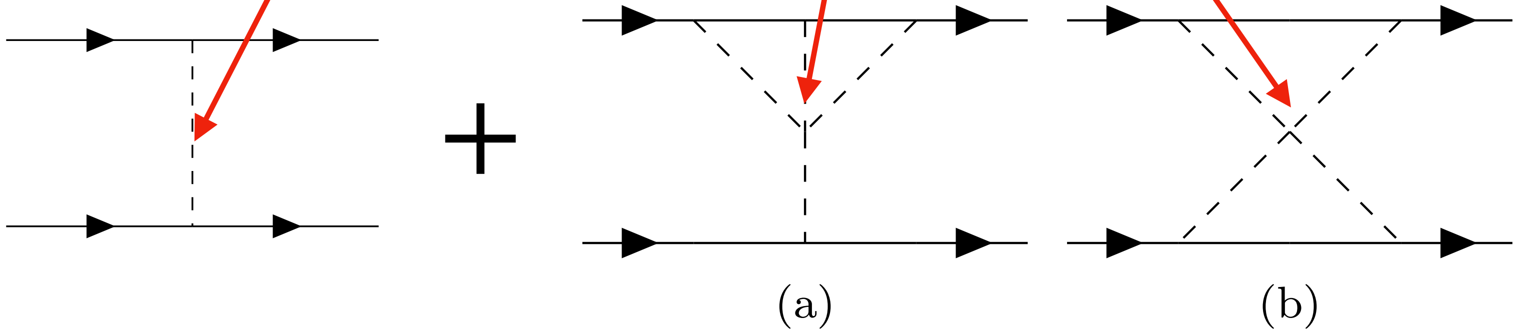
$$e^{iS_{\text{eff}}[\mathbf{x}_1,\mathbf{x}_2]} = \int \mathscr{D}[\varphi]\, e^{iS[\mathbf{x}_1,\mathbf{x}_2,\varphi]}$$

$$S_\varphi = \int \mathrm{d}^4x \left[-\frac{1}{2}(\partial\varphi)^2 - \frac{1}{4\Lambda^4}(\partial\varphi)^4 \right] + \frac{\varphi T}{M_P}$$

but

$$r < r_* \Leftrightarrow \frac{(\partial\varphi)^2}{\Lambda^4} \gg 1$$

$S_{\text{eff}} =$



$+\ldots$ DIVERGES

Screening mechanisms

TWO-BODY PROBLEM

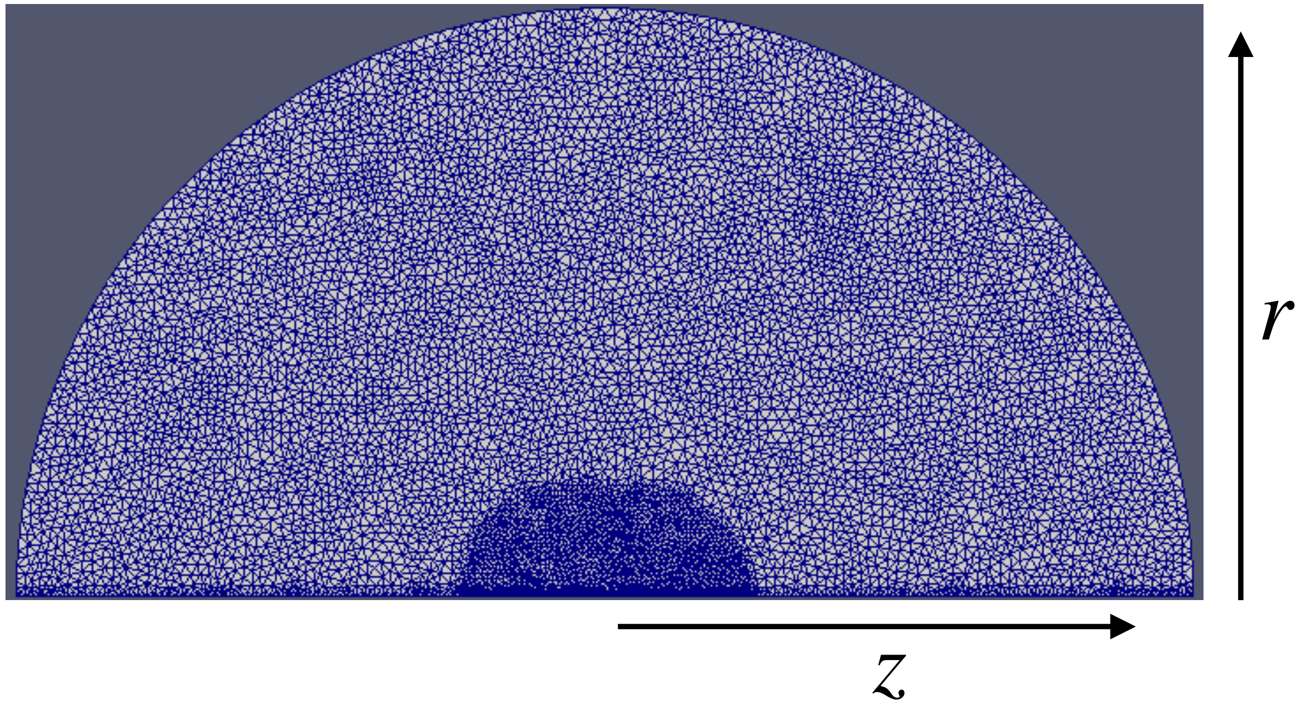
A. Kuntz (PRD) 19

A NUMERICAL SOLUTION:

$$\partial_i \left[\partial^i \varphi + \frac{1}{\Lambda^4} \left(\partial_i \partial^i \varphi \right)^2 \partial^i \varphi \right] = \frac{T}{M_P}$$

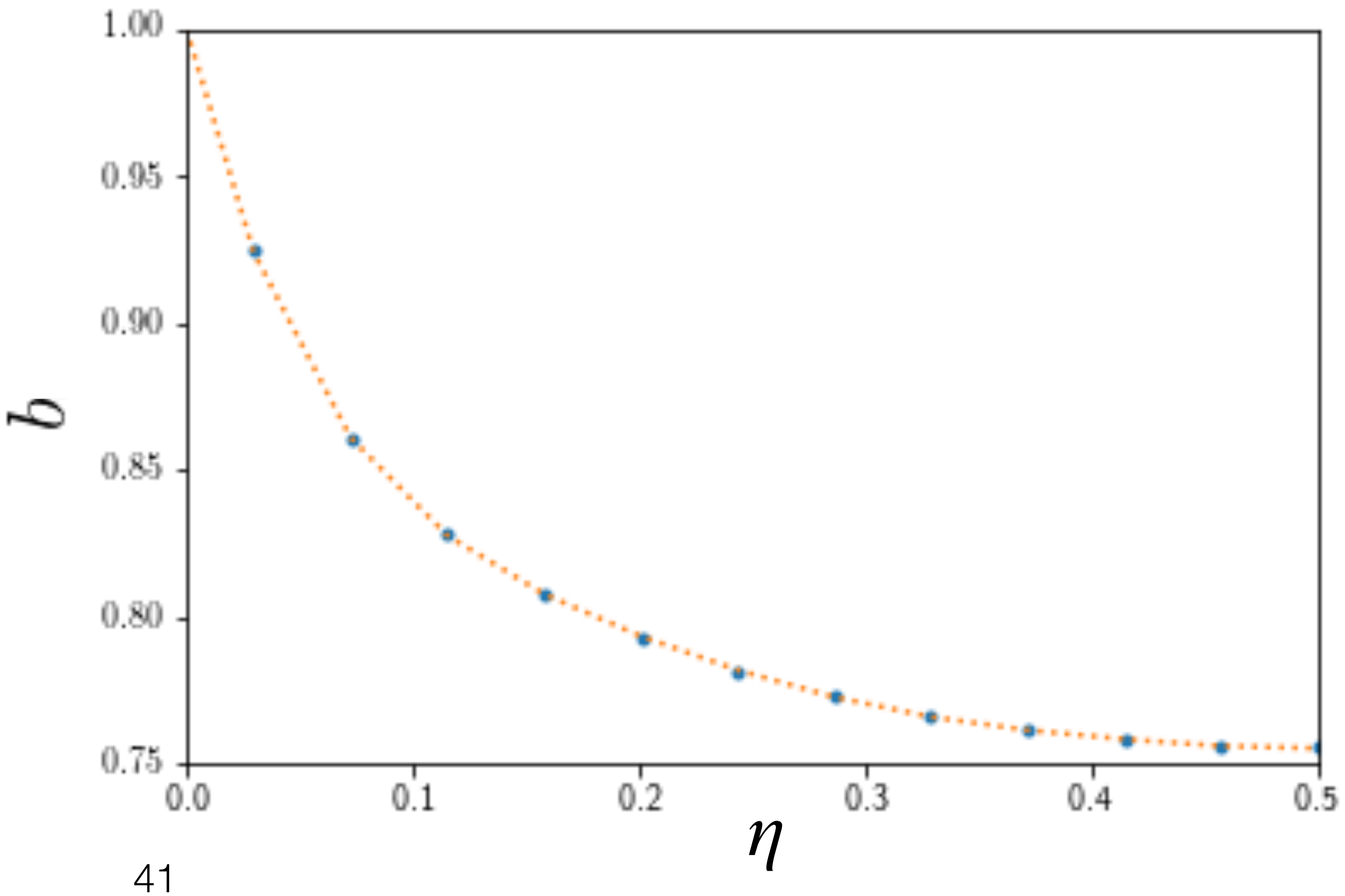
$$\Downarrow$$

$$\frac{E_{2\text{ body}}}{\mu \varphi_0(r)} = b(\eta) \quad (= 1 \text{ in Newton's gravity})$$



$$\mu = \frac{m_1 m_2}{m_1 + m_2}$$

$$\eta = \frac{m_1}{m_1 + m_2}$$



EP VIOLATION

A. Kuntz (PRD) 19

$$E = \mu\, b(\eta)\, \varphi_0(r)$$

$$\Rightarrow \quad \vec{a} = b(\eta)\, \vec{\nabla}\, \varphi_0(r)$$

EP VIOLATION

A. Kuntz (PRD) 19

$$E = \mu\, b(\eta)\, \varphi_0(r)$$

$$\Rightarrow \quad \vec{a} = b(\eta)\, \vec{\nabla}\, \varphi_0(r)$$

$$\delta r_{EM} \simeq 3 \times 10^{12} \left| \eta_{SE} \left(\frac{r}{r_*} \right)^{4/3} \right| \ \text{cm}$$

This gives a constraint :

$$\eta_{SE} \left(\frac{r}{r_*} \right)^{4/3} \lesssim 10^{-13}$$

Since $\eta_{\rm SE} \simeq 10^{-6}$, the perihelion constraint is better:

$$\left(\frac{r}{r_*} \right)^{4/3} \lesssim 10^{-11}$$

CONCLUSIONS PART 3

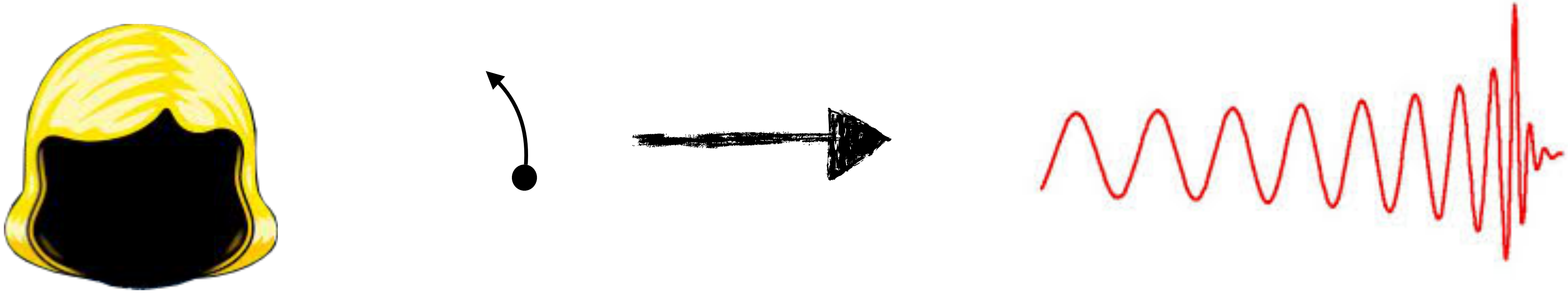
- Screening mechanisms naturally recover GR inside the solar system
- They lead to violations of the Equivalence Principle

There remains an important question:

HOW IS THE (TWO-BODY) MOTION OF BLACK HOLES MODIFIED IN SCALAR-TENSOR THEORIES ?

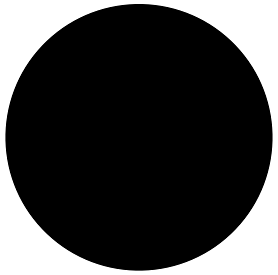
PLAN

1. THE TWO-BODY PROBLEM IN GR : AN EFT APPROACH
2. THE TWO-BODY PROBLEM IN SCALAR-TENSOR THEORIES
3. TWO-BODY PROBLEM AND SCREENING MECHANISMS
4. EXTREME MASS RATIO INSPIRALS AND SCALAR HAIR



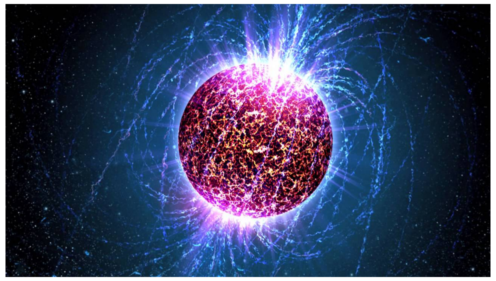
THE NO-HAIR THEOREM

In GR, BH are very simple objects!



M, J, Q

VS

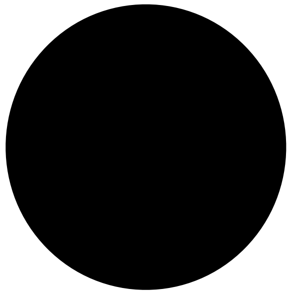


A ton of complicated physics
(composition, EoS…)

This can be generalised to modified gravity:

$$L = \frac{M_P^2}{2} R - (\partial\varphi)^2 - V(\varphi)$$

$$\Downarrow$$



$$\bar{\varphi}(r) = 0$$

(also valid for more complicated Lagrangians)

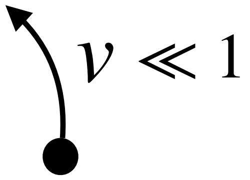
THE NO-HAIR THEOREM

However, it is easy to circumvent the assumptions of the theorem

	I Jacobson '99	II Babichev Esposito-Farèse '13	III Sotiriou et al. '14
Hair type	Environmental	Environmental	Secondary
Lagrangian	$L_1 = \frac{M_P^2}{2}R - \frac{1}{2}(\partial\varphi)^2$	$L = L_1 - \frac{1}{2\Lambda^3}(\partial\varphi)^2 \Box \varphi$	$L = L_1 + \bar{\alpha}\phi\big(R_{\mu\nu\rho\lambda}R^{\mu\nu\rho\lambda}$ $-4R_{\mu\nu}R^{\mu\nu} + R^2\big)$
Field	$\varphi(t,r) = qt + \beta_{\rm eff}\ \varphi_0(r)$	$\varphi(t,r) = qt + \beta_{\rm eff}\ \varphi_0(r)$	$\varphi(r) = \frac{Q}{r} + \mathcal{O}\left(\frac{1}{r^2}\right)$

The GW signals would then be quite different than in GR!

HAIR EXAMPLE II: CUBIC GALILEON



P. Brax, L. Heisenberg, A. Kuntz (JCAP) 20

$$\varphi = qt + \bar{\varphi}(r) + \delta\varphi$$

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + \delta g_{\mu\nu}$$

QUADRATIC ACTION for fluctuations:

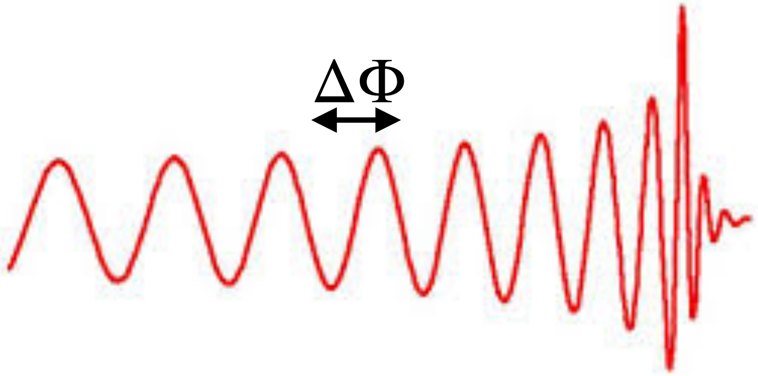
$$S = \int d^4x \frac{1}{2}\left[K_t(\partial_t\delta\varphi)^2 - K_r(\partial_r\delta\varphi)^2 - K_\Omega(\partial_\Omega\delta\varphi)^2\right] + \frac{\beta_{\rm eff}}{M_P}\delta\varphi T$$

$$K_t = 3\left(\frac{r_*}{r}\right)^{3/2}$$

$$K_r = 4\left(\frac{r_*}{r}\right)^{3/2}$$

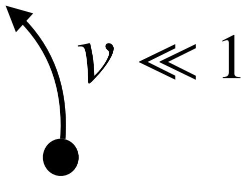
$$K_\Omega = \left(\frac{r_*}{r}\right)^{3/2}$$

Solve for the field using Green's function



$$\Delta\Phi \simeq 3.5 \times 10^{-7}\beta_{\rm eff}^{3/2}\left(\frac{\Lambda}{10^{-12}{\rm eV}}\right)^{3/2}\left(\frac{m_1}{50M_\odot}\right)^{-1}\left(\frac{m_0}{10^6M_\odot}\right)^{-3/2}\left(\frac{\Omega_{\rm in}}{10^{-3}{\rm Hz}}\right)^{-21/6}$$

A SYSTEMATIC APPROACH



$$\varphi = \bar{\varphi}(r) + \delta\varphi$$

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + \delta g_{\mu\nu}$$

UNITARY GAUGE : $\varphi(t, x) \to \bar{\varphi}(r)$ i.e $\delta\varphi = 0$

EMRI & scalar hair

A SYSTEMATIC APPROACH

$$\varphi = \bar{\varphi}(r) + \delta\varphi$$

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + \delta g_{\mu\nu}$$

UNITARY GAUGE : $\varphi(t,x) \to \bar{\varphi}(r)$ i.e $\delta\varphi = 0$

EFFECTIVE ACTION : $S = \int \mathrm{d}^4x\sqrt{-g}\left[\frac{1}{2}M_1^2(r)R - \Lambda(r) - f(r)g^{rr} - \alpha(r)\bar{K}^{\mu}{}_{\nu}K^{\nu}{}_{\mu}\right] + S^{(2)}$

G. Franciolini et al. 19

- Λ, f and α uniquely determined by the background $\bar{g}_{\mu\nu}$
- M_1^2 removable by a conformal transformation

$$g^{(\mathrm{E})}_{\mu\nu}(x) = g^{(\mathrm{J})}_{\mu\nu}(x)M_1^2(r)$$

$$S_{\mathrm{pp}} = -\int dt\, \mu\, \sqrt{-\bar{g}_{\mu\nu}v^{\mu}v^{\nu}} \to -\int dt\, \mu(r)\, \sqrt{-\bar{g}_{\mu\nu}v^{\mu}v^{\nu}}$$

THE METRIC

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + \delta g_{\mu\nu}$$

Background:

$$\bar{g}_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu = -\,a^2(r)\mathrm{d}t^2 + \frac{\mathrm{d}r^2}{b^2(r)} + c^2(r)\left(\mathrm{d}\theta^2 + \sin^2\theta \mathrm{d}\phi^2\right)$$

E.g. for Gauss-Bonnet:

$$a^2(r) = 1 - \frac{2M}{r} + \frac{MQ^2}{6r^3} + \mathcal{O}(r^{-4}) \quad b^2(r) = 1 - \frac{2M}{r} + \frac{Q^2}{2r^2} + \mathcal{O}(r^{-3}) \qquad c^2(r) = r^2 \quad \text{(gauge choice)}$$

THE METRIC

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + \delta g_{\mu\nu}$$

Background:

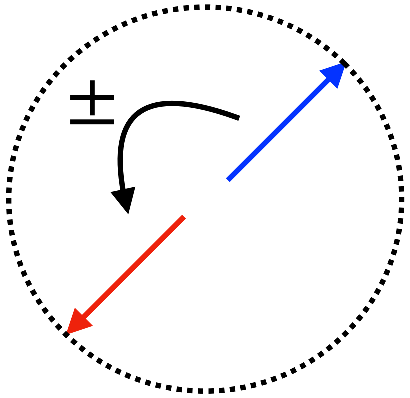
$$\bar{g}_{\mu\nu}\mathrm{d}x^{\mu}\mathrm{d}x^{\nu} = -a^2(r)\mathrm{d}t^2 + \frac{\mathrm{d}r^2}{b^2(r)} + c^2(r)\left(\mathrm{d}\theta^2 + \sin^2\theta\mathrm{d}\phi^2\right)$$

E.g. for Gauss-Bonnet:

$$a^2(r) = 1 - \frac{2M}{r} + \frac{MQ^2}{6r^3} + \mathcal{O}(r^{-4}) \quad b^2(r) = 1 - \frac{2M}{r} + \frac{Q^2}{2r^2} + \mathcal{O}(r^{-3}) \qquad c^2(r) = r^2 \quad \text{(gauge choice)}$$

Perturbations:

$\delta g_{\mu\nu}$ transforms under $(i, j) = (\theta, \phi)$ diffs and under PARITY:



$$\delta g_{\mu\nu}^{\text{odd}} \qquad \Leftrightarrow \qquad \Psi$$

THE ODD SECTOR

A. Kuntz, R. Penco, F. Piazza (JCAP) 20

GENERALIZED RW EQUATION

$$\frac{d^2\Psi}{d\tilde{r}^2} + (\omega^2 + V(\tilde{r}))\Psi = S$$

$$\frac{d\tilde{r}}{dr} = 1 + (\ldots)\frac{M}{r} + \mathcal{O}\left(\frac{M}{r}\right)^2$$

GENERALIZED TORTOISE COORDINATE

$$V(\tilde{r}(r)) = -\frac{1}{r^2}\left(1 - \frac{2M}{r}\right)\left(l(l+1) - (\ldots)\frac{M}{r} + \mathcal{O}\left(\frac{M}{r}\right)^2\right)$$

GENERALIZED RW POTENTIAL

THE ODD SECTOR

A. Kuntz, R. Penco, F. Piazza (JCAP) 20

GENERALIZED RW EQUATION

$$\frac{d^2\Psi}{d\tilde{r}^2} + (\omega^2 + V(\tilde{r}))\Psi = S$$

$$\frac{d\tilde{r}}{dr} = 1 + (\ldots)\frac{M}{r} + \mathcal{O}\left(\frac{M}{r}\right)^2$$ GENERALIZED TORTOISE COORDINATE

$$V(\tilde{r}(r)) = -\frac{1}{r^2}\left(1 - \frac{2M}{r}\right)\left(l(l+1) - (\ldots)\frac{M}{r} + \mathcal{O}\left(\frac{M}{r}\right)^2\right)$$ GENERALIZED RW POTENTIAL

Solution to the RW equation:

Poisson 93
Sasaki 94

$$\Psi(r) = \Psi_0(r) + (M\omega)\Psi_1(r) + (M\omega)^2\Psi_2(r) + \ldots$$

$$P \propto \sum_{l,m} \left|\frac{d\Psi}{dt}\right|^2$$

DISSIPATED POWER

A. Kuntz, R. Penco, F. Piazza (JCAP) 20

$$\frac{P}{P_N} = p_0 + p_1 v^2 + p_2 v^4 + \dots$$

GR Quadrupole

ppE parameters

We go up to 3.5PN !

N. Yunes, F. Pretorius 09

DISSIPATED POWER

A. Kuntz, R. Penco, F. Piazza (JCAP) 20

GR Quadrupole

$$\frac{P}{P_N} = p_0 + p_1 v^2 + p_2 v^4 + \dots$$

ppE parameters

We go up to 3.5PN !

N. Yunes, F. Pretorius 09

Our approach bridges the gap between ppE and theory:

- GIVE ME YOUR METRIC, I WILL GIVE YOU YOUR WAVEFORM !
- MODELED SEARCH WITH ADDITIONAL NON-GR COEFFICIENTS !

The even sector now needs to be done…

OUTLOOK

- We have investigated on the TWO-BODY PROBLEM in several types of SCALAR-TENSOR THEORIES, often adopting an EFFECTIVE FIELD THEORY viewpoint.

- GRAVITATIONAL WAVES astronomy still in infancy. Interesting physics ahead !

- EFFECTIVE FIELD THEORIES are fantastic tools to compare theory & experiment

THE OLDEST ACADEMIC PROBLEM OF PHYSICS IS STILL A SOURCE OF INSPIRATION!

OUTLOOK

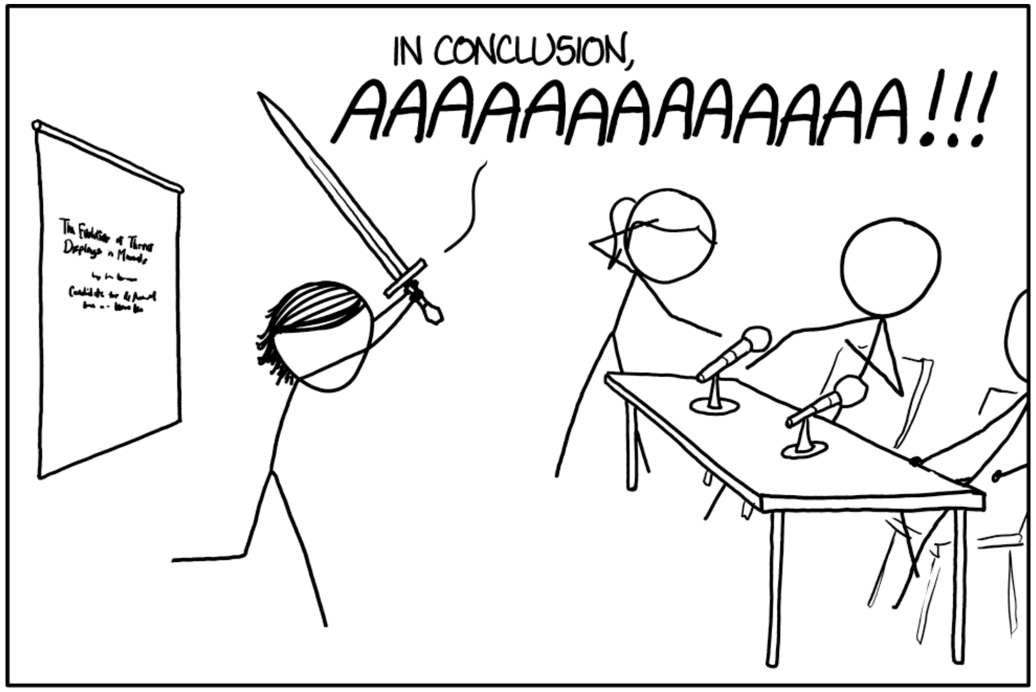
- We have investigated on the TWO-BODY PROBLEM in several types of SCALAR-TENSOR THEORIES, often adopting an EFFECTIVE FIELD THEORY viewpoint.

- GRAVITATIONAL WAVES astronomy still in infancy. Interesting physics ahead !

- EFFECTIVE FIELD THEORIES are fantastic tools to compare theory & experiment

THE OLDEST ACADEMIC PROBLEM OF PHYSICS IS STILL A SOURCE OF INSPIRATION!

THANK YOU !



THE BEST THESIS DEFENSE IS A GOOD THESIS OFFENSE.