The two-body problem in modified gravity

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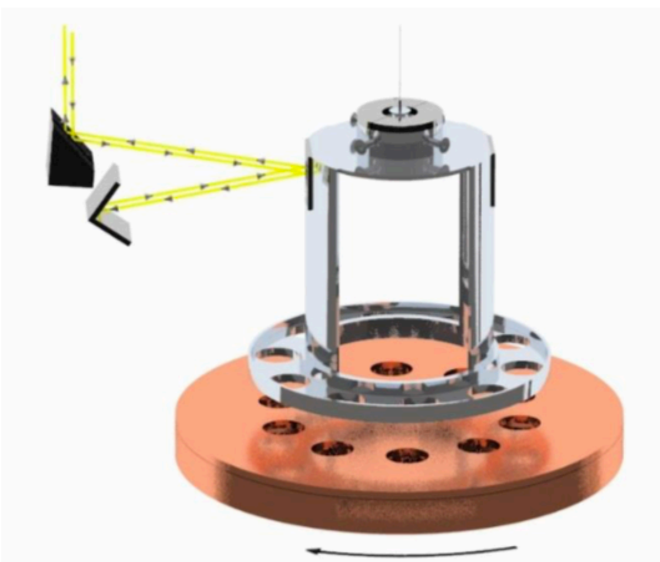
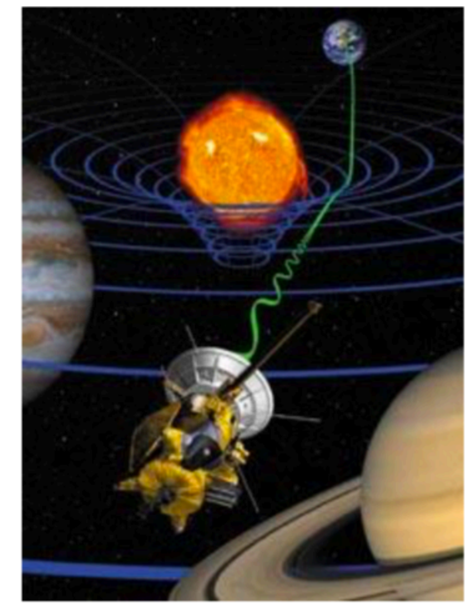
Motivation

GR is amazingly tested on small scales :

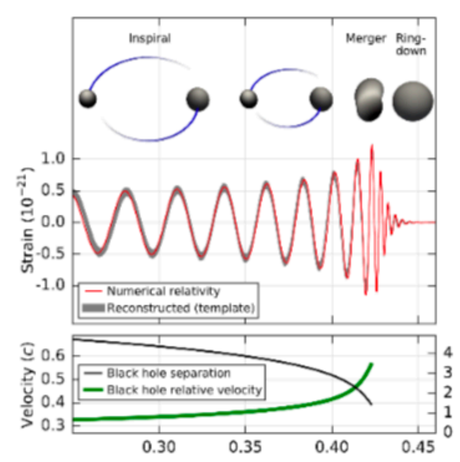


Laboratory experiments (Eotwash) tests of fifth forces and equivalence principle
0.1 mm

Cassini probe test of fifth forces
1 a.u., 150 million km.



Lunar ranging tests of strong equivalence principle and time variation of Newton's constant,
400 000 km

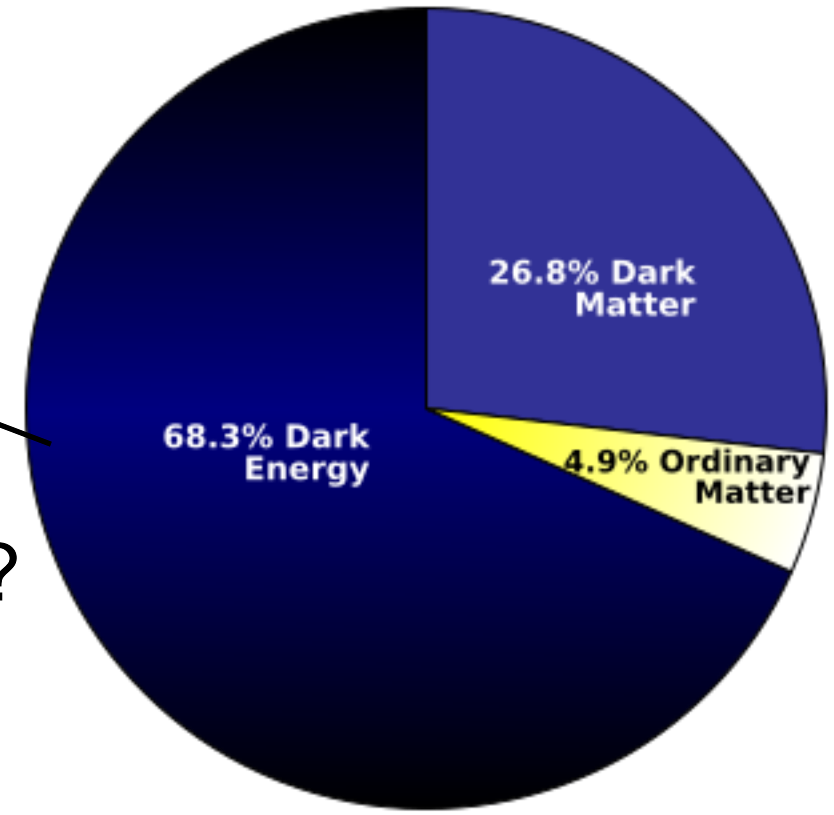


Gravitational wave emissions from black hole and neutron star mergers
50 Mpc

Motivation

But it is still quite mysterious on large scales !

$$S_{\Lambda CDM} = \frac{1}{16\pi G}\int d^4x\sqrt{-g}\,(R - 2\Lambda)$$



What if Dark Energy was not so simple as Λ?

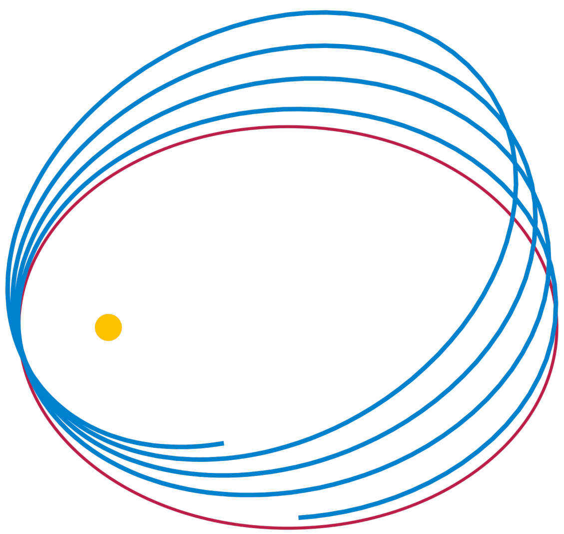
$$\Lambda \rightarrow \rho_\phi = \frac{\dot{\phi}^2}{2} + V(\phi)$$

This generically violates small-scale tests ! (via the coupling to matter)

Often a screening mechanism is invoked

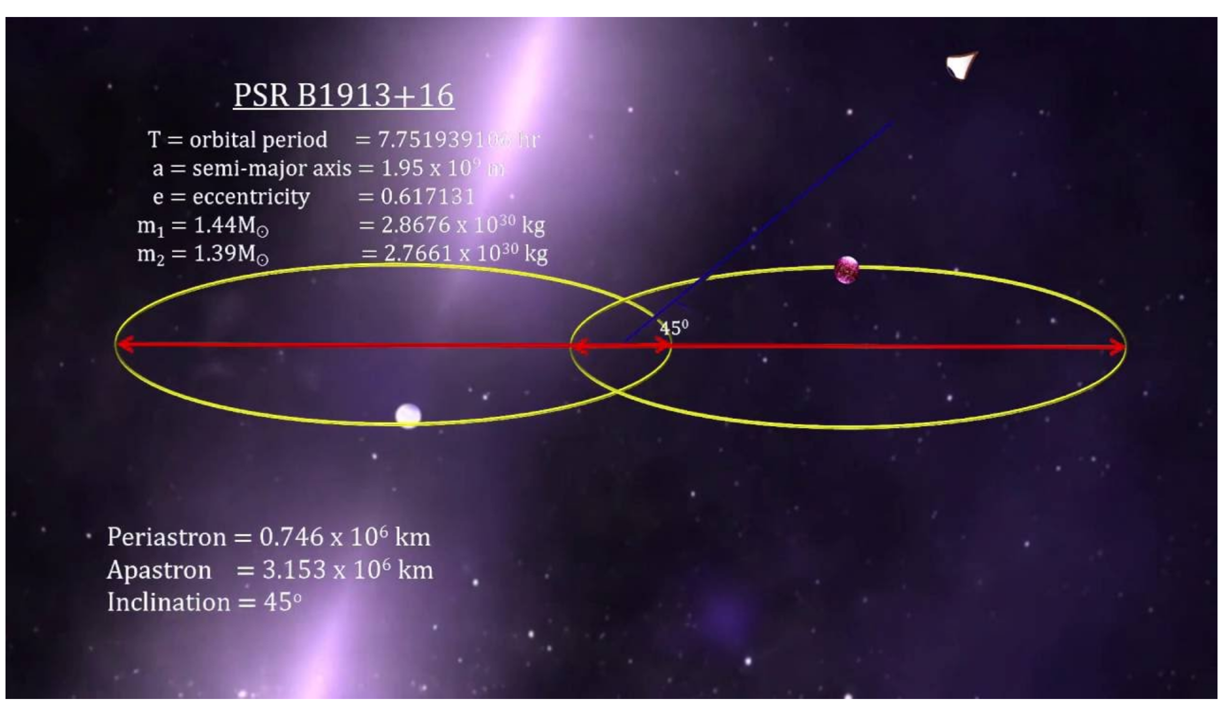
Motivation

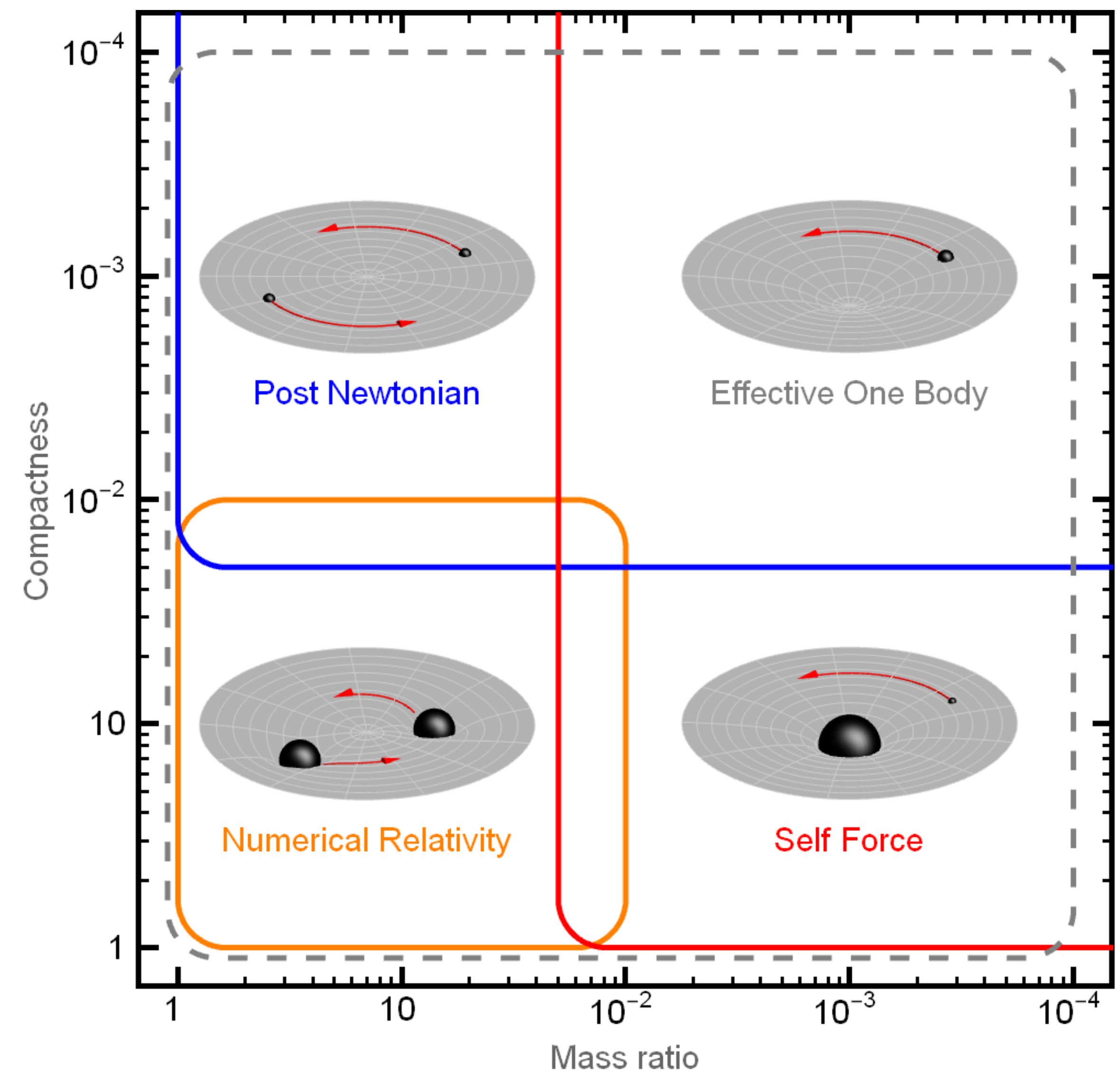
The two-body problem is an ideal playground for testing modifications of gravity



Perihelion precession

Binary pulsars





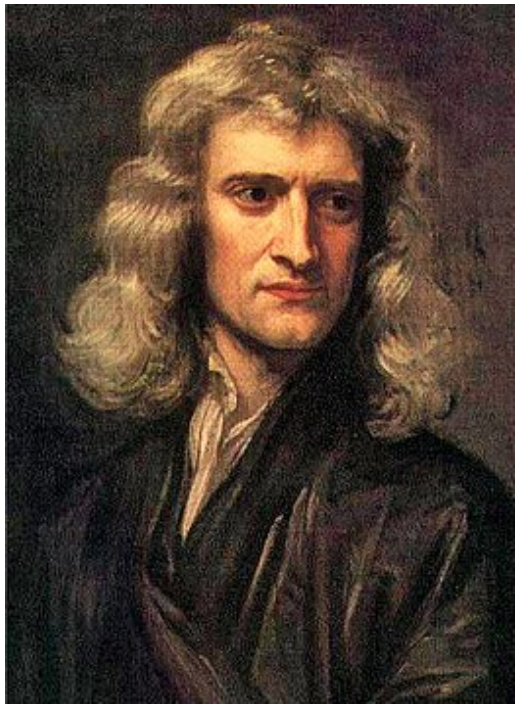
Gravitational waves

Plan

- The two-body problem in GR
- Scalar-Tensor theories and disformal couplings
- Vainshtein screening

The two-body problem in GR

Newtonian two-body problem



$$m_1 \frac{d^2\mathbf{r}_1}{dt^2} = -\frac{Gm_1m_2}{r^3}\mathbf{r}$$

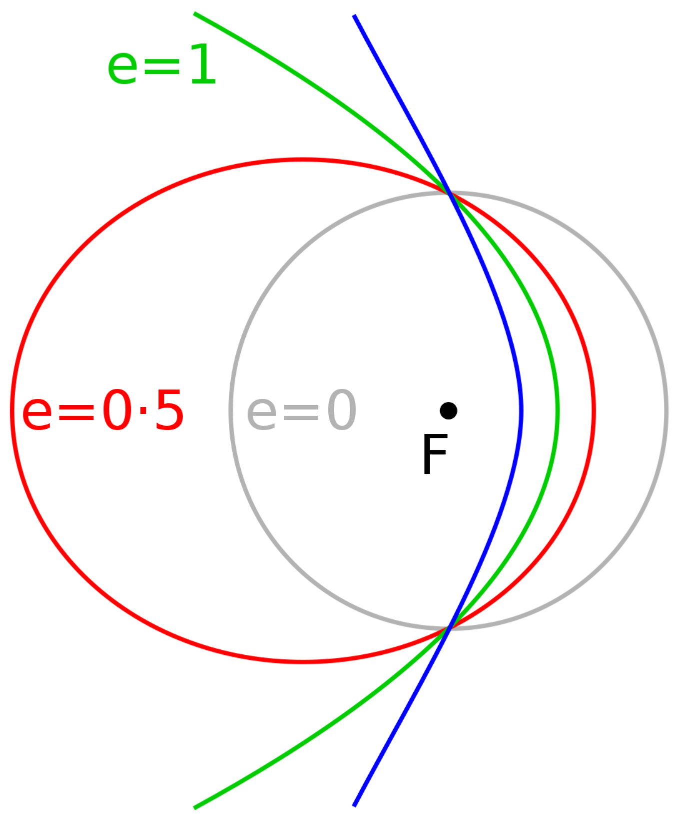
$$m_2 \frac{d^2\mathbf{r}_2}{dt^2} = \frac{Gm_1m_2}{r^3}\mathbf{r}$$

$$\Leftrightarrow$$

$$\frac{d^2\mathbf{R}}{dt^2} = 0$$

$$\frac{d^2\mathbf{r}}{dt^2} = -\frac{GM}{r^3}\mathbf{r}$$

$$\Downarrow$$



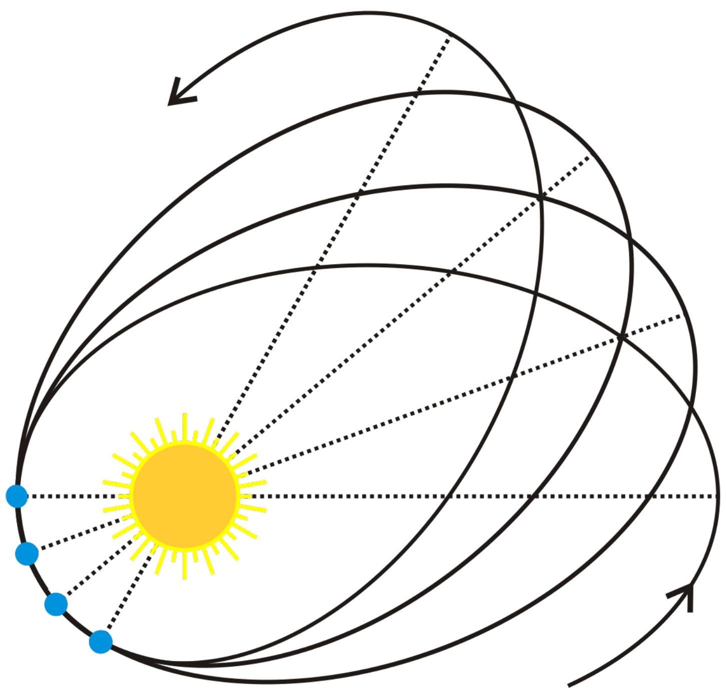
$$r(\theta) = \frac{p}{1 + e\cos(\theta)}$$

Virial theorem : $v^2 \sim \frac{GM}{r}$

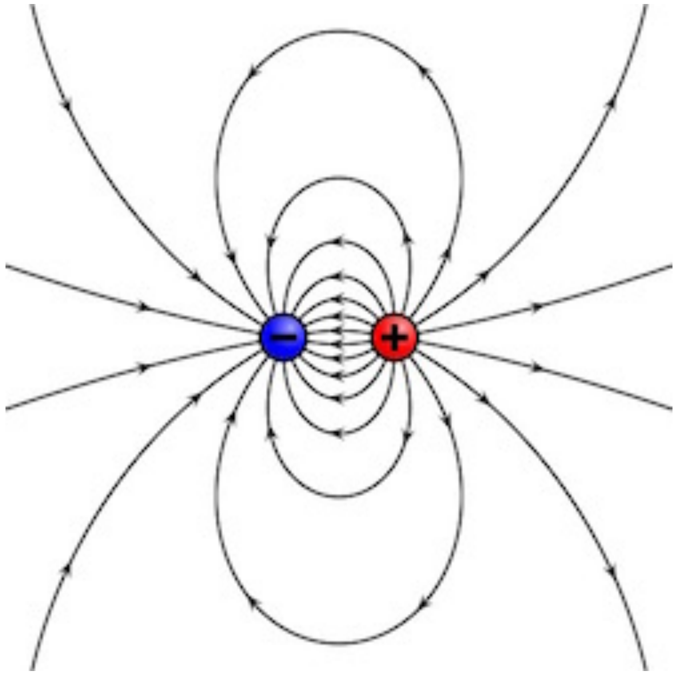
The two-body problem in GR : Intro

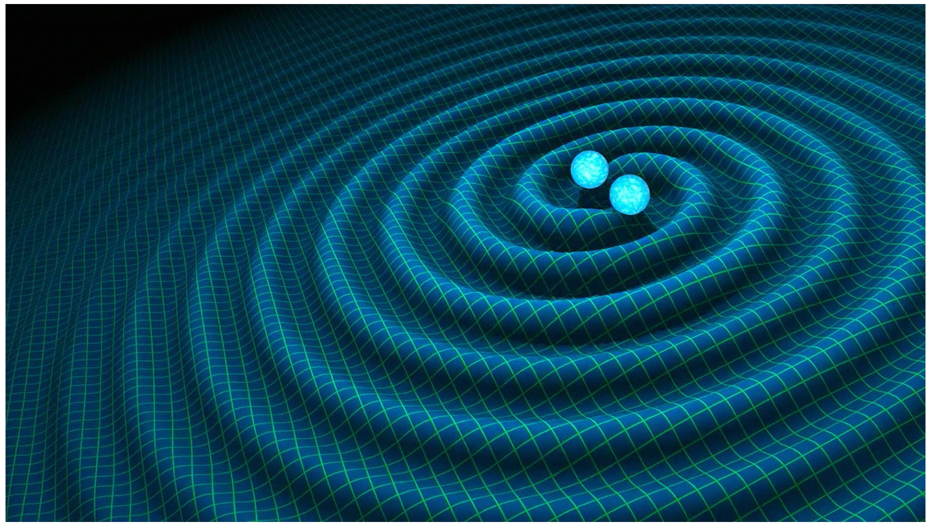
If one takes into account relativistic effects :

$$\mathbf{F} = -\frac{Gm_1m_2}{r^3}\mathbf{r}\left(1 + \mathcal{O}\left(\frac{v^2}{c^2}\right)\right)$$



Also, a moving object can generate waves !





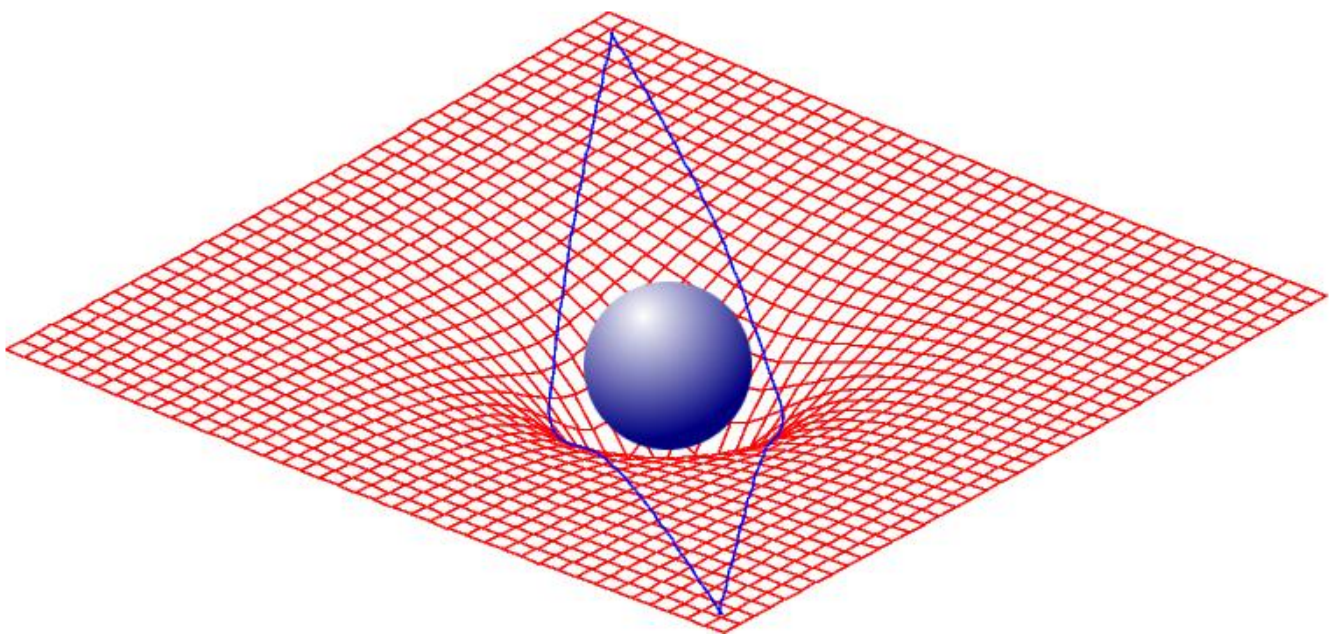
The two-body problem in GR

Basic ingredient of GR : the metric $g_{\mu\nu}$

Action principle (in vacuum) : $S = \frac{1}{16\pi G}\int d^4x\sqrt{-g}R \quad \Rightarrow \quad G_{\mu\nu} = 0$

A point-particle in GR : $S_{\text{pp,A}} = -\,m_A \int d\tau_A \;= -\,m_A \int dt \sqrt{-g_{\mu\nu}\frac{dx_A^\mu}{dt}\frac{dx_A^\nu}{dt}}$

$$\Downarrow$$



$$\frac{d^2x_A^\mu}{d\tau^2} + \Gamma^\mu_{\nu\rho}\frac{dx_A^\nu}{d\tau}\frac{dx_A^\rho}{d\tau} = 0$$

The two-body problem in GR

EFT approach : use field theory tools

Goldberger and Rothstein (2006)
Porto (2006)
+ many others…

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \quad \Rightarrow \quad S = S^{(2)} + S_{\text{int}}$$

$$\mathcal{O}(v^2) = \mathcal{O}\left(\frac{GM}{r}\right)$$

$$S^{(2)} = -\frac{1}{8}\int d^4x \left[-\frac{1}{2}(\partial_\mu h^\alpha_\alpha)^2 + (\partial_\mu h_{\nu\rho})^2\right]$$

Green function or propagator

$$S_{\text{int}} = m\int dt\left(\frac{v^2}{2} + \frac{h_{00}}{2} + \dots\right)$$

Expand the propagator : $\mathbf{k} \sim 1/r, \quad k_0 \sim v/r$

$$\frac{-i}{k^2 + i\epsilon} = -\frac{i}{\mathbf{k}^2}\left(1 + \frac{k_0^2}{\mathbf{k}^2} + \dots\right)$$

The two-body problem in GR

The two-body dynamics is encoded in the effective action :

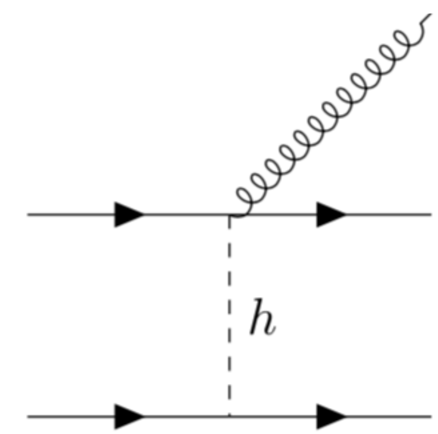
$$e^{iS_{\text{eff}}[\mathbf{x}_1(t),\mathbf{x}_2(t)]} = \int \mathcal{D}h_{\mu\nu} e^{iS[\mathbf{x}_1(t),\mathbf{x}_2(t),h_{\mu\nu}]}$$

It has real and imaginary parts :

$$\Re(S_{\text{eff}}) = \int dt L[\mathbf{x}_a, \mathbf{v}_a]$$

$$L = \frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2 + \frac{G_N m_1 m_2}{|\mathbf{x_1}(t) - \mathbf{x_2}(t)|} + L_{1\text{PN}} + \dots$$

$$\Im(S_{eff}) = \frac{T}{2}\int dE d\Omega \frac{d^2\Gamma}{dE d\Omega}$$



$$P = \frac{G}{5}\left\langle \dddot{I}_h^{ij\,2} \right\rangle + \dots$$

Scalar-tensor theories and disformal couplings

Modifying GR : scalar-tensor theories

GR works amazingly well for all our observations. But to which precision can we claim to be consistent with GR ?

$\Rightarrow$ We must parametrize deviations from GR

It is often instructive to look at simple modifications of GR

Scalar-Tensor theories : $g_{\mu\nu} \quad + \quad \phi$

$$S_\phi = -\frac{1}{2}\int d^4x\sqrt{-g}\,g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$$

Modifying GR : scalar-tensor theories

In GR, the coupling of the metric to matter follows from the equivalence principle :

$$S = \frac{1}{16\pi G}\int d^4x\sqrt{-g}R + S_m[g_{\mu\nu}, \psi_i]$$

Same

We generically expect a coupling of ϕ with matter. The most generic compatible with causality and the equivalence principle is : **Bekenstein 92**

$S_m[\tilde{g}_{\mu\nu}, \psi_i]$ with $\tilde{g}_{\mu\nu} = A(\phi, X)g_{\mu\nu} + B(\phi, X)\partial_\mu\phi\partial_\nu\phi$ $\quad X = g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$

Conformal Disformal

Conformal coupling

Focus first on $\tilde{g}_{\mu\nu} = A(\phi) g_{\mu\nu}$ **Dicke, Will, Wagoner, Nordtvedt, Damour, Esposito-Farèse…**

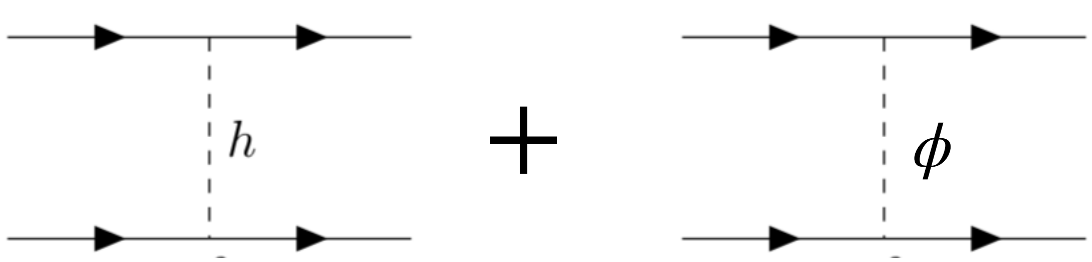
$$S_{\rm pp} = -m_A \int dt \sqrt{-\tilde{g}_{\mu\nu} \frac{dx_A^\mu}{dt} \frac{dx_A^\nu}{dt}}$$

$$\Rightarrow \quad S_{\rm pp} = -\int d\tau_A \, m_A(\phi) = m_A \int d\tau \left(-1 + \alpha_A \frac{\phi}{M_P} + \beta_A \left(\frac{\phi}{M_P} \right)^2 + \dots \right)$$

AK, F. Vernizzi, F. Piazza 19

Conservative

Dissipative



$$\tilde{G}_N = G_N \left(1 + 2\alpha_1 \alpha_2 \right)$$

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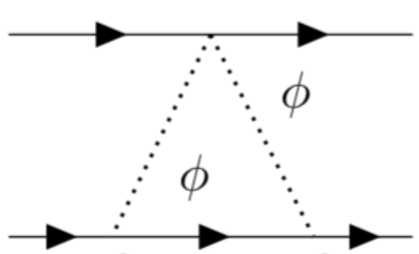
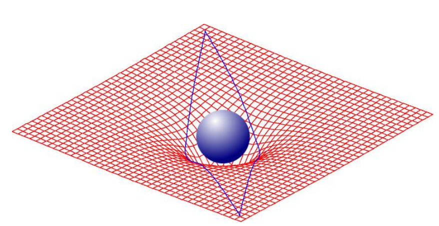


$+\dots$

PPN parameters :

$$\gamma_{AB} = 1 - 4 \frac{\alpha_A \alpha_B}{1 + 2\alpha_A \alpha_B}$$



$$\beta_{AB} = 1 - 2 \frac{\alpha_A^2 \alpha_B^2 + f_{AB}}{(1 + 2\alpha_A \alpha_B)^2}$$

Dissipative

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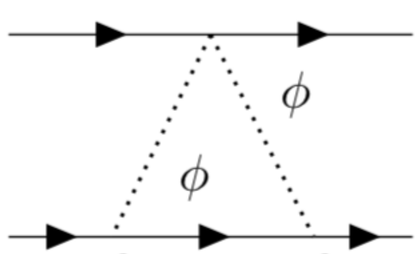

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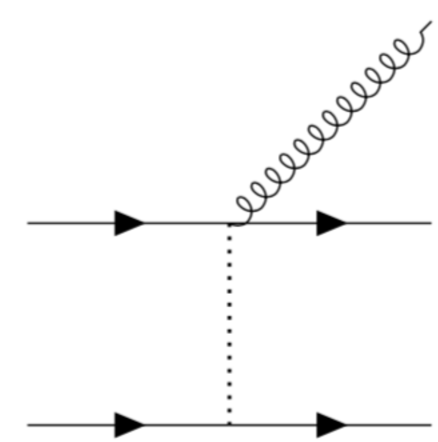
Dissipative



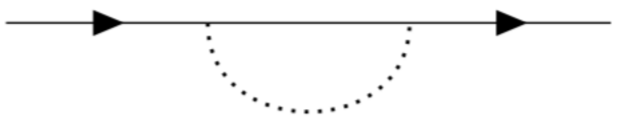
$$P_\phi = 2 G_N \left(\left\langle \dot{I}_\phi^{\,2} \right\rangle + \frac{1}{3} \left\langle \ddot{I}_\phi^{i\,2} \right\rangle + \frac{1}{30} \left\langle \dddot{I}_\phi^{ij\,2} \right\rangle + \dots \right)$$

Monopole Dipole Quadrupole

Conformal coupling

$$S_{\rm int} = -\int d\tau_A \, m_A(\phi) = m_A \int d\tau \left(-1 + \alpha_A \frac{\phi}{M_P} + \beta_A \left(\frac{\phi}{M_P} \right)^2 + \ldots \right)$$

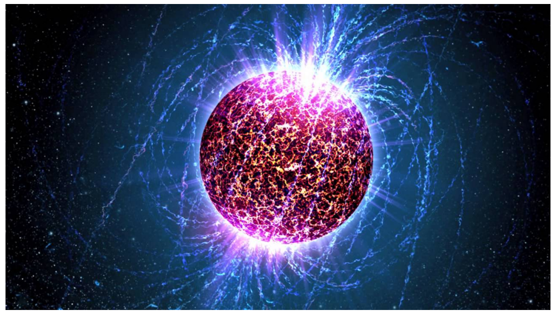
Mass renormalization :



$$-m_{\rm bare} \int dt \quad \rightarrow \quad -\left(m_{\rm bare} + E(\Lambda)\right) \int dt$$

$$\Lambda \sim \frac{1}{r_s}$$

$E = 0$



$E \neq 0$

$$E = -\frac{\tilde{G}}{2} \int d^3x d^3y \frac{\rho(\mathbf{x})\rho(\mathbf{y})}{|\mathbf{x} - \mathbf{y}|}$$

Conformal coupling

$$S_{\text{int}} = -\int d\tau_A\, m_A(\phi) = m_A \int d\tau \left(-1 + \alpha_A \frac{\phi}{M_P} + \beta_A \left(\frac{\phi}{M_P}\right)^2 + \dots \right)$$

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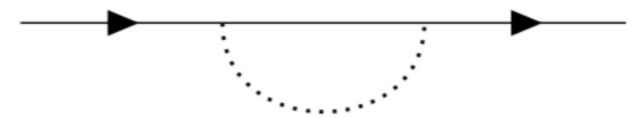

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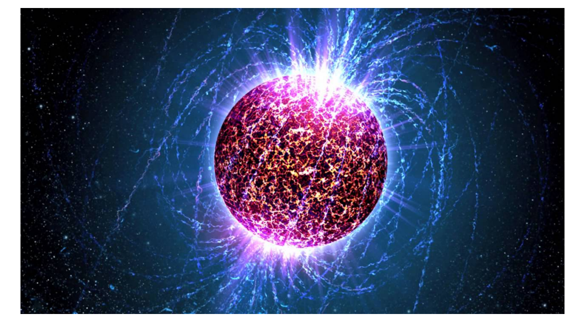
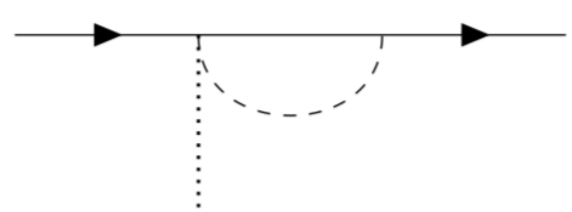
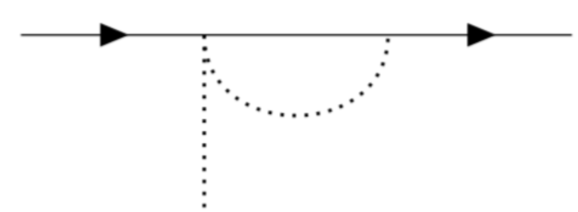
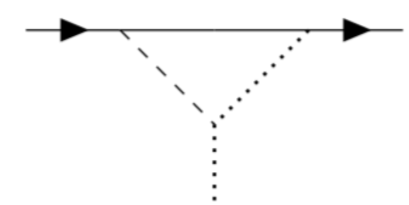


$E \neq 0$

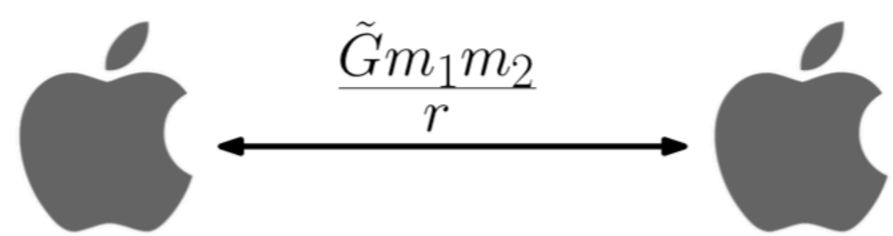
Charge renormalization :



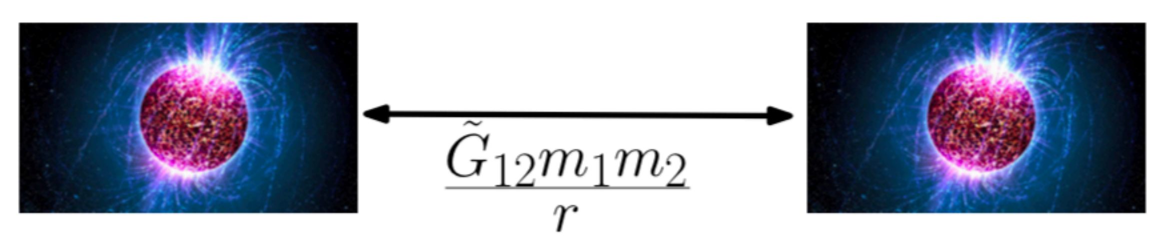




$$\alpha_{\text{bare}} m_{\text{bare}} \int dt\, \phi \quad \rightarrow \quad \alpha(\Lambda) m(\Lambda) \int dt\, \phi$$



and $\tilde{G}_N = G_N\left(1 + 2\alpha_1\alpha_2\right)$



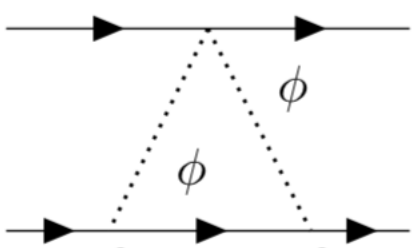
Disformal coupling

$$\tilde{g}_{\mu\nu} = A(\phi)g_{\mu\nu} + B(\phi)\partial_\mu\phi\partial_\nu\phi \qquad S_{\rm pp} = -m_A\int dt\sqrt{-\tilde{g}_{\mu\nu}\frac{dx_A^\mu}{dt}\frac{dx_A^\nu}{dt}}$$

$$\Rightarrow \quad S_{\rm pp} = m_A\int d\tau\left(-1+\alpha_A\frac{\phi}{M_P}+\beta_A\left(\frac{\phi}{M_P}\right)^2+\ldots\right)\left(1+\frac{b_A}{M^2M_P^2}(\partial_\mu\phi v_A^\mu)^2+\ldots\right)$$

AK, P. Brax, AC Davis 19

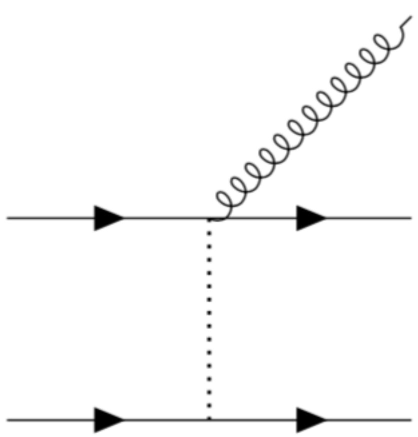
Conservative



$+\ldots$

$$L_{\rm dis} = 4\alpha^2 b\frac{G^2m_1m_2(m_1+m_2)}{M^2}\left(\frac{d}{dt}\frac{1}{r}\right)^2$$

Dissipative



Monopole $\quad I_{\rm dis} = 8\alpha b\frac{Gm_1m_2}{M^2}\frac{d^2}{dt^2}\frac{1}{r}$

$$r = |\mathbf{x_1}(t) - \mathbf{x_2}(t)|$$

Disformal coupling

Circular trajectory

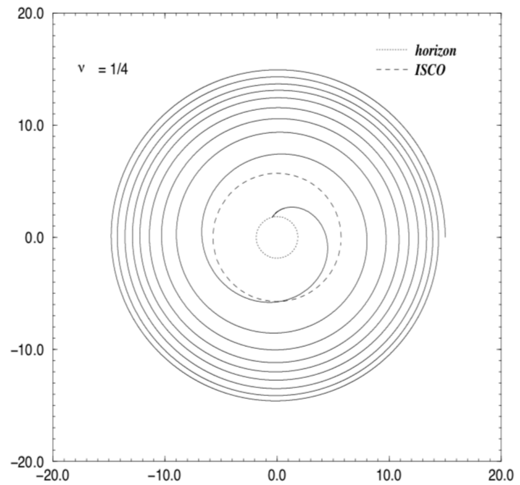
$$L_{\rm dis} = 4\alpha^2 b \frac{G^2 m_1 m_2 (m_1+m_2)}{M^2}\left(\frac{d}{dt}\frac{1}{r}\right)^2 \qquad I_{\rm dis} = 8\alpha b \frac{G m_1 m_2}{M^2}\frac{d^2}{dt^2}\frac{1}{r}$$

For circular orbits : $\dot{r} = 0\,!$

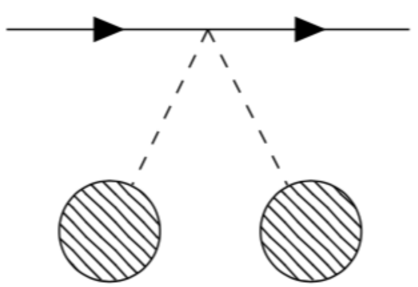
No contribution of the disformal coupling. This is intuitive because :

$$\int d\tau (\partial_\mu \phi v_A^\mu)^2 = \int d\tau \left(\frac{d\phi}{d\tau}\right)^2$$

In this case we showed that only radiation reaction effects contribute



$$\Rightarrow \quad L_{\rm dis} = \mathcal{O}(v^{14}), \quad I_{\rm dis} = \mathcal{O}(v^{12})$$



Disformal coupling

Elliptic trajectory

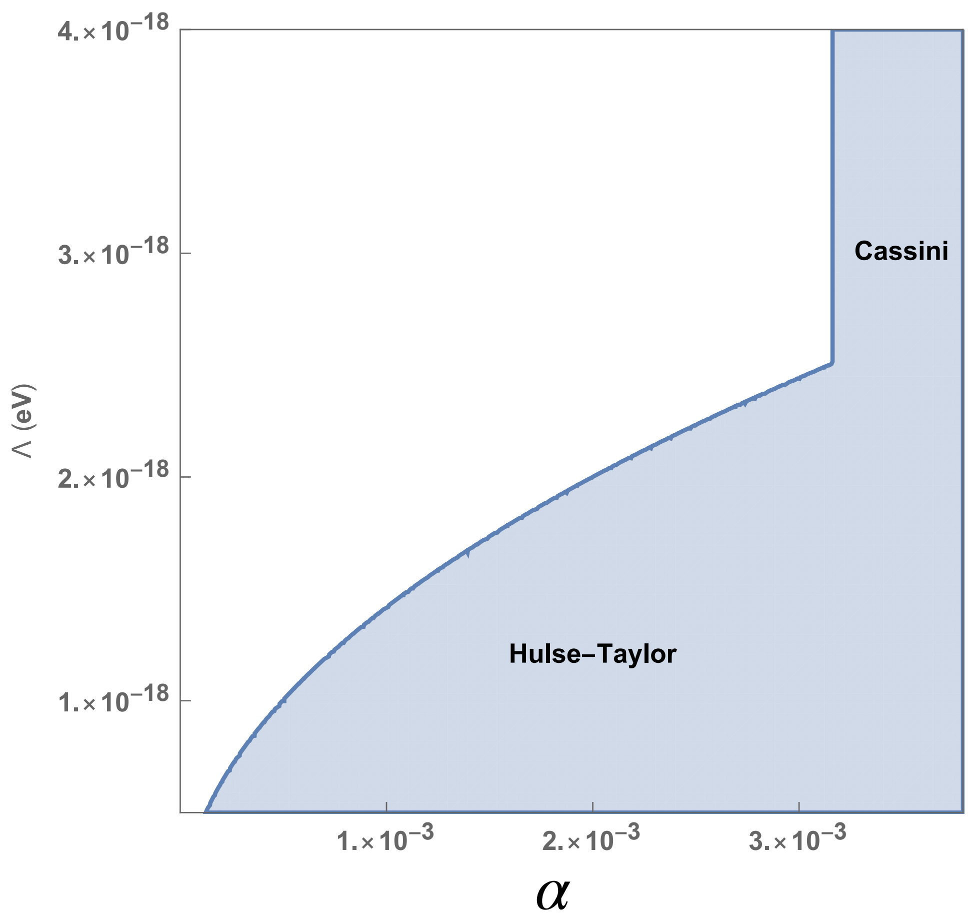
Monopole $$I_{\rm dis} = 8\alpha b \frac{G m_1 m_2}{M^2} \frac{d^2}{dt^2} \frac{1}{r}$$

$$\Rightarrow \quad P_\phi^{\rm mono} \simeq \frac{64}{9G} \alpha^2 (G M_c \omega)^{10/3} \left(f_2(e) - 12 y f_3(e) + 36 y^2 f_4(e) \right)$$

$$f_2(e) = \frac{e^2}{(1-e^2)^{7/2}} \left(1 + \frac{1}{4} e^2\right)$$

$$f_3(e) = \frac{e^2}{(1-e^2)^{13/2}} \left(1 + \frac{37}{4} e^2 + \frac{59}{8} e^4 + \frac{27}{64} e^6\right)$$

$$f_4(e) = \frac{e^2}{(1-e^2)^{19/2}} \left(1 + \frac{217}{4} e^2 + \frac{1259}{4} e^4 + \frac{11815}{32} e^6 + \frac{11455}{128} e^8 + \frac{1125}{512} e^{10}\right)$$



$$y \simeq \left(\frac{\omega}{M}\right)^2$$

$$\omega_{\rm Hulse-Taylor} \sim 10^{-18}\ {\rm eV}$$

Vainshtein screening

Motivation

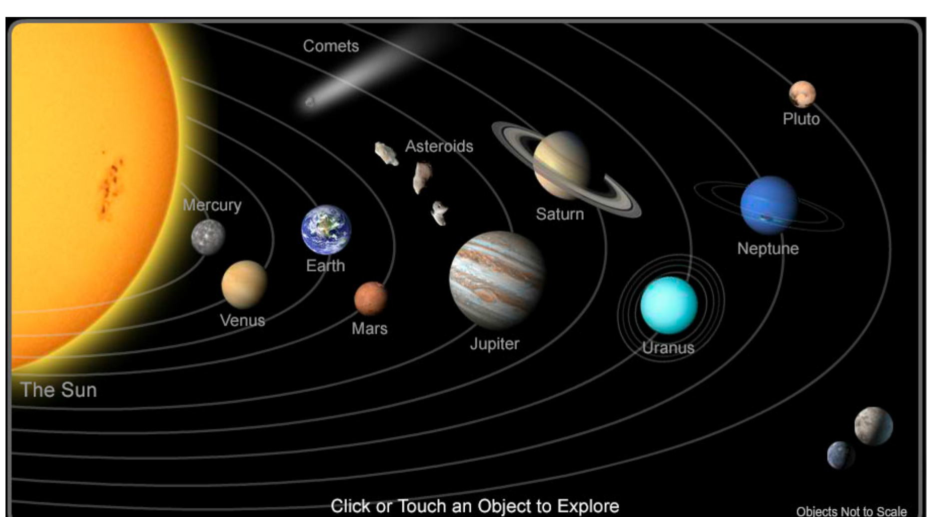
In the theories presented above, GR tests force the couplings to be small

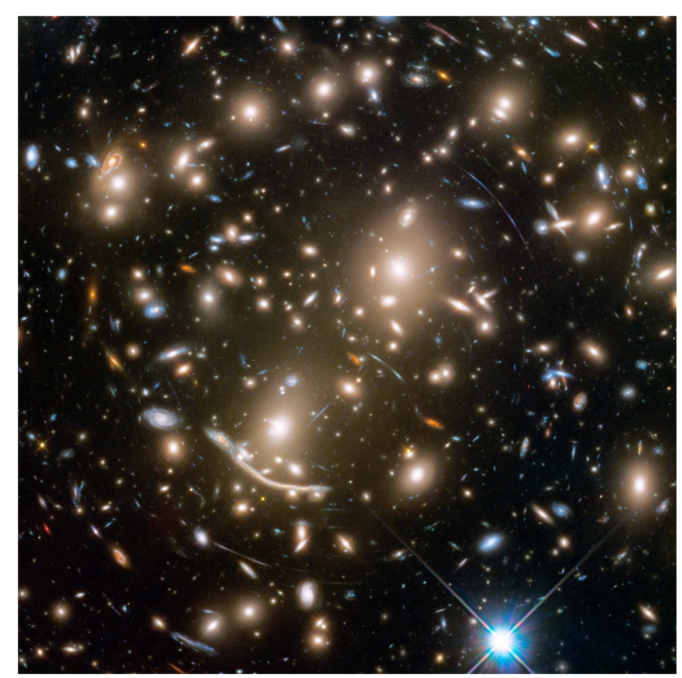
$$\alpha \lesssim 10^{-2}$$

Often a screening mechanism is invoked to have interesting deviations on cosmological scales

$$\phi(r) \simeq 0 \qquad \phi(r) \sim GM/r$$

r_* $\qquad$ r





Spherically symmetric screening

Take the simplest Vainshtein screening :

(K-Mouflage)

$$S = \int d^4x \left[-\frac{(\partial\phi)^2}{2} - \frac{1}{4\Lambda^4}(\partial\phi)^4 + \frac{\phi T}{M_P} \right]$$

Barreira et al 2015

Spherically symmetric screening

Take the simplest Vainshtein screening : $\tilde{S} = \int dt d^3x \left[-\frac{1}{2}(\nabla \tilde{\phi})^2 - \frac{1}{4}(\nabla \tilde{\phi})^4 + \tilde{\phi}\tilde{T} \right]$

(K-Mouflage)

Barreira et al 2015

$$\phi' + (\phi')^3 = \frac{M}{r^2}$$

Spherically symmetric screening

Take the simplest Vainshtein screening : $\tilde{S} = \int dt d^3x \left[-\frac{1}{2}(\nabla\tilde{\phi})^2 - \frac{1}{4}(\nabla\tilde{\phi})^4 + \tilde{\phi}\tilde{T} \right]$

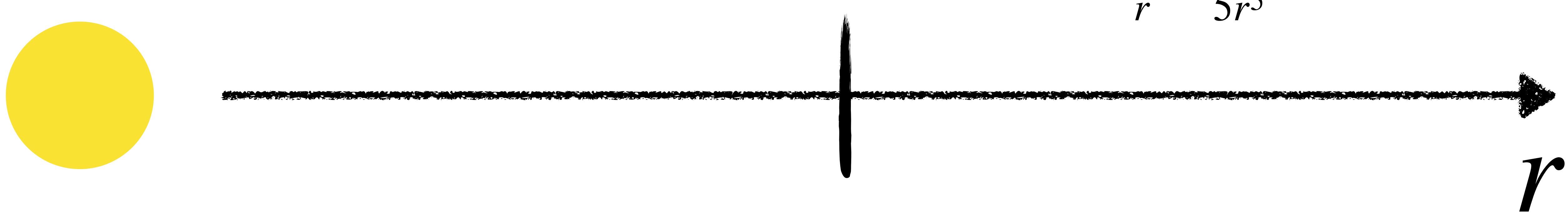
(K-Mouflage)

Barreira et al 2015

$$\phi' + (\phi')^3 = \frac{M}{r^2}$$

$$\Rightarrow \phi(r) = -\frac{M}{r}\,{}_3F_2\left(\frac{1}{4},\frac{1}{3},\frac{2}{3};\frac{5}{4},\frac{3}{2};-\frac{27M^2}{4r^4}\right)$$

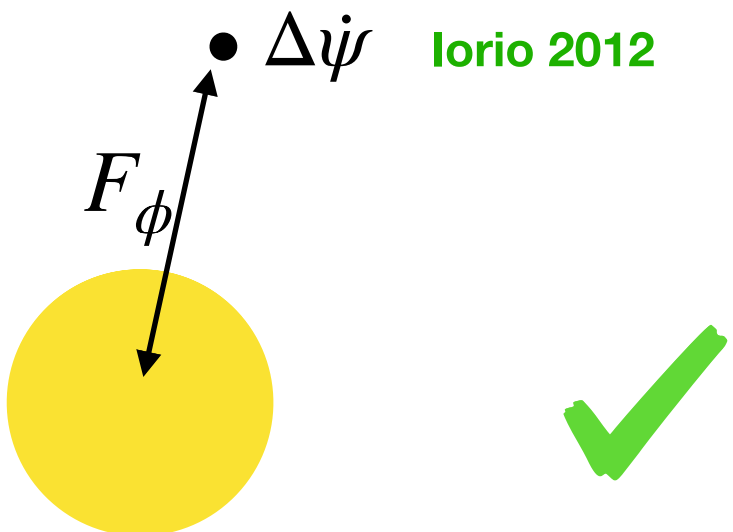
$$\phi(r) = 3(Mr)^{1/3} + \dots \qquad\qquad \phi(r) = -\frac{M}{r} + \frac{M^3}{5r^5} + \dots$$



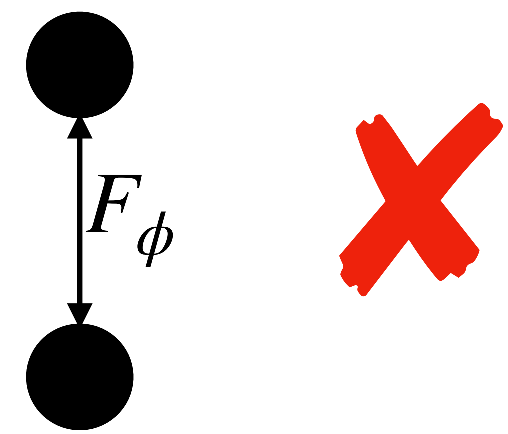
$r_* = \sqrt{M}(\simeq 1\mathrm{Pc}$ for the Sun)

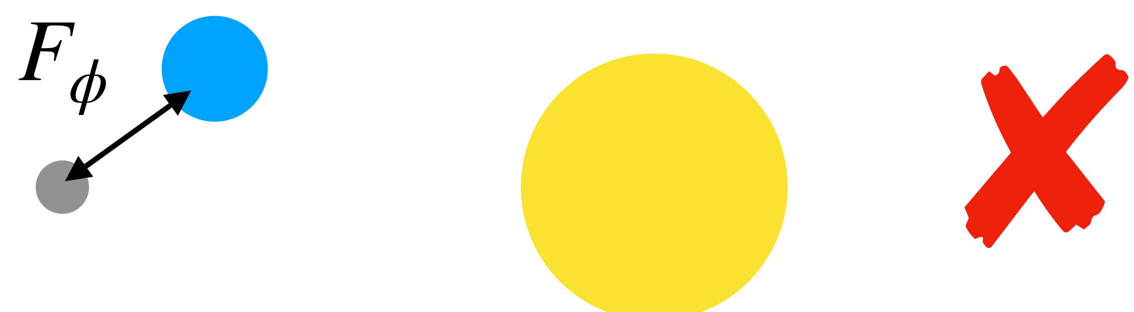
Motivation

Still small-scale tests of GR are very precise !



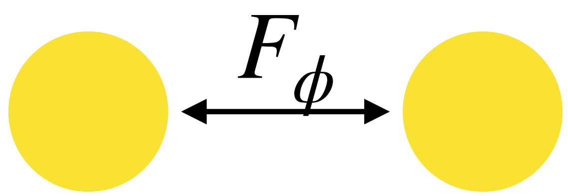
Iorio 2012





$$\Delta\dot{\psi} \sim \frac{\phi}{\phi_{\rm Newt}} \sim \left(\frac{r}{r_*}\right)^{4/3} \leq 10^{-11}$$

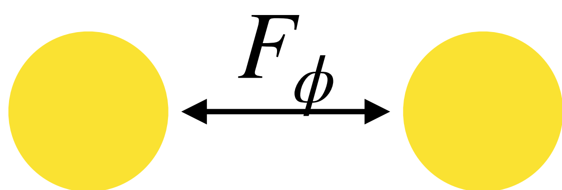
The two-body problem : outside



$$\tilde{S} = \int dt d^3x \left[-\frac{1}{2}(\nabla \tilde{\phi})^2 - \frac{1}{4}(\nabla \tilde{\phi})^4 + \tilde{\phi}\tilde{T} \right]$$

$$\tilde{T} = -m_1 \delta^3(\mathbf{x} - \mathbf{x}_1) - m_2 \delta^3(\mathbf{x} - \mathbf{x}_2)$$

The two-body problem : outside

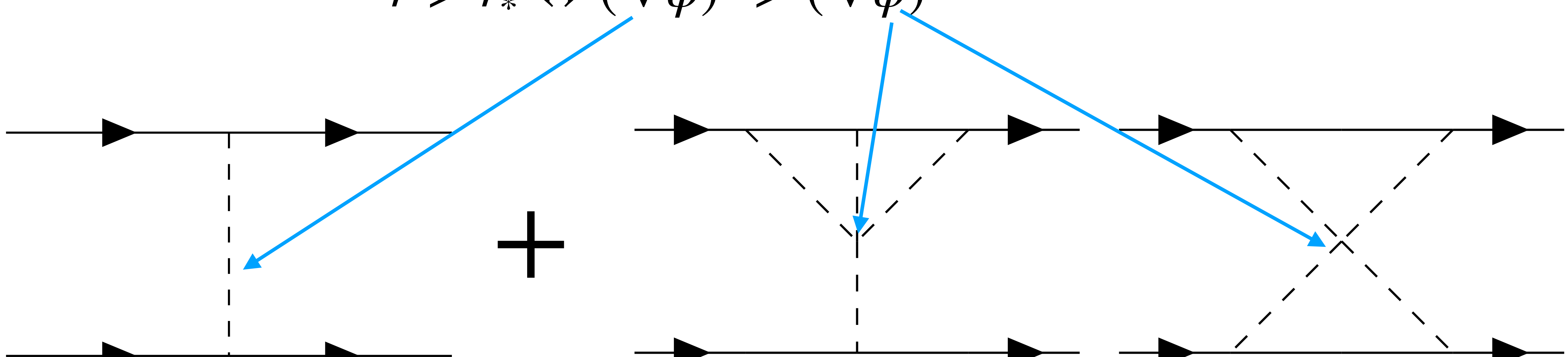


$$\tilde{S} = \int dt d^3x \left[-\frac{1}{2}(\nabla \tilde{\phi})^2 - \frac{1}{4}(\nabla \tilde{\phi})^4 + \tilde{\phi}\tilde{T} \right]$$

$$e^{iS_{\rm cl}[\mathbf{x}_1,\mathbf{x}_2]} = \int \mathscr{D}[\phi] e^{iS[\mathbf{x}_1,\mathbf{x}_2,\phi]}$$

$$r > r_* \Leftrightarrow (\nabla \phi)^2 > (\nabla \phi)^4$$

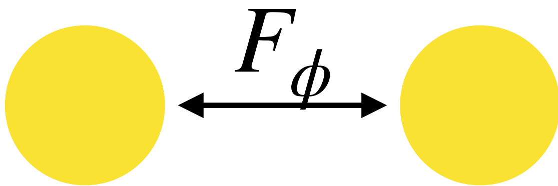
$S_{\rm cl} =$



(a) (b)

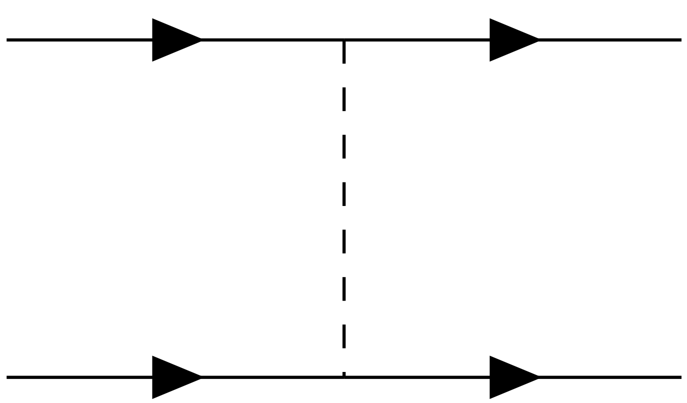
$+ \ldots$

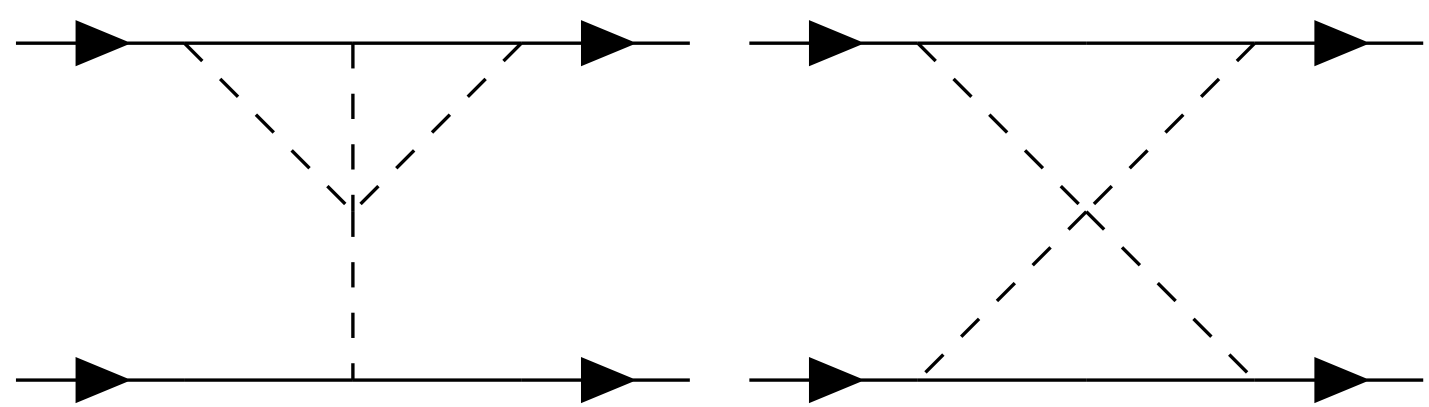
The two-body problem : outside



$$\tilde{S} = \int dt d^3x \left[-\frac{1}{2}(\nabla\tilde{\phi})^2 - \frac{1}{4}(\nabla\tilde{\phi})^4 + \tilde{\phi}\tilde{T} \right]$$

$$\int dtE = -S_{\rm cl} \Rightarrow \qquad E = -\frac{m_1 m_2}{r} + \frac{m_1 m_2 (m_1^2 + m_2^2)}{5r^5} + \dots$$

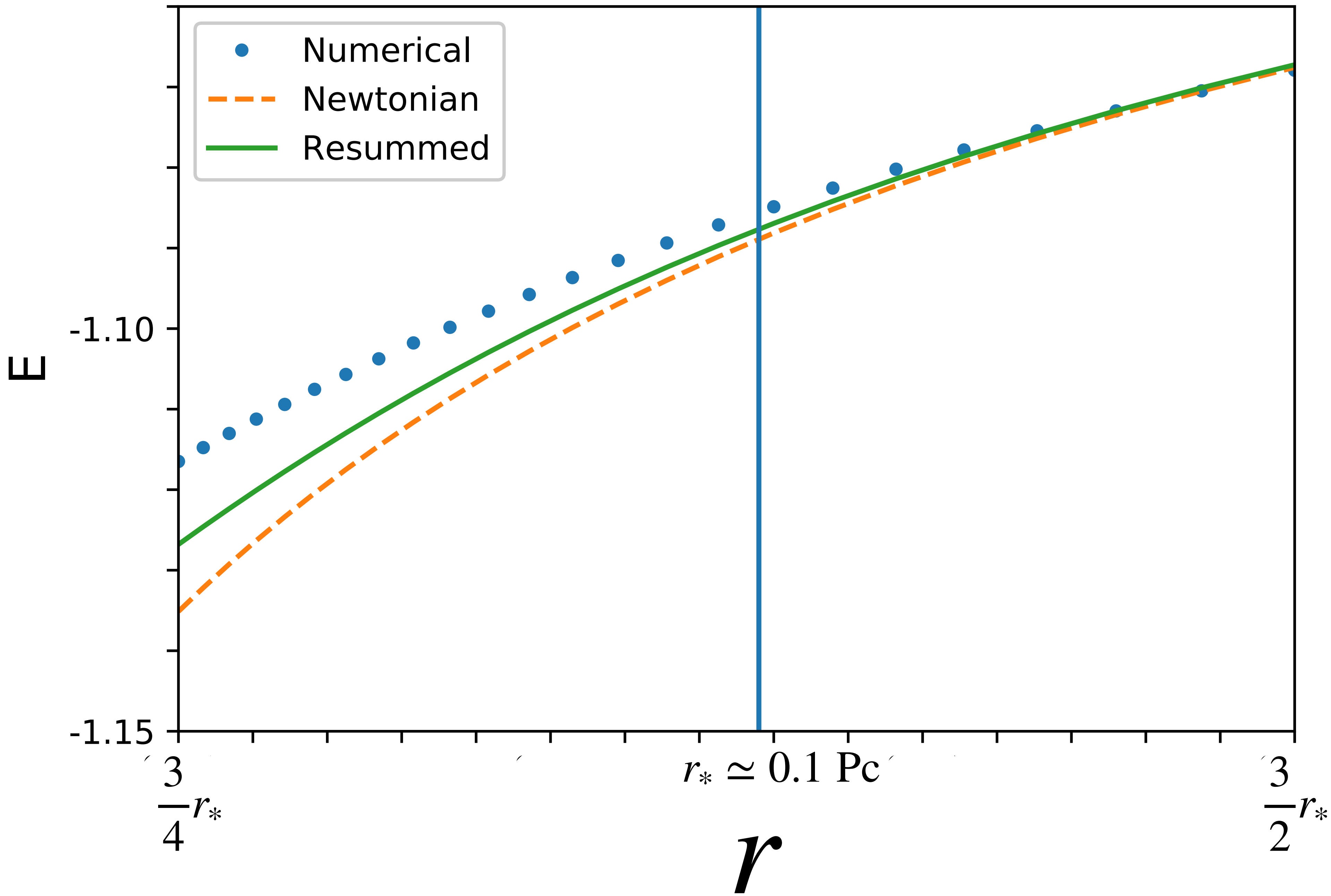




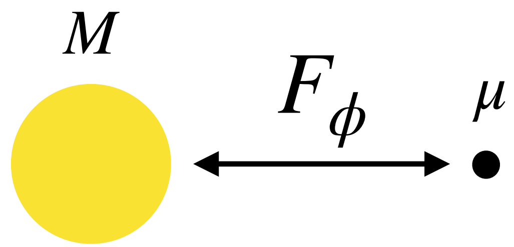
(a) (b)

The two-body problem : outside

$$E = -\frac{m_1 m_2}{r} + \frac{m_1 m_2 (m_1^2 + m_2^2)}{5 r^5} + \ldots$$

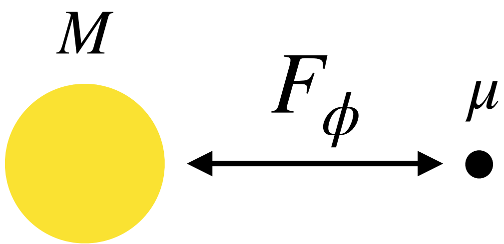


Effective One-Body (EOB) : outside

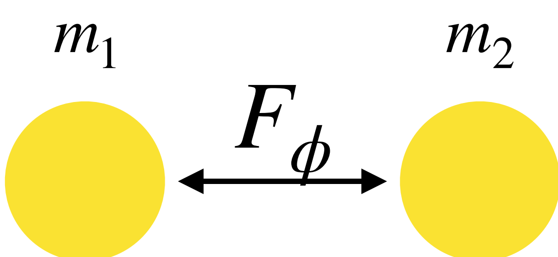


$$E_{\rm tm} = \mu \phi_{\rm tm}(r) = \mu \left(-\frac{M}{r} + \frac{M^3}{5r^5} + \ldots \right)$$

Effective One-Body (EOB) : outside

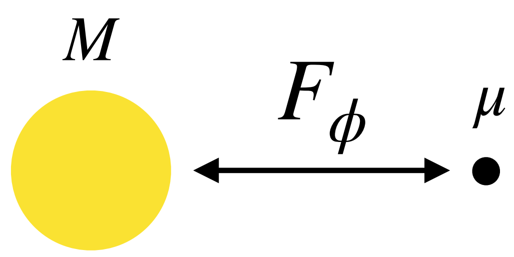


$$E_{\rm tm} = \mu \phi_{\rm tm}(r) = \mu \left(-\frac{M}{r} + \frac{M^3}{5r^5} + \ldots \right)$$

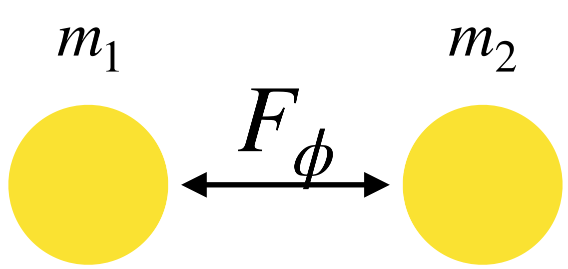


$$E = -\frac{m_1 m_2}{r} + \frac{m_1 m_2 (m_1^2 + m_2^2)}{5r^5} + \ldots$$

Effective One-Body (EOB) : outside



$$E_{\rm tm} = \mu\phi_{\rm tm}(r) = \mu\left(-\frac{M}{r} + \frac{M^3}{5r^5} + \ldots\right)$$



$$E = -\frac{m_1 m_2}{r} + \frac{m_1 m_2 (m_1^2 + m_2^2)}{5r^5} + \ldots$$

$$= \mu\left(-\frac{M}{r} + \frac{M^3}{5r^5}(x^2 + (1-x)^2) + \ldots\right)$$

$$\mu = \frac{m_1 m_2}{m_1 + m_2}$$

$$M = m_1 + m_2$$

$$x = \frac{m_1}{m_1 + m_2}$$

The two-body energy is a deformation of the test-mass energy

Damour & Buonanno 99

Energy map outside

Idea : resum nonlinearities by using only E_{tm}

$$E_{\rm tm} = \mu\left(-\frac{M}{r} + \frac{M^3}{5r^5} + \ldots\right) \quad E = \mu\left(-\frac{M}{r} + \frac{M^3}{5r^5}(x^2 + (1-x)^2) + \ldots\right)$$

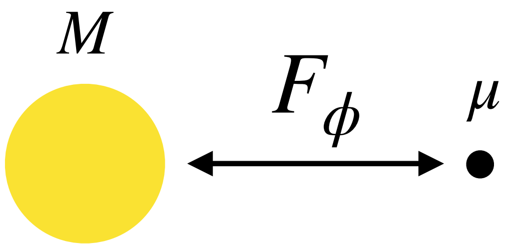
Energy map outside

Idea : resum nonlinearities by using only E_{tm}

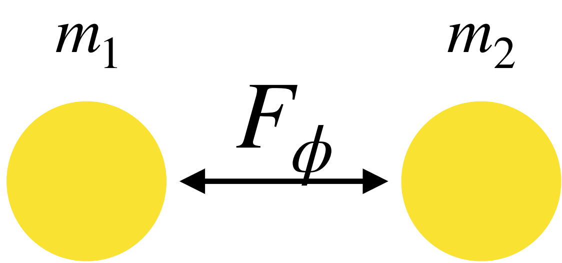
$$E_{\rm tm} = \mu\left(-\frac{M}{r} + \frac{M^3}{5r^5} + \ldots\right) \quad E = \mu\left(-\frac{M}{r} + \frac{M^3}{5r^5}(x^2 + (1-x)^2) + \ldots\right)$$

$$\frac{E}{E_{\rm tm}} = a_0 + a_1\left(\frac{E_{\rm tm}}{E_N} - 1\right) + a_2\left(\frac{E_{\rm tm}}{E_N} - 1\right)^2 + \ldots$$

Inside the nonlinear radius

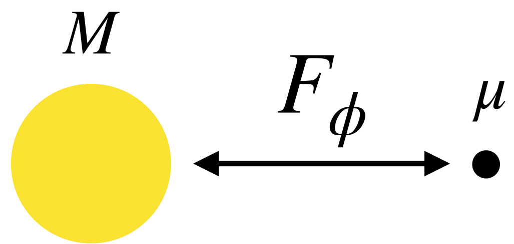


$$E_{\rm tm} = 3\mu\,(Mr)^{1/3} + \ldots$$

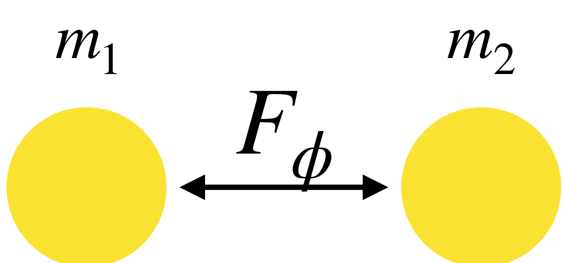


?

Inside the nonlinear radius

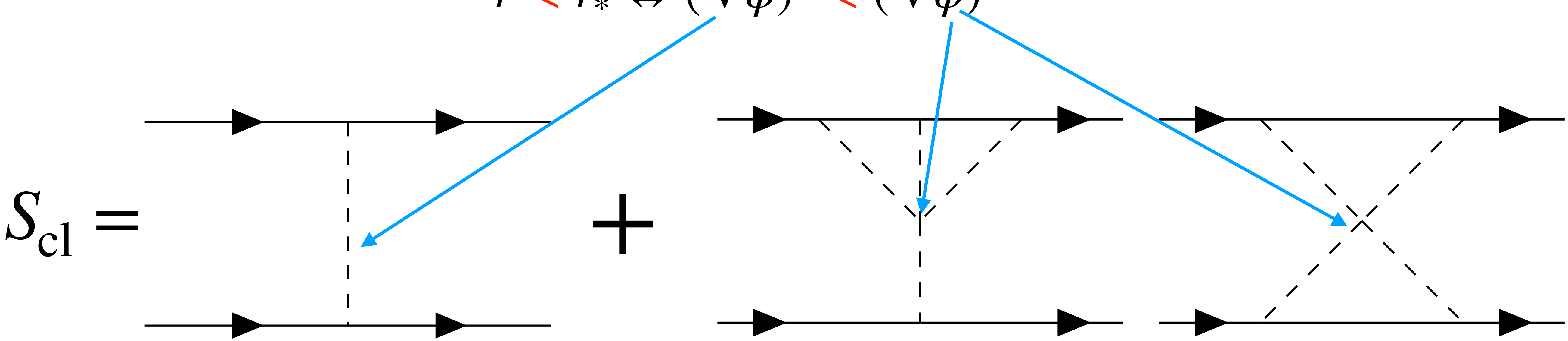


$$E_{\mathrm{tm}} = 3\mu\,(Mr)^{1/3} + \ldots$$



?

$$r < r_* \Leftrightarrow (\nabla\phi)^2 < (\nabla\phi)^4$$



(a) (b)

$$+\ldots$$

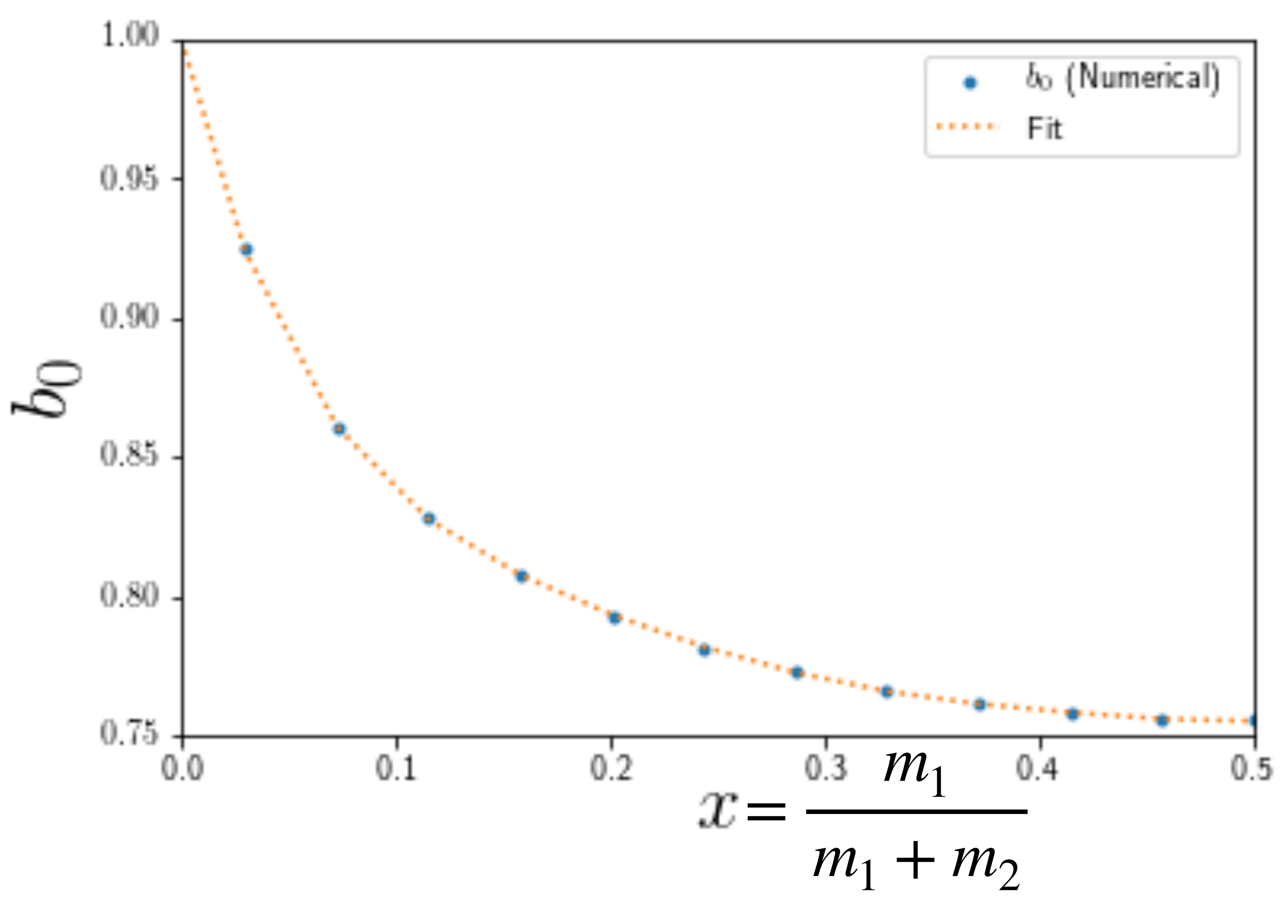
Diverges

Inside the nonlinear radius

Idea: Postulate that the energy map is valid inside

$$\frac{E}{E_{\rm tm}} = \frac{E}{\mu\phi_{\rm tm}(r)} = b_0(x) + \ldots$$

One cannot compute b_0, but one can get it with a numerical simulation !



EP violation

$E = \mu b_0(x)\phi_{\mathrm{tm}}(r)$

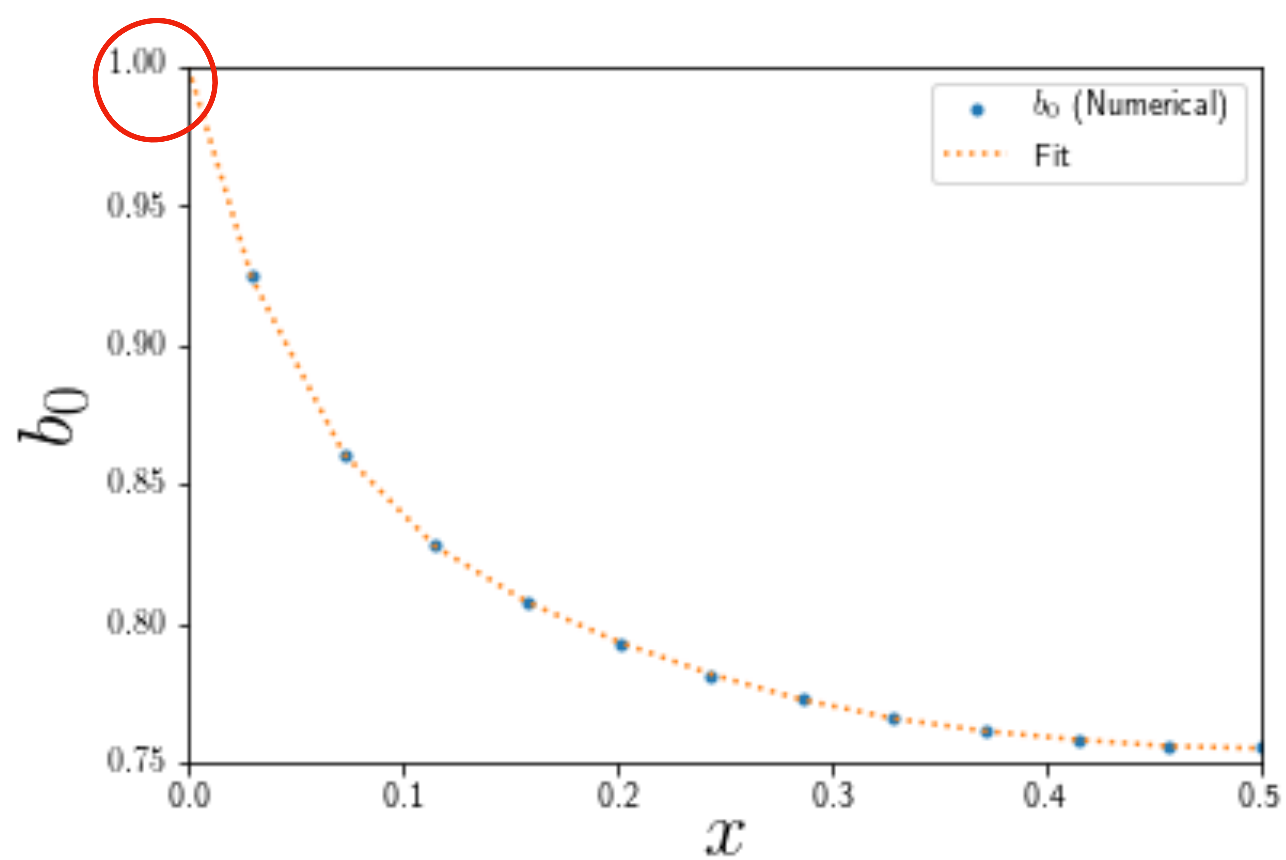
The Moon is a test-mass :

$$\vec{F}_M = -\vec{\nabla} E_M \simeq -m_M \vec{\nabla}\phi_S(r)$$

and

$$\vec{F}_M = m_M \vec{a}_M$$

$\Rightarrow$ $\vec{a}_M = \vec{\nabla}\phi_S(r)$ does not depend on m_M



EP violation

$$E = \mu b_0(x)\phi_{\rm tm}(r)$$

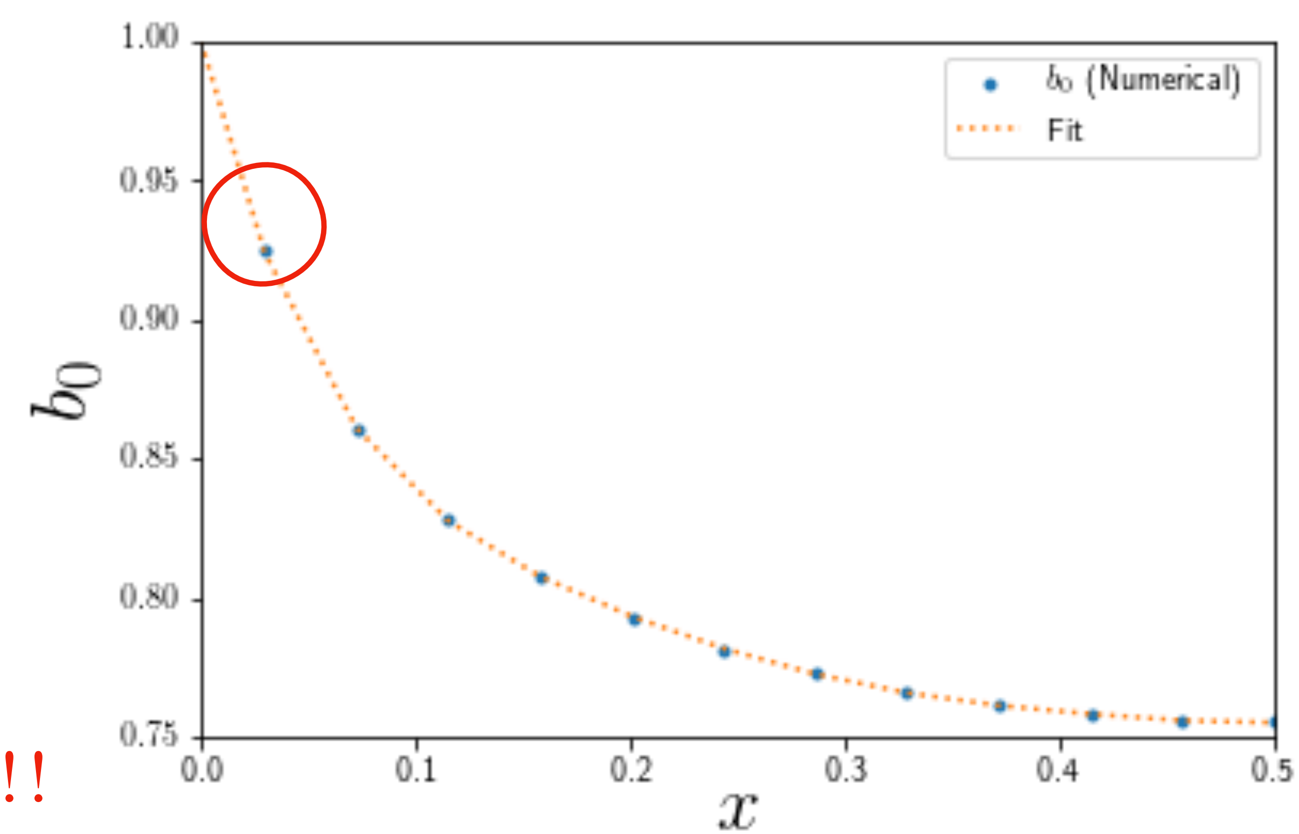
The Earth is not a test-mass :

$$\vec{F}_E \simeq -m_E\, b_0(x_{SE})\,\vec{\nabla}\,\phi_S(r)$$

and

$$\vec{F}_E = m_E \vec{a}_E$$

$\Rightarrow \quad \vec{a}_E = b_0(x_{SE})\,\vec{\nabla}\,\phi_S(r)$ depends on x_{SE} !!



The Sun-Earth-Moon system

$$\delta r_{EM} \simeq 3 \times 10^{12} \left| x_{SE} \left(\frac{r}{r_*} \right)^{4/3} \right| \text{ cm}$$

This gives a constraint :

$$x_{SE} \left(\frac{r}{r_*} \right)^{4/3} \lesssim 10^{-13}$$

Since $x_{\rm SE} \simeq 10^{-6}$, the perihelion constraint is better :

$$\left(\frac{r}{r_*} \right)^{4/3} \lesssim 10^{-11}$$

Conclusions

- The two-body problem triggers lots of developments in analytic GR. They can be transposed to modified gravity.

- Future directions : conservative and dissipative dynamics of inspiralling compact objects with screening ; GW from hairy black holes (analytic results in the extreme mass-ratio case)