EFFECTIVE FIELD THEORIES

L'Agape 2018
Adrien KUNTZ

Or why in real world we do not care about high-energy physics

<u>Foreword</u> : → EFT is the formalization of the idea that we do not have to know the precise form of the interaction between quarks to do solid ~~state~~ ~~physics~~ mechanics.

→ To do so, we should eliminate the degrees of freedom (dofs) with energy higher than the dofs we are interested in. These high-energy dofs are allowed to exist on short time scales by quantum mechanics $\Delta E \Delta t \sim \hbar$

→ A very important principle is to use symmetries to constrain low-energy physics

<u>Plan</u> : I Introduction : Electromagnetism (EM) in materials.

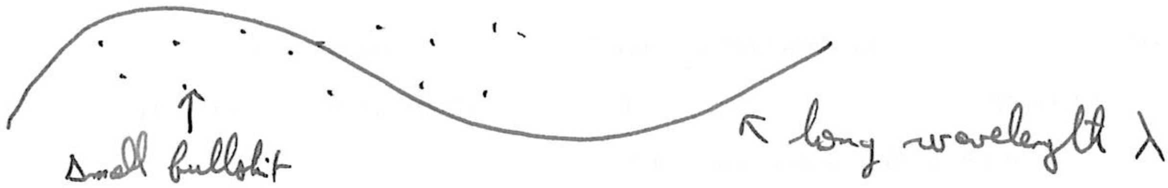
II Fermi weak interactions

~~III~~ Euler - Heisenberg hamiltonian

~~IV~~ Rayleigh scattering & the blue sky

V "Quantum" gravity

Intro EFT: EM in dielectrics



How to describe this?

Observat°: the speed of light is not c but $\frac{c}{n}$

Let's construct an EM theory invariant under the Lorentz group of velocity $\frac{c}{n}$

$x^\mu = \left(\frac{ct}{n}, \vec{x}\right) \qquad p^\mu = \left(\frac{nE}{c}, \vec{p}\right) \qquad A^\mu = \left(\frac{n\phi}{c}, \vec{A}\right)$

$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is gauge-invariant

Effective Lagrangian: $\mathcal{L} = -\frac{a}{4} F_{\mu\nu} F^{\mu\nu}$ with $a = \frac{1}{\mu}$ a cst

$\Rightarrow \mathcal{L} = +\frac{1}{2\mu}\left(\frac{n^2}{c^2} E^2 - B^2\right)$

Write $\frac{n^2}{c^2} = \varepsilon\mu \Rightarrow \boxed{\mathcal{L} = +\frac{1}{2}\left(\varepsilon_0 E^2 - \frac{B^2}{\mu}\right)}$

Conclusion: Only a change in $\left| \begin{array}{l} \varepsilon_0 \to \varepsilon \\ \mu_0 \to \mu \end{array} \right.$ ie renormalisat° of long wavelength observable parameters.

This is the essence of an EFT:

→ Focus on long-wavelength physics

→ Use the symmetries to constrain the operators that appear in the Lagrangian

→ The modificat° brought by short-distance physics appears in a set of coefficients (ε, μ) that you can measure from experiment <u>OR</u> calculate if you know short-dist physics

3 examples where the full theory is too complicated at low energy to make any prediction and so we have to resort to EFT:

→ Beyond the Standard Model: we don't know of a possible UV completion

→ Quarks at low energy: We cannot make any prediction using the QCD Lagrangian because it is strongly coupled (highly nonlinear & nonperturbative). Using EFT arguments (chiral perturbation theory χPT), we find that quarks form bound objects named baryons and mesons at low E.

complicated physics

low E χPT perturbative — high E QCD perturbative → E

→ In cosmology, we describe the universe's matter by a background + small perturbation energy density $\rho = \bar{\rho}\,(1+\delta)$.

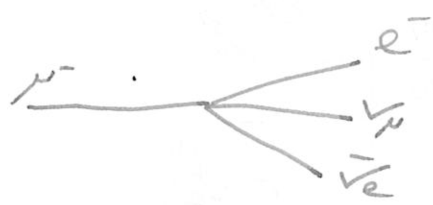
This evidently fails on small scales, where you have stars and void, but this is quite accurate on large scales (millions of galaxies).

Taking into account the effects of short-distance physics on the large-scale density contrast δ is the programme of the EFT of large-scale structures.

It works exactly like EM in materials, by replacing atoms with galaxies.

II Fermi weak interactions.

This is the usual textbook example of an EFT, and will introduce the methodology involved.



Experimental observations:

1) Interactions at short-distances: We can use a point-like approximation

2) We observe only left-handed neutrinos ($\to$ use $P_L = \frac{1-\gamma^5}{2}$)

3) The process has constant strength in energy ($\to$ no derivatives)

4) The interactions preserve chirality

Recall the formalism of Dirac spinors:

$$\psi = \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix} \quad \bar\psi = \begin{pmatrix} \bar\psi_R \\ \bar\psi_L \end{pmatrix} \quad \gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu \\ \bar\sigma^\mu & 0 \end{pmatrix} \quad \begin{array}{l} \sigma^\mu = (\mathbb{1}, \sigma^i) \\ \bar\sigma^\mu = (\mathbb{1}, -\sigma^i) \end{array}$$

$P_L = \frac{1-\gamma^5}{2}$ with $\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3$, is the left-handed projector.

Let's try some combinations:

$\bar\psi\psi = \bar\psi_R\psi_L + \bar\psi_L\psi_R$ do not preserve chirality 4)

$\bar\psi P_L \psi = \bar\psi_R \psi_L$

$\bar\psi\gamma^\mu\psi = \bar\psi_R\sigma^\mu\psi_R + \bar\psi_L\bar\sigma^\mu\psi_L$ preserves chirality but involves right-handed particles (contradict 2))

$\bar\psi\gamma^\mu P_L\psi = \bar\psi_L\bar\sigma^\mu\psi_L$ is OK!

So a term allowed in the Lagrangian is:

$$\mathcal{L}_{eff} \supset -4G_F(\bar{\nu}_\mu \gamma^\alpha P_L \mu)(\bar{e}\gamma_\alpha P_L \nu_e).$$

$\boxed{\text{Rq: dimension of } G_F \quad [G_F] = -2 \ (\text{in mass})}$

and the matrix element for the process is $i\times$ Vertex

Let's now try to derive this vertex from the short-distance physics, which we know is the theory of weak interactions:

$$\mathcal{L}_{UV} \supset \frac{g}{\sqrt{2}} \bar{\nu}_\mu \gamma^\alpha P_L \mu W_\alpha^\pm + \frac{g}{\sqrt{2}} \bar{e}\gamma^\alpha P_L \nu_e W_\alpha^\pm$$

We will "integrate out" the heavy W boson which does not appear at low energy:

$$\mathcal{M} = \left(\frac{ig}{\sqrt{2}}\right)^2 (\bar{\nu}_\mu \gamma^\alpha P_L \mu)(\bar{e}\gamma^\beta P_L \nu_e) \frac{-i g_{\alpha\beta}}{p^2 - M_W^2}$$

Since $p \sim$ Energy of the process $\ll M_W^2$, $\frac{1}{p^2-M_W^2} = \frac{-1}{M_W^2}\left(1 + \frac{p^2}{M_W^2} + \dots\right)$

$$\Rightarrow \mathcal{M} = -\frac{ig}{2M_W^2}(\bar{\nu}_\mu \gamma^\alpha P_L \mu)(\bar{e}\gamma_\alpha P_L \nu_e) + \mathcal{O}\left(\frac{1}{M_W^4}\right)$$

$$\Rightarrow \boxed{G_F = \frac{g^2}{8M_W^2}}$$

- We see that the size of the high-energy physics (M_W) appears in the negative dimension of G_F. This is a generic feature of EFTs and we will use it heavily from now on.

• We can also compute the $\frac{p^2}{M_W^4}$ correction, that will depend on the energy of the process, contrary to the lowest-order term (according to point 3)).

II Euler - Heisenberg Hamiltonian - Schwinger effect

In EM we describe photons by a vector field A_μ subject to a gauge invariance $A_\mu \to A_\mu + \partial_\mu \alpha$.

2 gauge-invariant quantities are $\left| \begin{array}{l} F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \\ \tilde{F}_{\mu\nu} = \varepsilon_{\mu\nu\rho\sigma} F^{\rho\sigma} \end{array} \right.$

The usual kinetic term of EM is

$$\boxed{\mathcal{L}_0 = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}}$$

Question: What other invariants can we put in $\mathcal{L}_{EM}$?

~~→ $F_{\mu\nu} \tilde{F}^{\mu\nu}$ is a surface~~

Recall: $F_{\mu\nu} F^{\mu\nu} \sim E^2 - B^2$ in term of the electric/magnetic fields $\vec{E}, \vec{B}$.
$F_{\mu\nu} \tilde{F}^{\mu\nu} \sim \vec{E} \cdot \vec{B}$

→ $F_{\mu\nu} \tilde{F}^{\mu\nu}$ is a surface term. Moreover, it is forbidden by parity (i.e looking in a mirror) in which $\left| \begin{array}{l} E \to E \\ B \to -B \end{array} \right.$ and which is a symmetry of EM.

Parity :

→ $F_{\mu\nu} \Box F^{\mu\nu}$ goes into the propagator

→ $F_{\mu\nu} F_{\nu\lambda} F_{\lambda\mu}$ is forbidden by charge conjugation in which $A_\mu \to -A_\mu$

→ $(F_{\mu\nu} F^{\mu\nu})^2$ and $(F_{\mu\nu} \tilde{F}^{\mu\nu})^2$ are 2 allowed operators

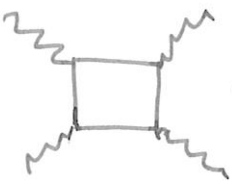
→ All other operators obtained by contracting differently the indices or changing $F \to \tilde{F}$ are either linear combinations of these 2 ones or forbidden by parity.

Since $[\mathcal{L}] = 4$ and $[F] = 2$, we can write the following interaction lagrangian:

$$\boxed{\mathcal{L}_{int} = \frac{\alpha^2}{m_e^4} \left(C_1 (F_{\mu\nu} F^{\mu\nu})^2 + C_2 (F_{\mu\nu} \tilde{F}^{\mu\nu})^2 \right).}$$

• The ~~size~~ high-energy particle that we are integrating out is the electron in QED, so there is $\frac{1}{m_e^4}$

• I've put an $\alpha^2 = e^4$ (electron charge) because these interaction physically come from this Feynman diagram in QED:


+perm

where there are 4 vertex, each proportional to e.

Calculating this diagram gives the value of the numerical coefficients C_1, C_2: $C_1 = \frac{1}{90}$ $C_2 = \frac{7}{90}$

(eq 4-125 in Itzykson - Zuber)

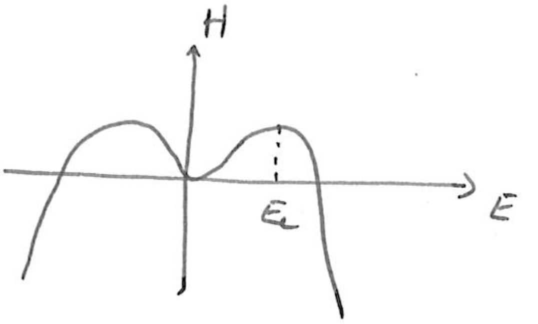
→ This 4-photon interaction has been observed recently at LHC (arxiv: 1702.01625).

Schwinger effect : instability of the vacuum

If we concentrate only on the electric field, the total Lagrangian is of the form:

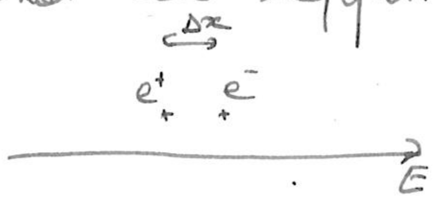
$$\mathcal{L} \sim \frac{E^2}{2} + \frac{\alpha^2}{m_e^4} C_1 E^4$$

So the Hamiltonian is: $H \sim \frac{E^2}{2} - \frac{\alpha^2}{m_e^4} C_1 E^4$ $\quad \underline{C_1 > 0}$



It seems that for $E > E_c$, the vacuum is unstable and the electric field grows to infinity!!

What does happen physically?



Fluctuations of e^+e^- in the vacuum.

To create a pair out of the vacuum, one should have

$$\underbrace{U}_{\text{potential energy of the pair}} \gtrsim \underbrace{2m}_{\text{rest energy of the pair}}$$

Since $\Delta x \sim m^{-1}$, $U = eE\Delta x = \frac{eE}{m}$ $\Rightarrow$ $\boxed{E \gtrsim \frac{m^2}{e} = E_c}$

Then the e^+e^- pair will follow the E-field and screen the charge distribution producing this E-field

$\Rightarrow$ <u>The maximum E-field that one can obtain is E_c</u>

It is way ~~below~~ above any experimental reach.

IV Rayleigh scattering

There are several energy scales in the problem:

→ Photon energy $E_\gamma \sim 2\,eV$

→ Excitation energy of the atom $\Delta E \sim 13{,}6\,eV$

→ Inverse size of the atom $r_0^{-1} \sim 2\,keV$

→ Mass of the atom $M_{atom} \sim 1\,GeV$.

We want to construct an effective theory for this scattering

1) Non-Relativistic scaling

The atom is modeled by a scalar field ϕ which is highly non-relativistic. This means that since $\phi \sim e^{iEt} a_k^\dagger + e^{-iEt} a_k$ and $E \sim M$, there is a rapid oscillation in ϕ that we can factor out. So let's write:

$$\phi(x,t) = \frac{1}{\sqrt{2M}} \left(e^{-iMt} \psi(x,t) + cc \right) \qquad (*)$$

where ψ is complex and $\dot\psi \ll M\psi$. Then:

$$\mathcal{L}_\phi = \frac{1}{2}\left(\dot\phi^2 - (\nabla\phi)^2 \pm M^2 \phi^2 \right) \Rightarrow \boxed{\mathcal{L}_\psi = \psi^* \left(i\partial_t + \frac{\nabla^2}{2M} \right) \psi}$$

(we drop quantities with unequal powers of ψ^* and ψ since they oscillate $\sim e^{iMt}$ and vanish once integrated).

From this we see that since $[\mathcal{L}] = 4$, $\boxed{[\psi] = \frac{3}{2}}$

Because of the rewriting $(*)$, $\partial_\mu \psi$ does not have a good transformation under Lorentz boost, contrary to $\partial_\mu \phi$.

To build out quantities that are Lorentz invariant in the interaction Lagrangian, we should rather use $v^\mu = (1,0,0,0)$ which is the velocity of the atom (expressed here in its rest frame).

2) <u>Building the interaction Lagrangian</u>

The kinetic Lagrangian for a NR atom + photons is:

$$\boxed{\mathcal{L}_0 = \psi^\dagger \left(i\partial_t + \frac{\nabla^2}{2m} \right) \psi - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}}$$

What are the interaction allowed?

We should respect gauge invariance ($\to F_{\mu\nu}$) and Lorentz invariance (contract $F_{\mu\nu}$ with v_μ and ∂_μ).

Moreover the atom number is conserved: we should use only $\psi^\dagger\psi$.

- $v^\mu v^\nu F_{\mu\nu} = 0$
- ① $\psi^\dagger \psi F_{\mu\nu} F^{\mu\nu}$ and ② $\psi^\dagger \psi\, v_\alpha F^{\alpha\mu} v^\beta F_{\beta\mu}$ are allowed
- ③ $\psi^\dagger \psi\, v_\alpha \partial^\alpha (F_{\mu\nu} F^{\mu\nu})$ is allowed but higher order in derivatives

Power counting : $[F] = 2 \quad [\psi] = \frac{3}{2} \quad [v] = 0$

$$\Rightarrow \left| \begin{array}{l} [①] = [②] = 7 \\ [③] = 8 \end{array} \right.$$

$$\boxed{\begin{aligned} \mathcal{L}_{int} = {} & c_1 \psi^\dagger \psi F_{\mu\nu} F^{\mu\nu} + c_2 \psi^\dagger \psi\, v^\alpha F_{\alpha\mu} v_\beta F^{\beta\mu} \\ & + c_3 \psi^\dagger \psi\, v^\alpha \partial_\alpha (F_{\mu\nu} F^{\mu\nu}) \end{aligned}}$$

$[c_1] = [c_2] = -3 \qquad [c_3] = -4.$

C_1, C_2 are built out of $\Delta E, r_0^{-1}, M$. ~~...~~ (not E_γ since this is a variable of the problem (incoming energy) and not related to an energy cutoff characteristic of the ~~..~~ high-energy physics of the atom that we are integrating out).

Which energy to choose?

- M would mean that we probe the ~~..~~ Compton wavelength of the atom. But we are not in a particle physics collider!
- If we refer to an EM course, we know that the process is classical, so it should not refer to the quantum ΔE.

Conclusion : $\boxed{C_1 \simeq C_2 \simeq r_0^3}$

The cross-section for this process is $\sigma \sim |\mathcal{M}|^2 \propto r_0^6$.

Since $[\sigma] = -2$, $\boxed{\sigma \propto E_\gamma^4 r_0^6}$

This is the origin of the blue sky...

C_3 brings a correction to this cross-section. What is its size?

$\mathcal{E} C_3 \sim \frac{C_{1,2}}{\Lambda}$ where Λ is an energy cutoff

Λ should be the lowest cutoff available, which is ΔE.

~~Th~~ So the effect of the last operator in $\mathcal{L}_{int}$ (containing one more derivative $\partial \sim E_\gamma$) on σ is:

$$\boxed{\sigma \propto E_\gamma^4 r_0^6 \left(1 + \mathcal{O}\left(\frac{E_\gamma}{\Delta E}\right)\right)}$$

V "Quantum" gravity.

1) Planck - suppressed operators

The usual kinetic term in GR is:

$$S_{EH} = m_p^2 \int d^4x \sqrt{-g}\, R .$$

Are there higher order operators allowed?
The ~~symmetry~~ that we can use to constrain them is of course reparametrisation invariance → use $R_{\mu\nu\rho\sigma}$

$$\Rightarrow \boxed{S_{int} = m_p^2 \int d^4x \sqrt{-g}\, \frac{R^2}{m_p^2}} + \text{(other quadratic combinations of } R_{\mu\nu\rho\sigma})$$

The scale of the high-energy physics that we integrate out is, of course, the quantum gravity scale m_P.
This means that if you ever find a good quantum gravity theory, it will have to fit in these operators at low energy.
Since m_P is so much larger than any energy scale we have access to ($m_P \sim 10^{19}$ GeV), the effect of these operators is totally unobservable at our energies.
This does not mean that effective theories are totally useless in ~~cosmology~~ gravity, and I will show a beautiful example: NRGR

2) Intermezzo: Why are loops quantum

Take a simple scalar field theory:

$$S = \int d^4x \left[(\partial\phi)^2 - m^2\phi^2 + V(\phi) + J\phi \right]$$

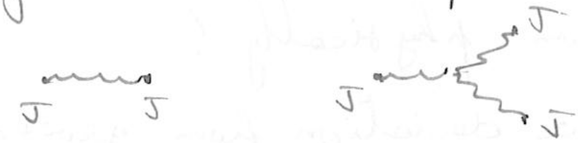
In this part, we will reintroduce $\hbar = 10^{-34}$ J.s.
What is the dimension of S?

↳ $S \sim \int dt\, L$ and L = Energy = J ⇒ $[S]$ = J.s.

In the path integral approach, we should consequently have a weight $e^{iS/\hbar}$

This means that each vertex ($\sim \frac{V(\phi)}{\hbar}$) is multiplied by $\frac{1}{\hbar}$ and each propagator (inverse of $\frac{(\partial\phi)^2}{\hbar}$) is multiplied by $\hbar$.

Diagrams without loops:



\# vertex = \# propo + 1

$\rightarrow$ multiply by $\frac{1}{\hbar}$

Diagrams with loops:

\# vertex = \# propo + 1 − \# loops

$\Rightarrow$ multiply by $\hbar^{\#\text{loops}-1}$

This is why we say that loops are quantum.

In the following, we will concentrate on "Tree-level" diagrams (without loops), which corresponds to take the classical action (obtained by solving the eq. of motion) in the path-integral approach.

3) Non-Relativistic General Relativity (NRGR) hep-th/0409156

Consider 2 neutron stars or black holes:

$\uparrow v$ $\qquad v^2 \sim \frac{GM}{r} << 1$

$v \downarrow$ $\qquad$ ↑ virial th

$S = S_{EH} + S_m$. How to build S_m?

Symmetries:

$\rightarrow$ Diffeo invariance: use $R_{\mu\nu\rho\sigma}$

→ Reparametrization invariance along the worldline of the particles : $\lambda_a \to \tilde{\lambda}_a(\lambda_a)$ → use $d\tau^2 = g_{\mu\nu}\, dx^\mu dx^\nu$

<u>Lowest order operator</u>: $\boxed{-m\int d\tau} = -m\int dt\, \sqrt{g_{\mu\nu}u^\mu u^\nu}$
where u^μ is the 4-velocity of the particle.

<u>Higher order</u>: $c_R\int d\tau\, R + c_V\int d\tau\, R_{\mu\nu}\dot{x}^\mu \dot{x}^\nu$

Where do these operator come from physically?

We can see that they induce a deviation from geodesic motion, since $\delta\left(-m\int d\tau\right) = 0 \Rightarrow \frac{Du^\nu}{d\tau} = 0$.

They come from the <u>finite-size</u> ($\Leftrightarrow$ not a point-particle) of the object under consideration.

Then we linearize around flat space: $\boxed{g_{\mu\nu} = \eta_{\mu\nu} + \frac{h_{\mu\nu}}{m_p}}$

(around an object, $\frac{h_{\mu\nu}}{m_p}$ can be identified with the Newtonian potential $\frac{GM}{r}$, and so this breaks down when $\frac{GM}{r} \sim 1 \Rightarrow r \sim r_s$ Schwartzchild radius).

Since $u^\mu = (1, \vec{v})$, $\eta_{\mu\nu}u^\mu u^\nu = 1 - v^2$

So the lowest-order operator can be written as:

$$S_m = -m\int dt\, \sqrt{1 - v^2 + \frac{h_{\mu\nu}}{m_p}u^\mu u^\nu}$$

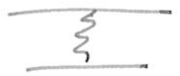
$$= -m\int dt + \underbrace{m\int dt\, \frac{v^2}{2}}_{\text{Kinetic energy}} - \underbrace{\frac{m}{2m_p}\int dt\, h_{00}}_{\text{Interaction 1-graviton 1-source}}$$

If we consider 2 bodies following a path $x_a(t)$ and we integrate out the graviton following the path-integral procedure:

$$e^{iS_{eff}[x_a]} = \int D[h_{\mu\nu}]\, e^{i S_{EH}[h_{\mu\nu}] + S_{pp,1} + S_{pp,2}}$$

Then this generate Feynman diagrams which correspond to an expansion in v of the energy of the 2-body system

The lowest-order diagram can be shown to be:

 $= i\int dt \, \frac{G m_1 m_2}{|x_1 - x_2|}$ which is the Newtonian energy.

At higher order in v, we obtain the Einstein-Infeld-Hoffmann Lagrangian which corresponds to the first relativistic correction to the Newtonian energy ($\rightarrow$ precession of Mercury perihelion).

At even higher order, we can treat in a nice way the effect of the finite-size operators that we mentioned earlier.

<u>Further reading</u>:

This course is heavily based on:

→ Kaplan, "Five lectures on EFT"

→ The course of Slava Rychkov, available on the google site QFT@ENS