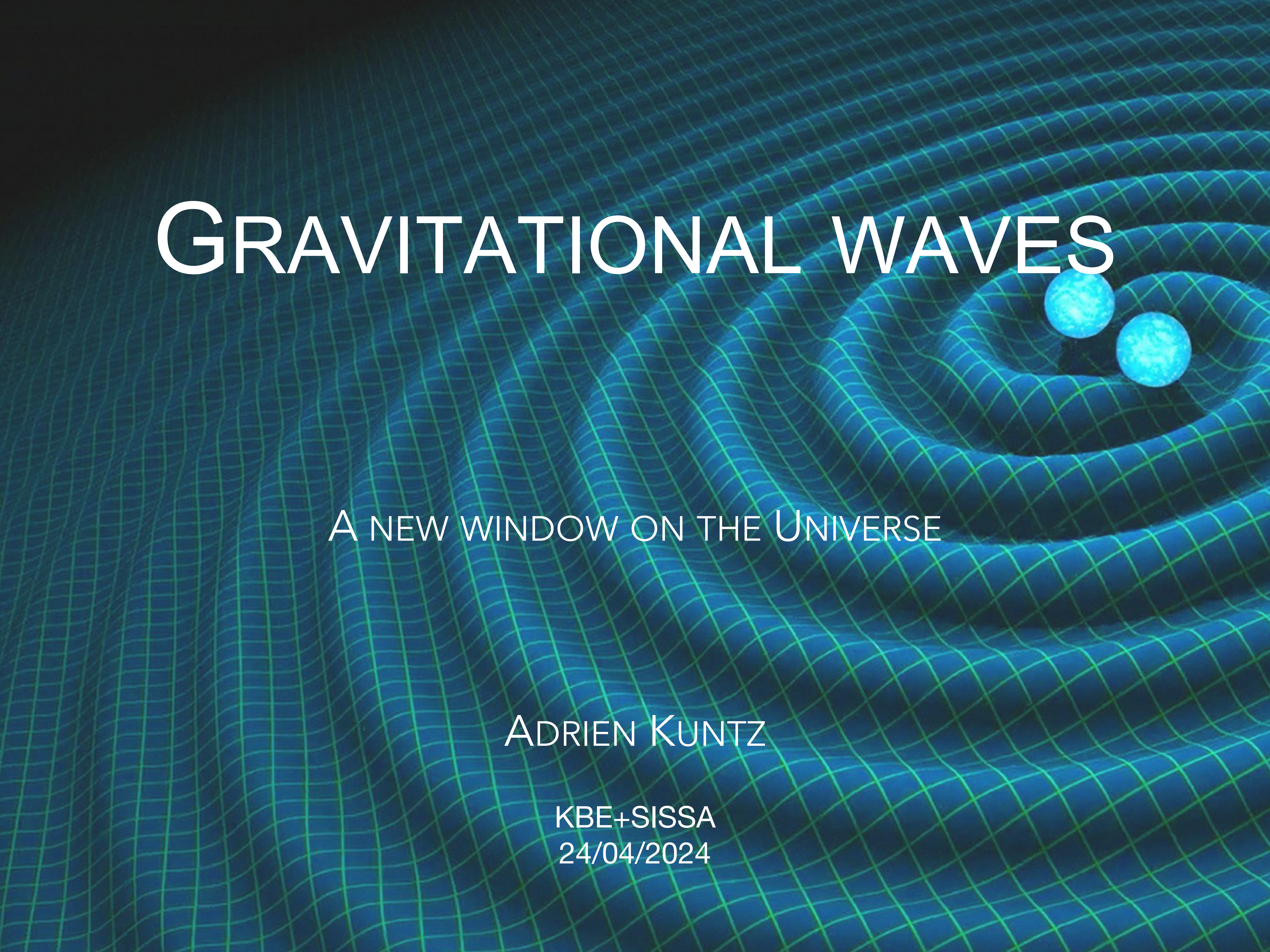


GRAVITATIONAL WAVES

A NEW WINDOW ON THE UNIVERSE

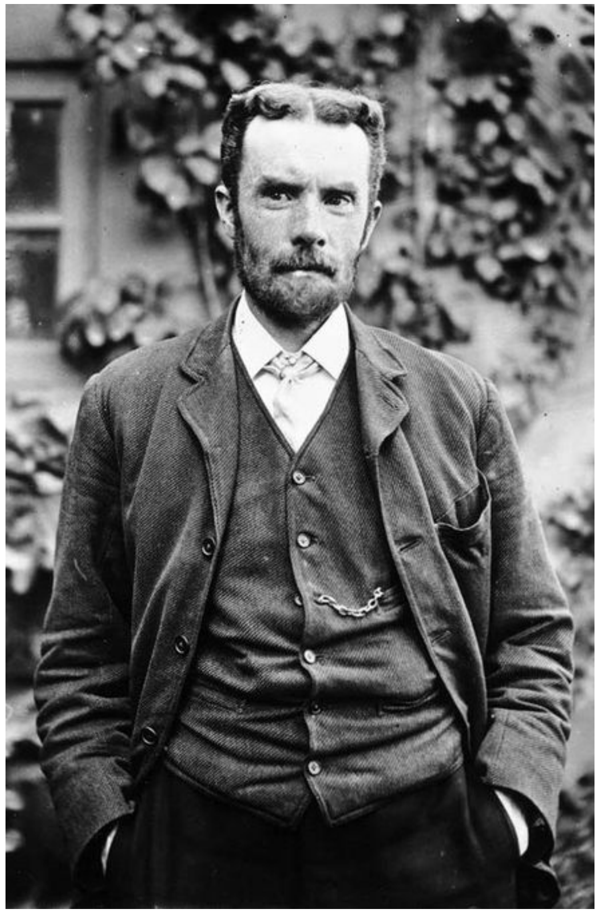
ADRIEN KUNTZ

KBE+SISSA

24/04/2024

Part I : Short history of GW

A BIT OF HISTORY



Gravity propagates (1893)

$$\nu^2 \nabla^2 \mathbf{e} = \frac{\partial^2 \mathbf{e}}{\partial t^2},$$

1905

Quand nous parlerons donc de la position ou de la vitesse du corps attirant, il s'agira de cette position ou de cette vitesse à l'instant où l'*onde gravifique* est partie de ce corps; quand nous parlerons de la position ou de la vitesse du corps attiré, il s'agira de cette position ou de cette vitesse à l'instant où ce corps attiré a été atteint par l'onde gravifique émanée de l'autre corps; il est clair que le premier instant est antérieur au second.

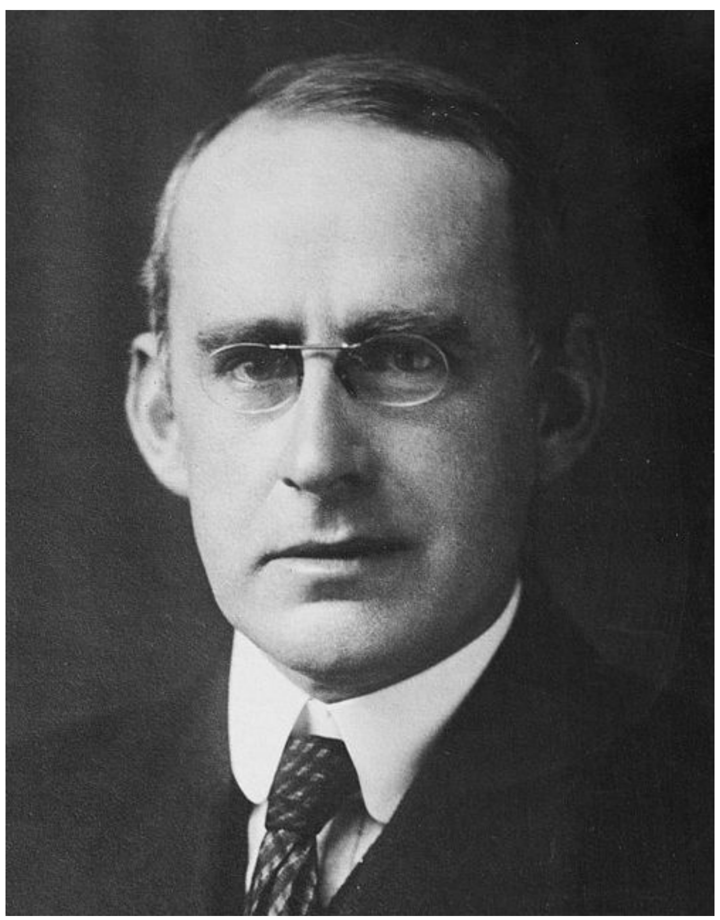
Accelerating mass should produce GW

A BIT OF HISTORY

1916: generation of GW is quadrupolar

$$h = \frac{2G}{c^4}\frac{1}{r}\frac{\partial^2 Q}{\partial t^2}$$

Transverse-transverse	h_{22}, h_{33}, h_{23}.
Longitudinal-transverse	h_{12}, h_{13}, h_{24}, h_{34}.
Longitudinal-longitudinal	h_{11}, h_{14}, h_{44}.



1922

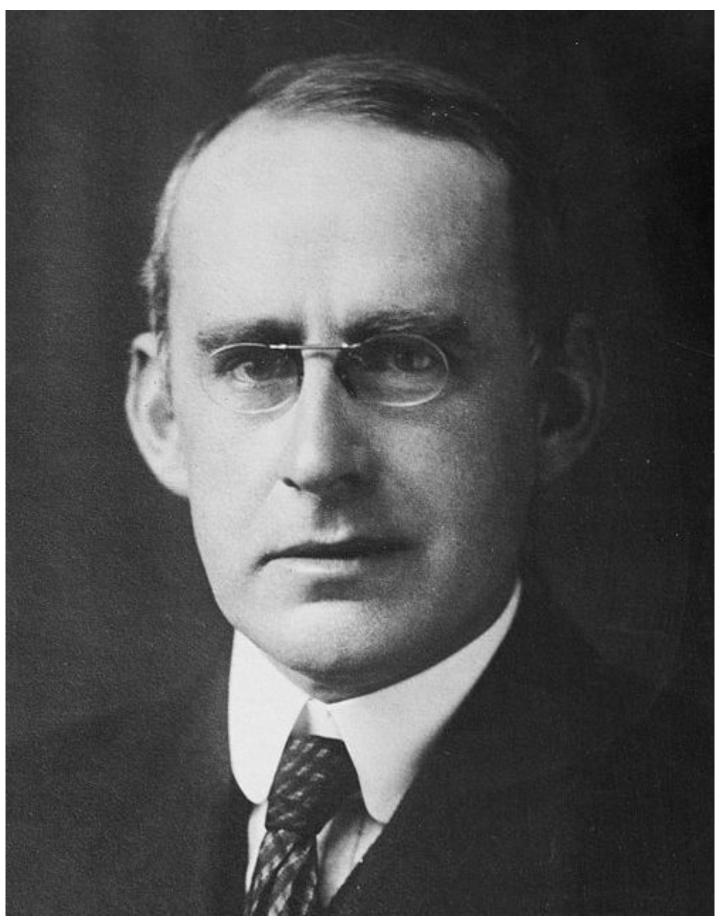
They are not objective, and (like absolute velocity) are not detectable by any conceivable experiment. They are merely sinuosities in the co-ordinate-system, and the only speed of propagation relevant to them is "the speed of thought."

A BIT OF HISTORY

$$h = \frac{2G}{c^4}\frac{1}{r}\frac{\partial^2 Q}{\partial t^2}$$

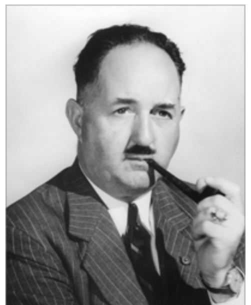
Transverse-transverse	h_{22}, h_{33}, h_{23}.
Longitudinal-transverse	$h_{12}, h_{13}, h_{24}, h_{34}$.
Longitudinal-longitudinal	h_{11}, h_{14}, h_{44}.

Together with a young collaborator, I arrived at the interesting result that gravitational waves do not exist, though they had been assumed a certainty to the first approximation. (1936)

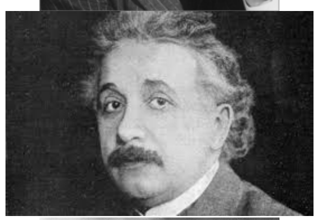


They are not objective, and (like absolute velocity) are not detectable by any conceivable experiment. They are merely sinuosities in the co-ordinate-system, and the only speed of propagation relevant to them is "the speed of thought."

TO CUT A LONG STORY SHORT…

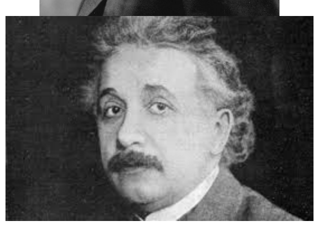


Robertson (referee) to Einstein and Rosen : your conclusion an artifact of your coordinate system



Einstein : I will never publish in Physical Review again

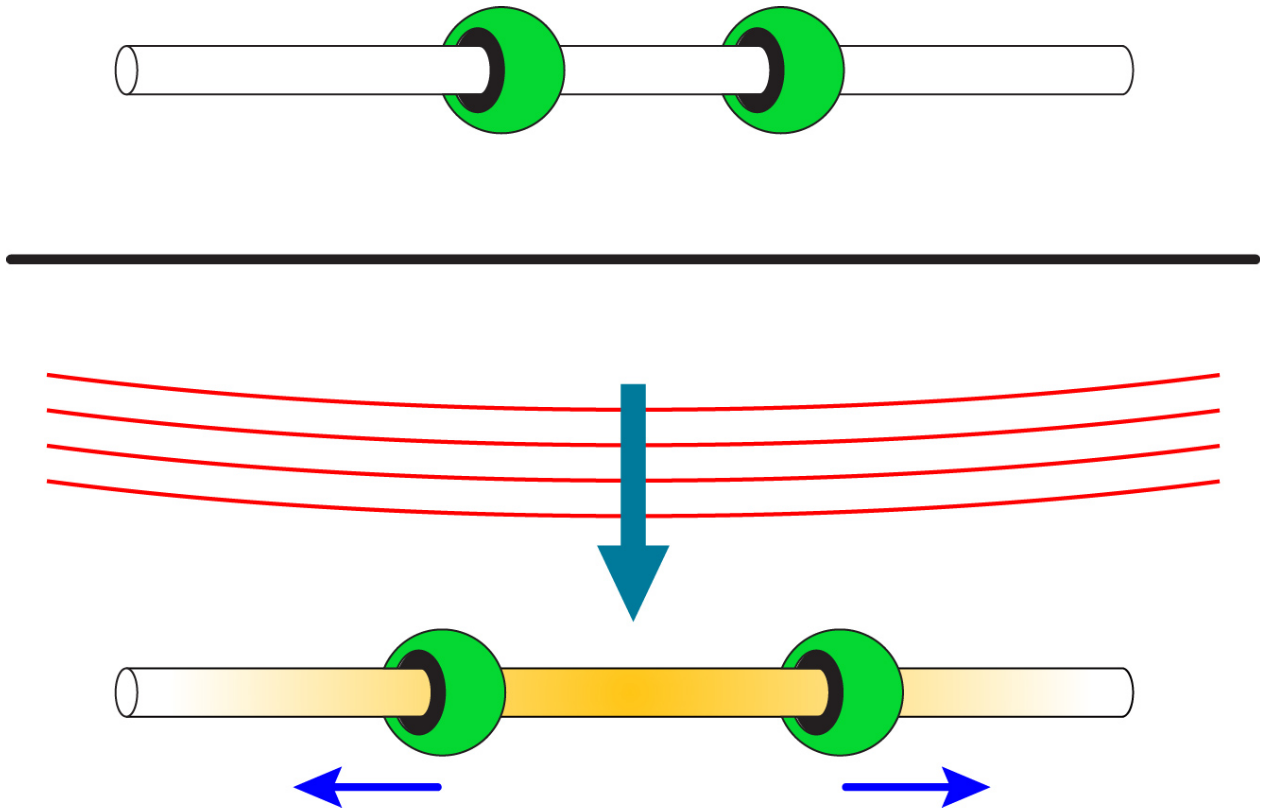
Infeld : Look, Einstein, Robertson was right



Einstein to Rosen (1937) : OK, so GW really exist. Let's just change the conclusion of our article.

Rosen : I still don't believe it and I prefer to publish my own version

LATER ON (1955)

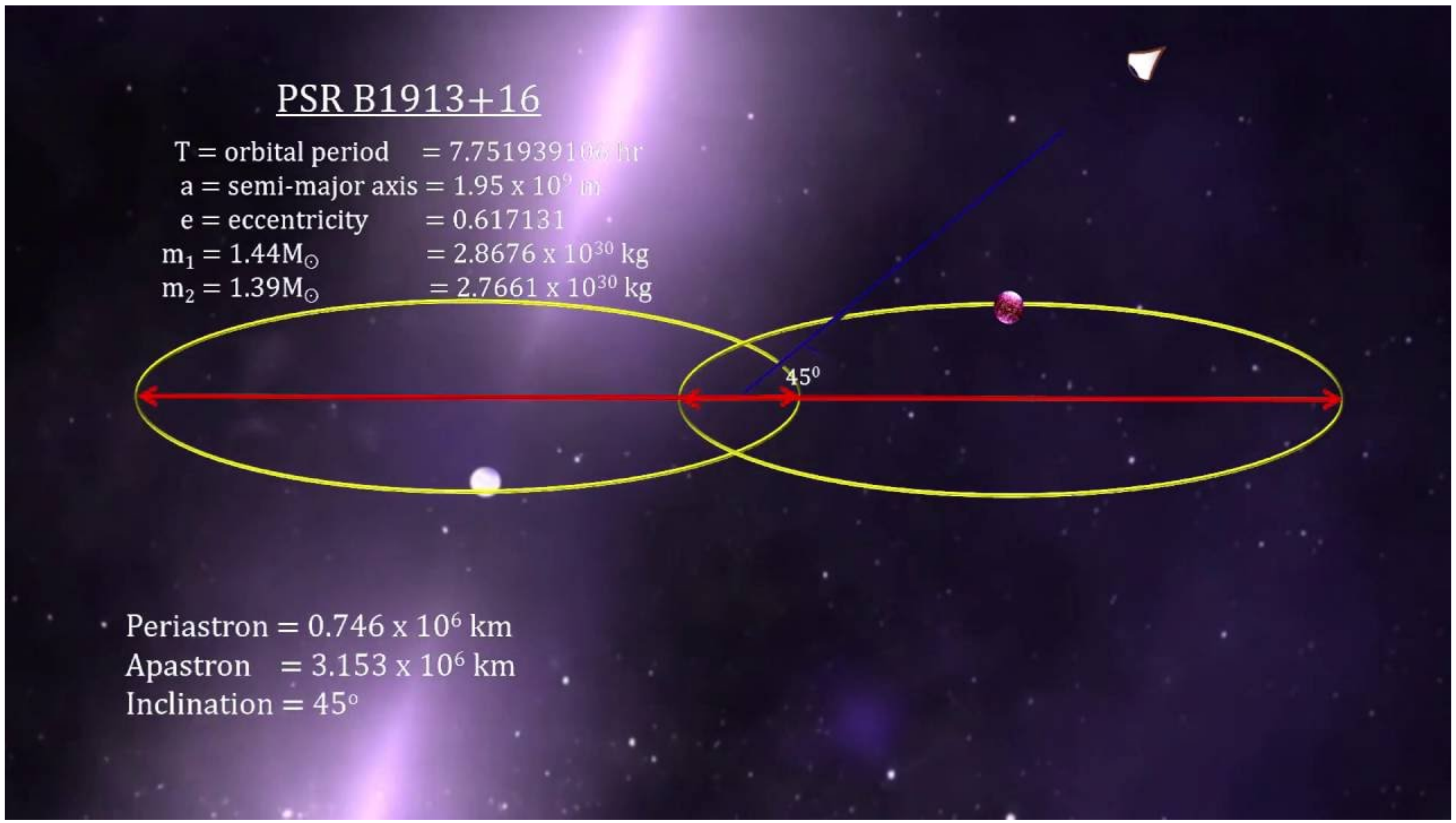


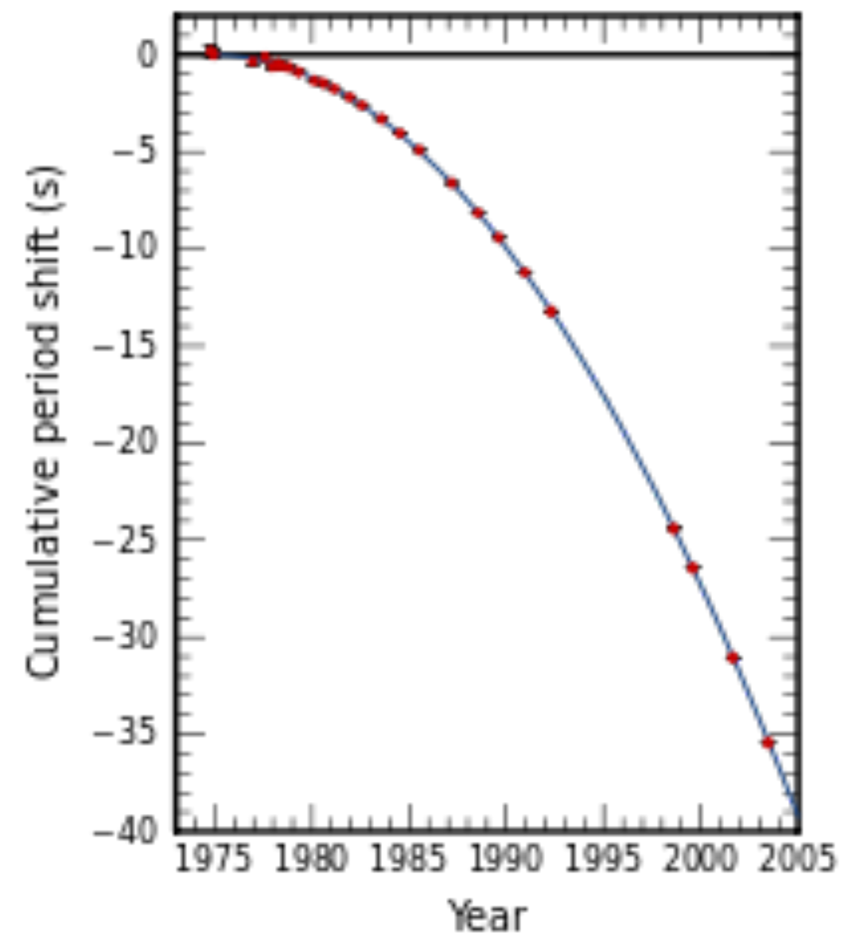
+ Pirani, Bondi, Wheeler..

Rosen still did not believe until the 70's…

THE HULSE-TAYLOR PULSAR

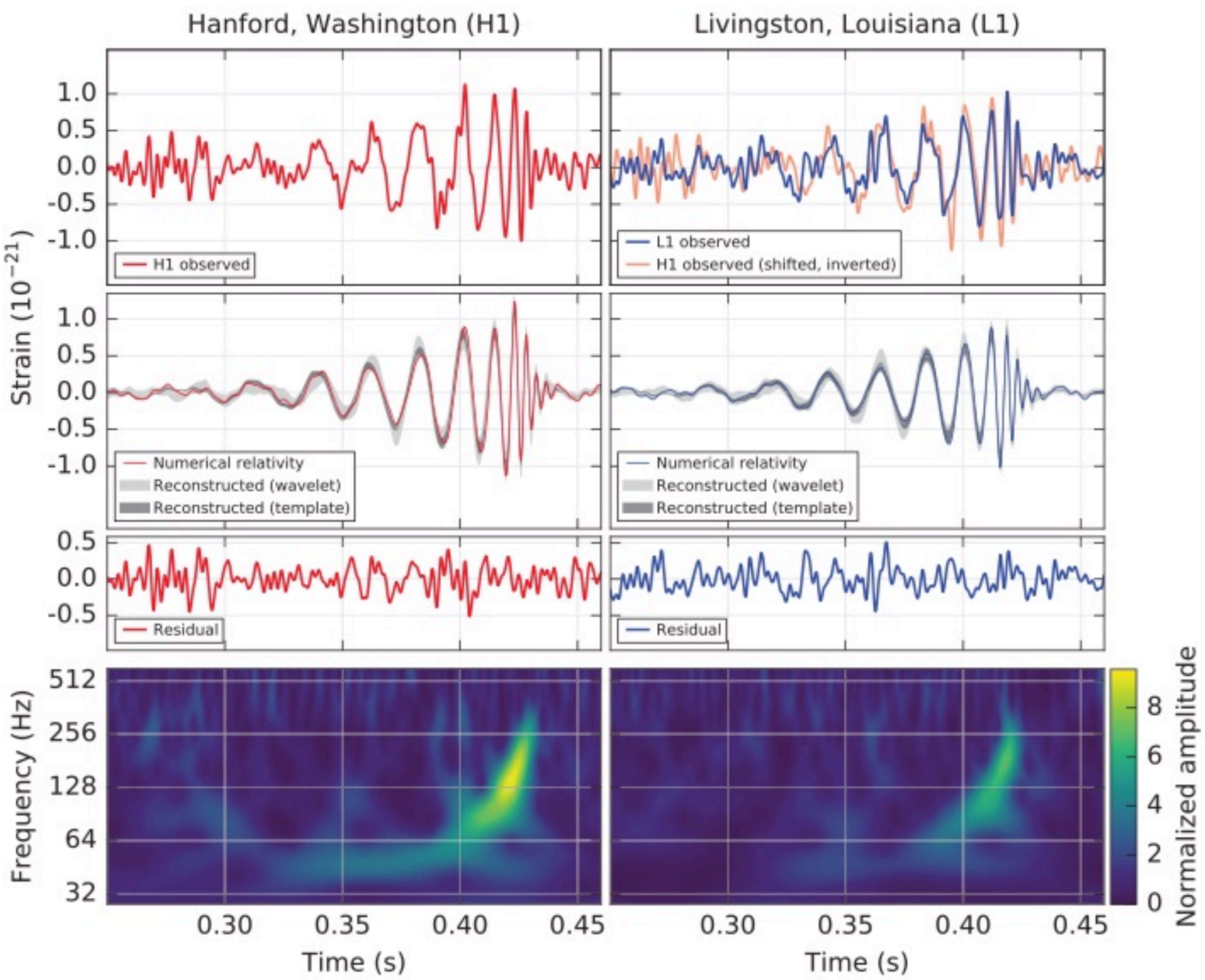
1974



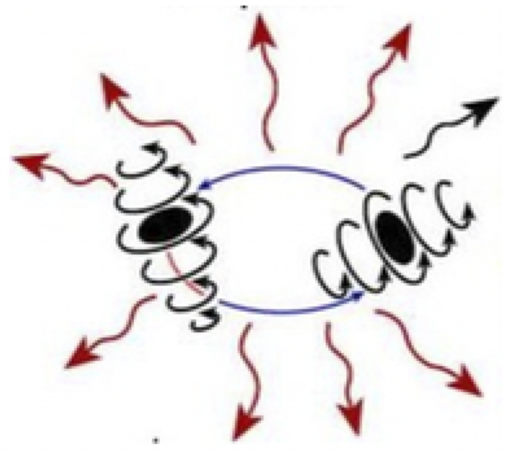


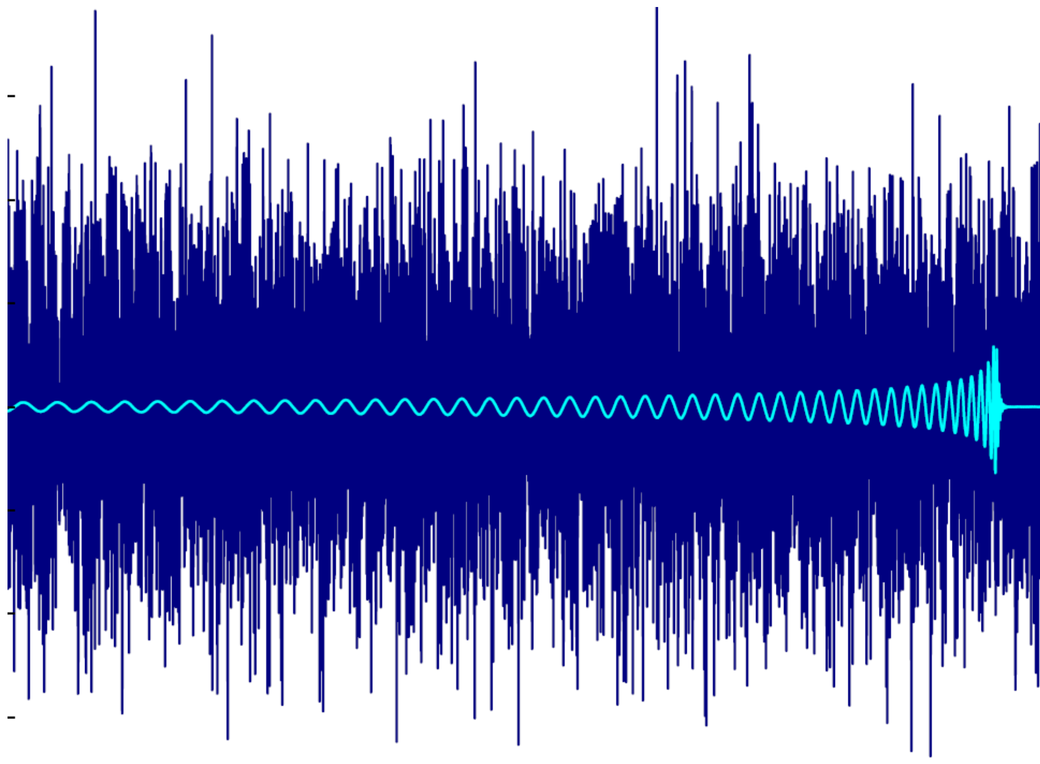
Consistent with GR at the 0.2% level !

TODAY



Hanford, Washington (H1)
Livingston, Louisiana (L1)
Strain (10^{-21})
H1 observed
L1 observed
H1 observed (shifted, inverted)
Numerical relativity
Reconstructed (wavelet)
Reconstructed (template)
Residual
Frequency (Hz)
Normalized amplitude
Time (s)



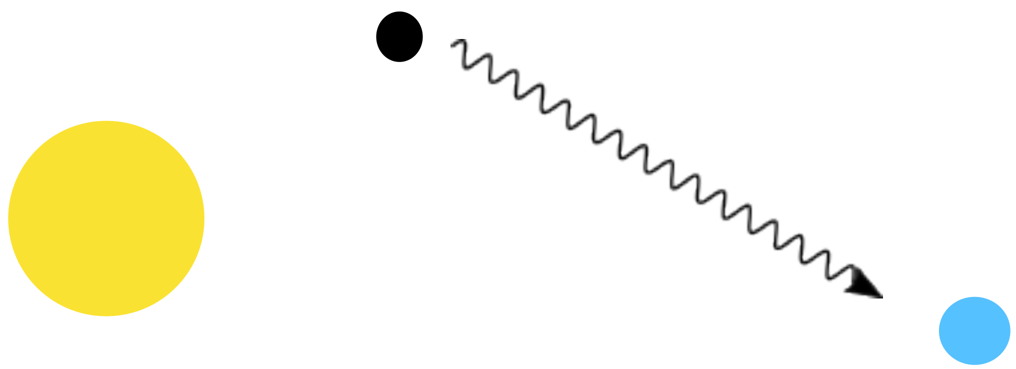


Part II : sources of GW

GW ARE WEAK

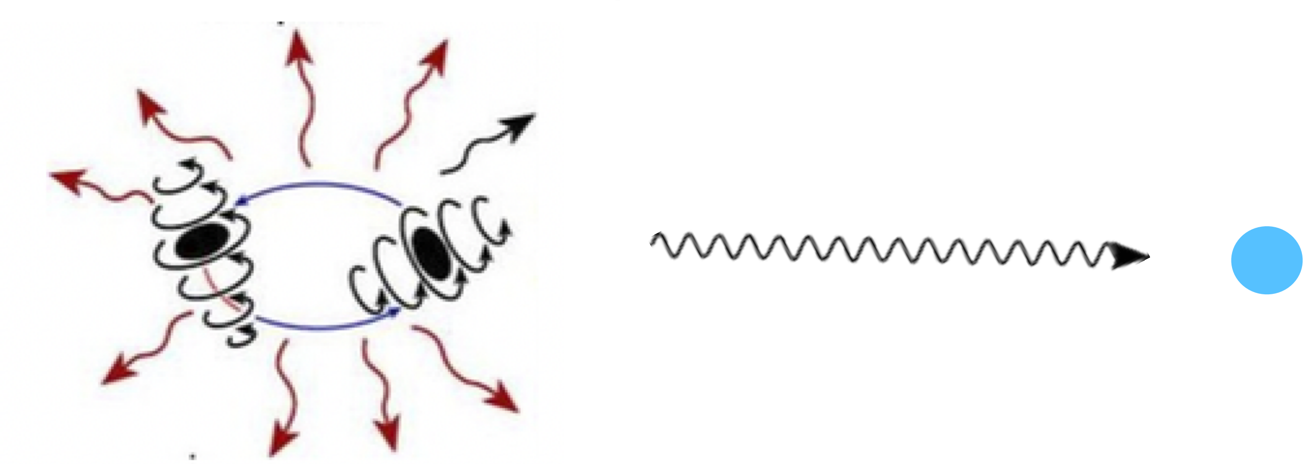
$$h = \frac{2G}{c^4}\frac{1}{r}\frac{\partial^2 Q}{\partial t^2}$$

Mercury



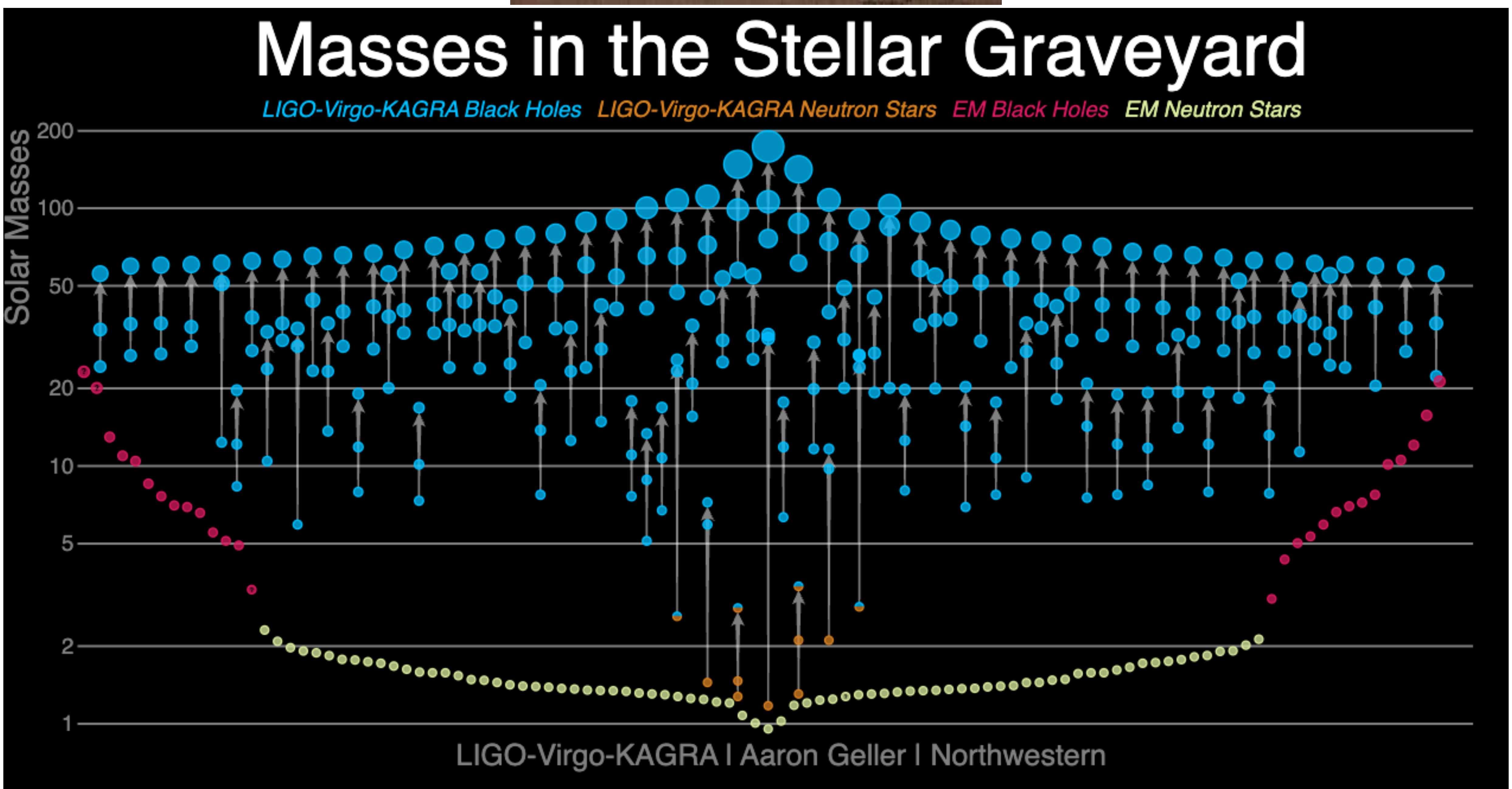
$$h \sim 10^{-23} \text{ at } 10^{-8}\text{Hz}$$

Black holes at 200 Mpc



$$h \sim 10^{-20} \text{ at } 100\text{Hz}$$

LIGO/VIRGO


Masses in the Stellar Graveyard
LIGO-Virgo-KAGRA Black Holes
LIGO-Virgo-KAGRA Neutron Stars
EM Black Holes
EM Neutron Stars
Solar Masses
200
100
50
20
10
5
2
1
LIGO-Virgo-KAGRA | Aaron Geller | Northwestern

LANDSCAPE

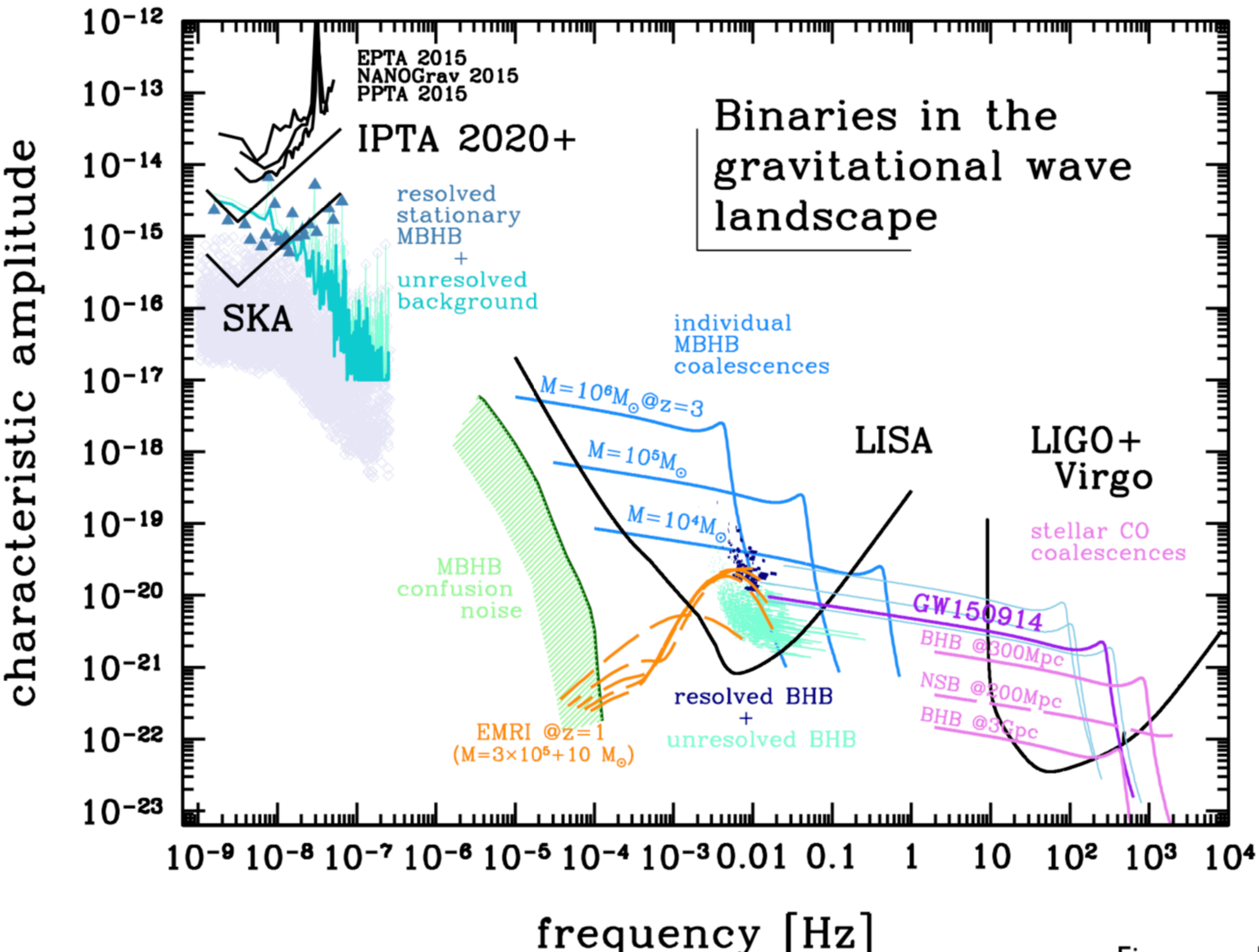
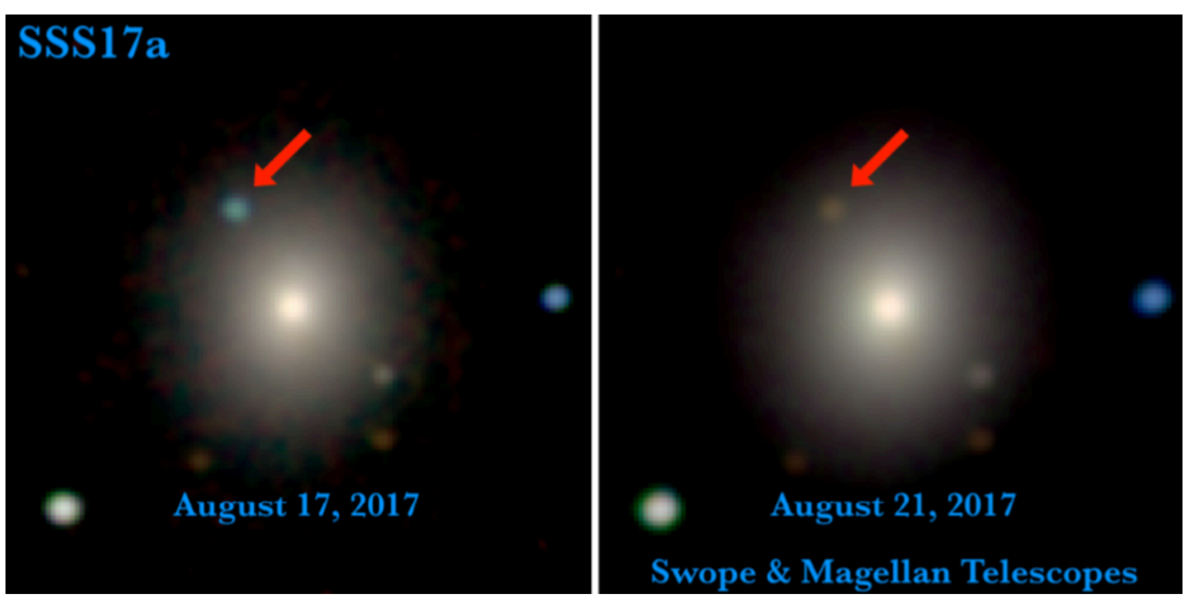
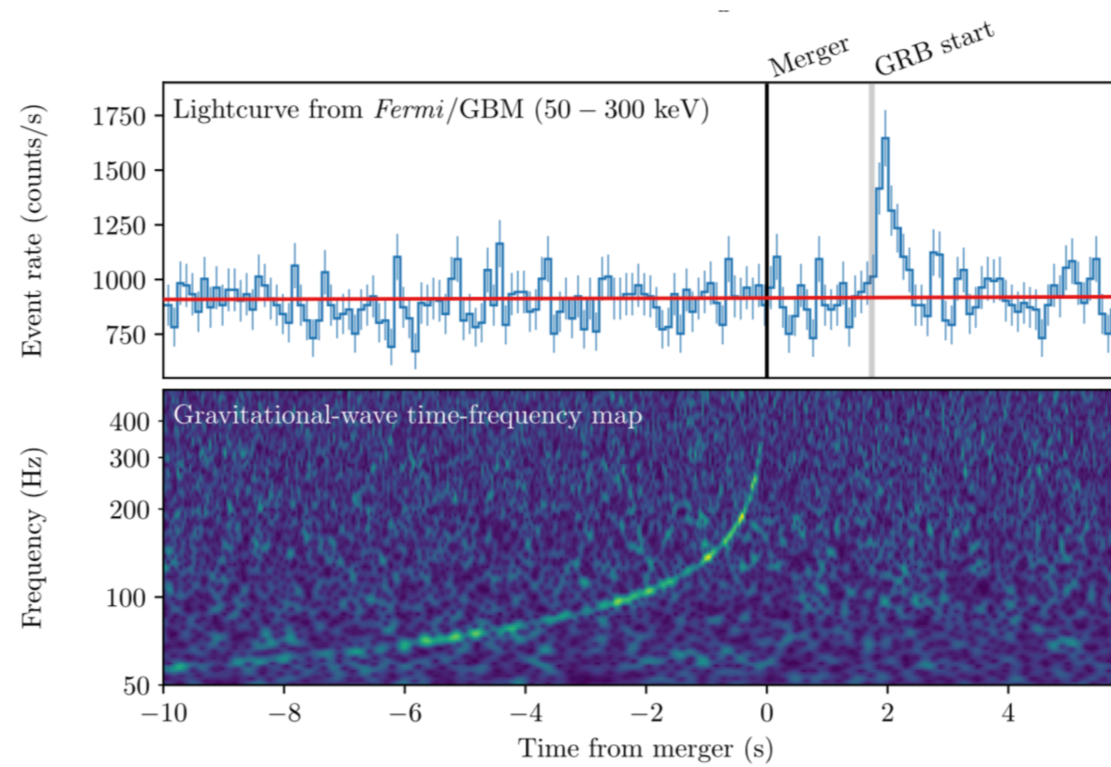


Figure credit: Alberto Sesana

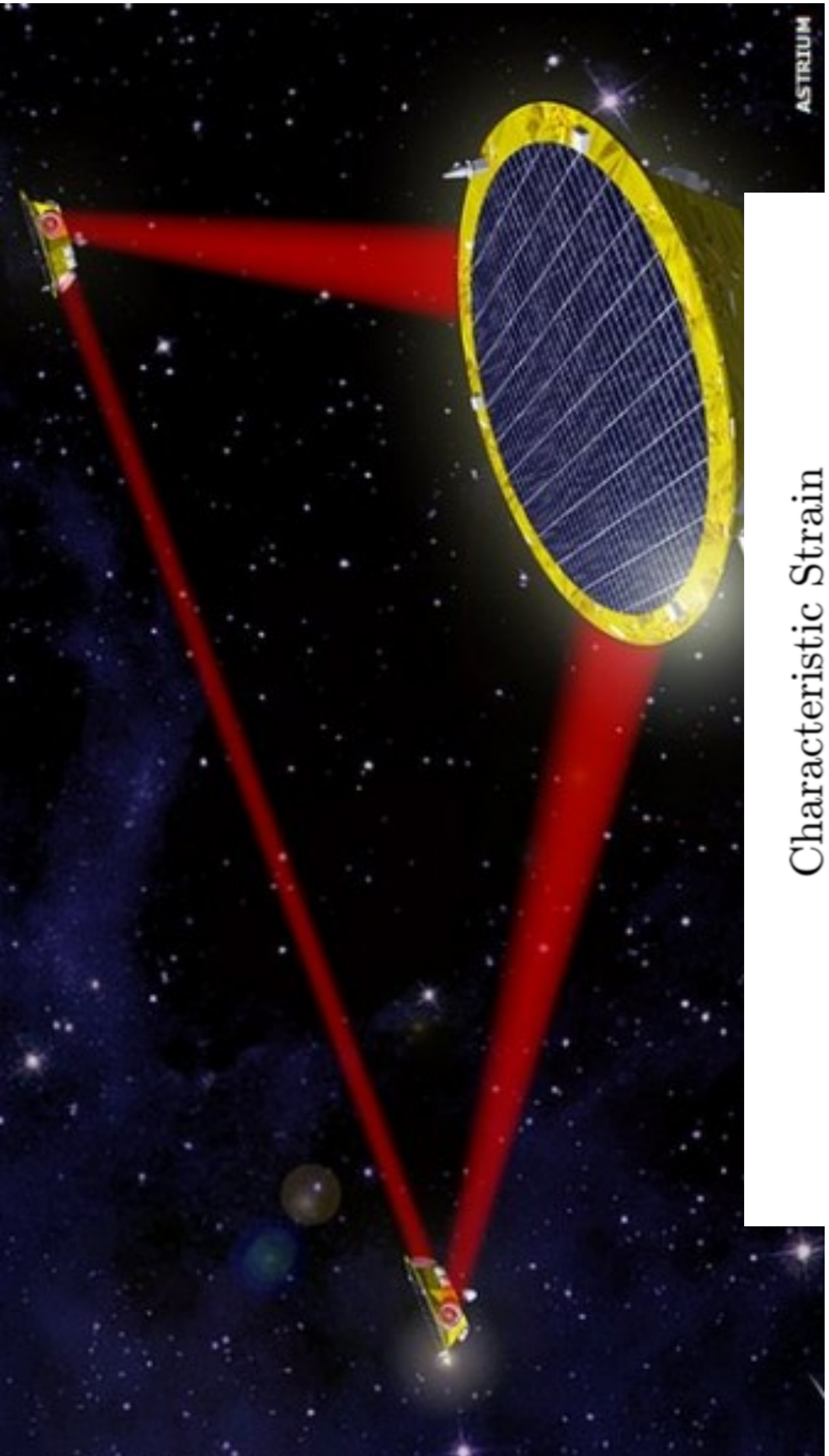
LIGO SOURCES

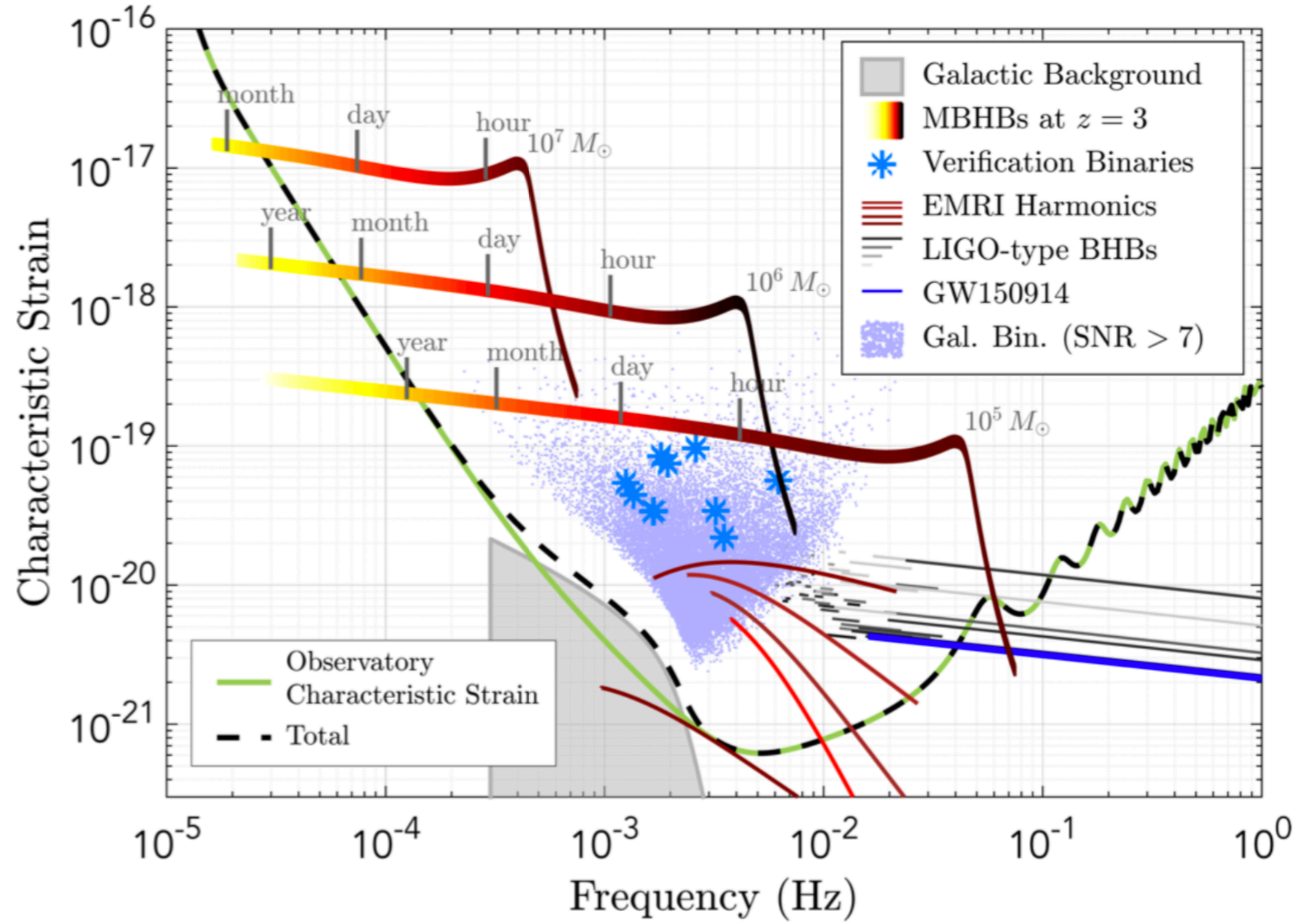
- Most sources at z ~ 0.1 (less than 1 GPc) : close universe
- BH-BH : spin and mass distribution, formation (star or primordial ?), no EM counterpart ?
- BH-NS : few events
- NS-NS : Few events. 1 EM counterpart : EOS of NS, speed of gravity, kilonova !





LISA



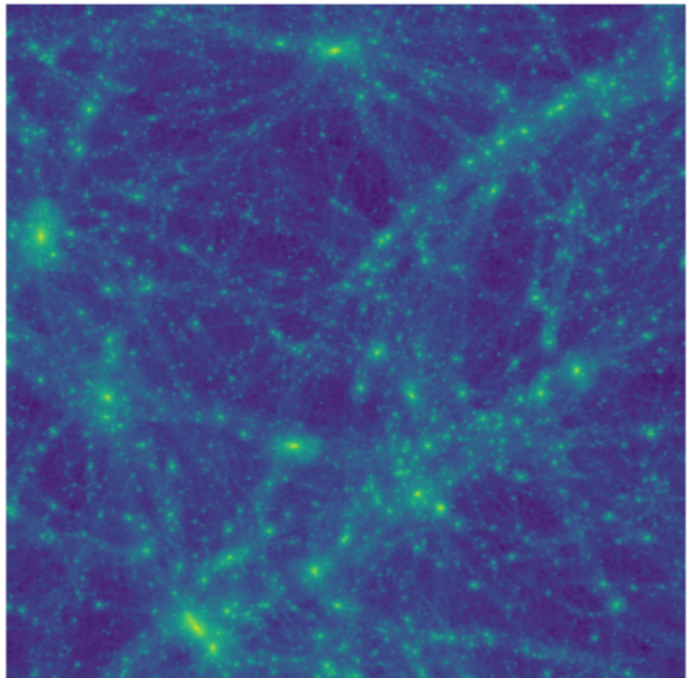


(Credit : Marta Volonteri)

LISA SOURCES

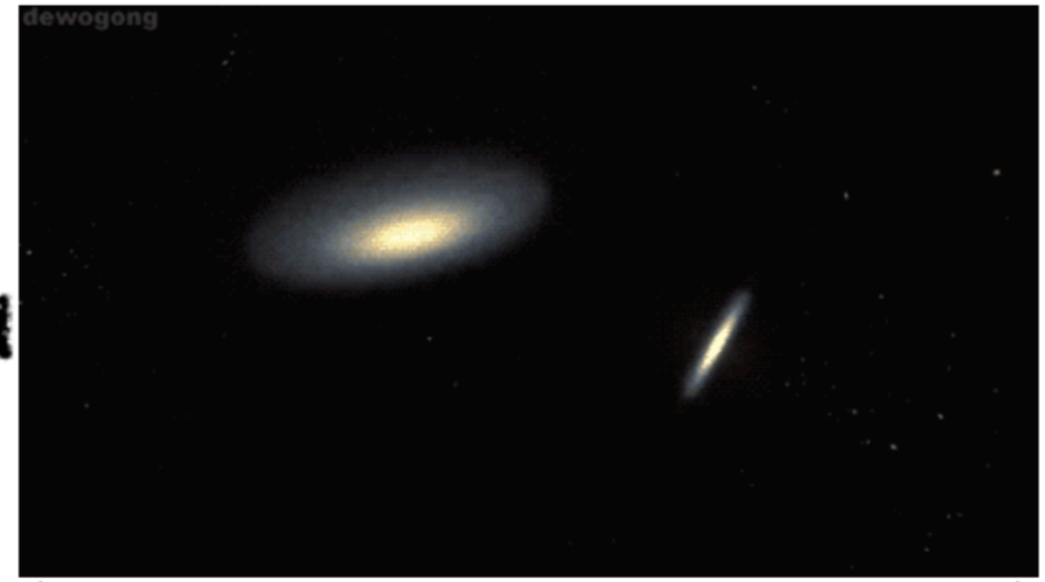
- Binary stars in our Galaxy (background noise + 25000 resolved)
- $50M_{\odot}$ BBH before they enter the LIGO band !
- Supermassive BH merger, up to z ~ 15 => population models, observation of the formation of a quasar in real time, H0 measurement and cosmology,…
- Extreme mass ratio inspiral (60 $M_{\odot}$ VS $10^5 M_{\odot}$) up to z=4 => Very accurate measurement of the spin, eccentricity, inclination and test of gravity. Few events per year
- Stochastic GW background (inflation, cosmic strings…)
- Core collapse supernovaes
- Exotic and unmodeled sources !

SOME QUESTIONS LISA WILL TRY TO ANSWER



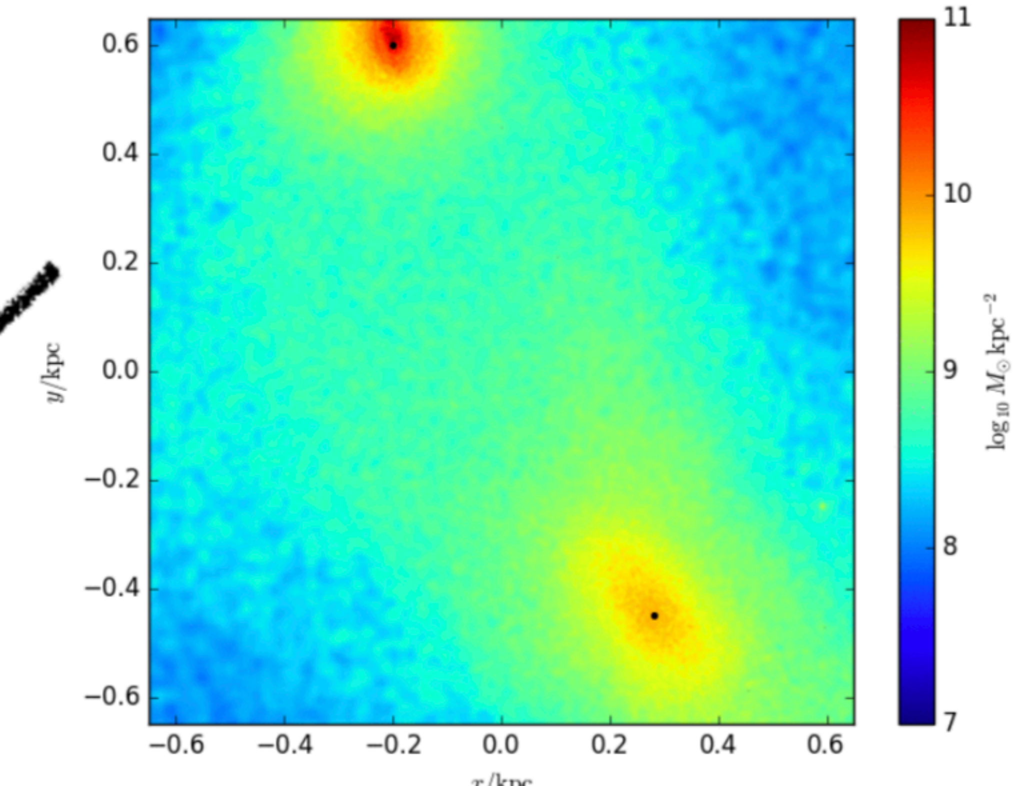
~100 Mpc (cosmology)

Gravity



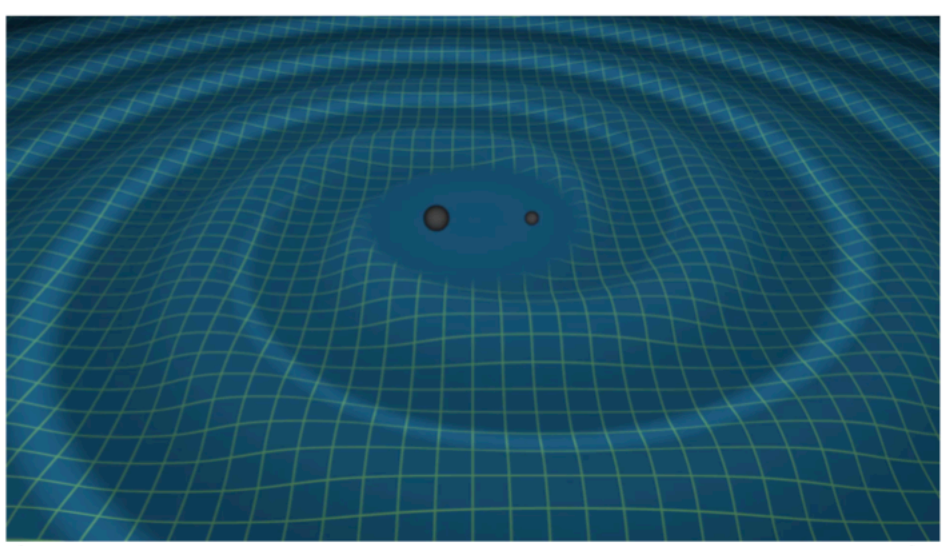
~100 kpc -1 kpc (galaxy mergers)

Dynamical Friction



~1 kpc-1pc (binary formation)

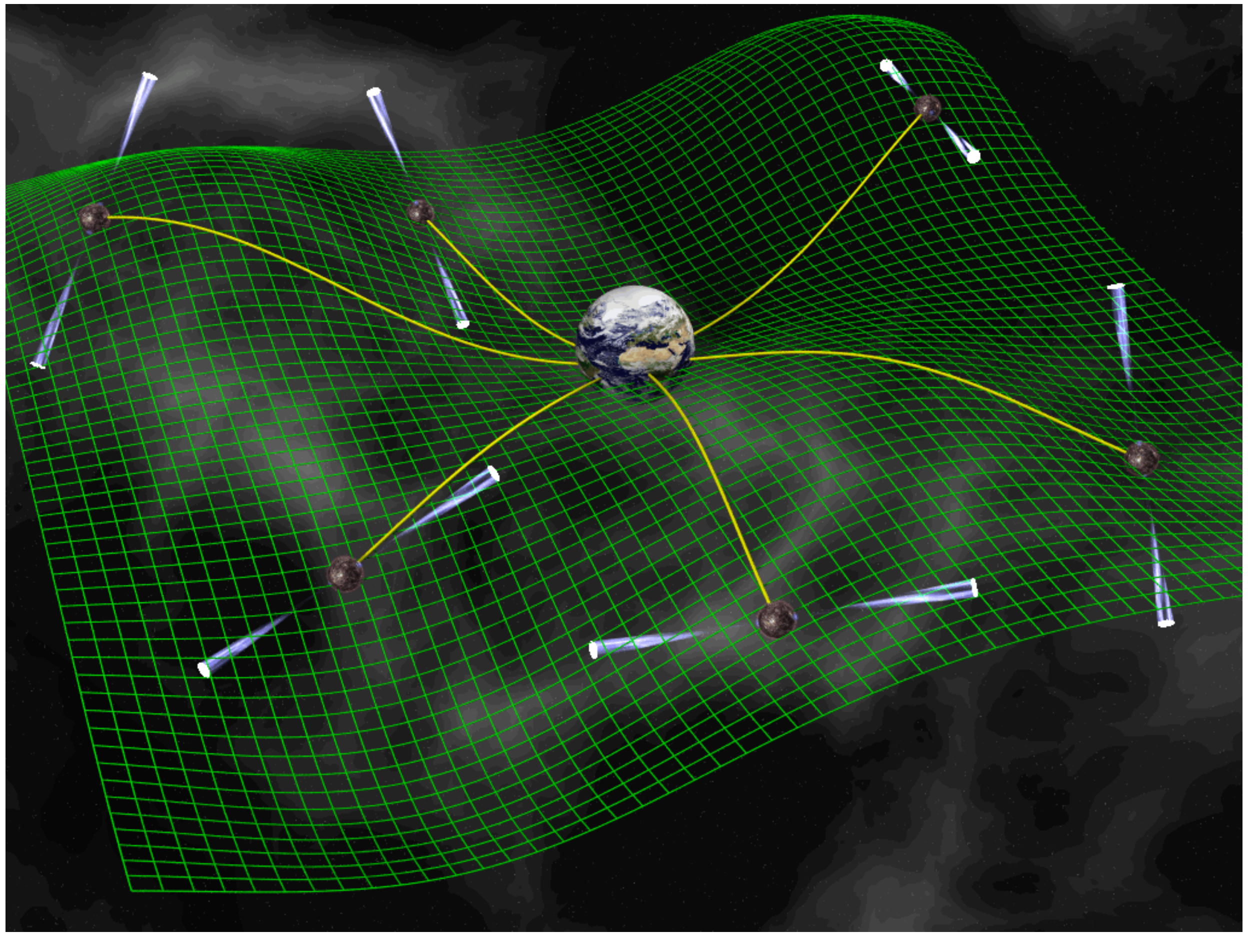
Gas torques?
Stellar scattering?
Last pc problem



~1 millipc(BH merger)

Courtesy of Hugo Pfister

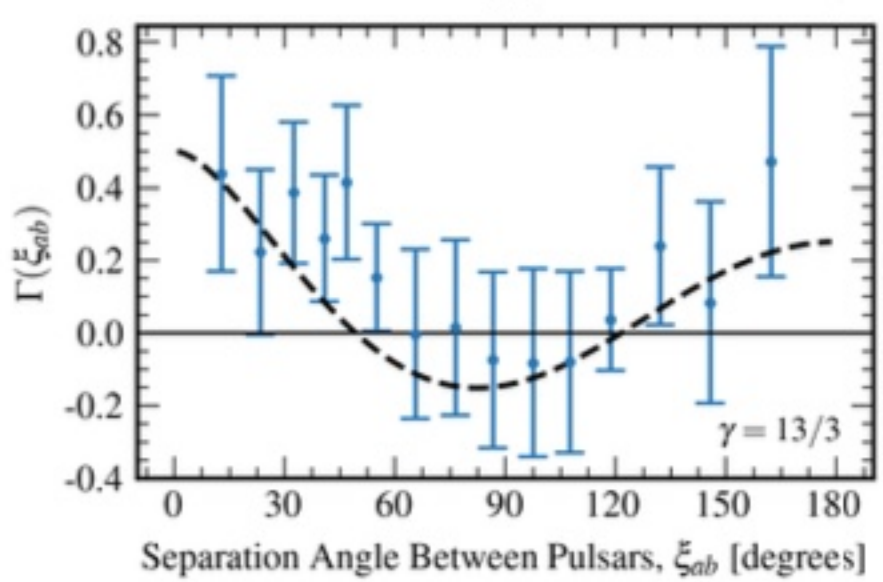
PULSAR TIMING ARRAY



- Supermassive BH binaries
- Inflation
- Cosmic strings

The NANOGrav 15-year Data Set: Evidence for a Gravitational-Wave Background

(c)

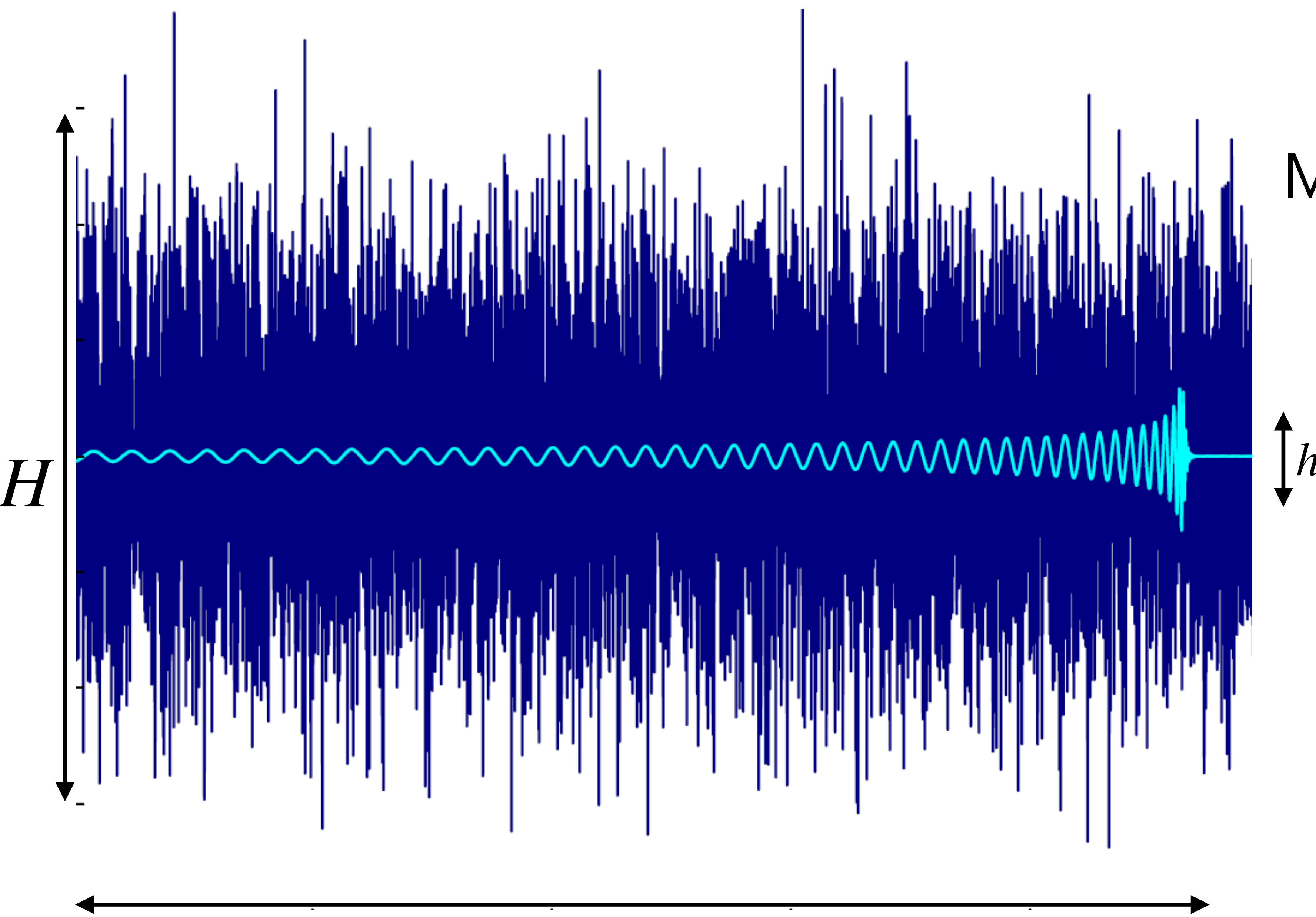


$$\Gamma(\xi_{ab}) = \frac{3}{2}x\ln(x) - \frac{1}{4}x + \frac{1}{2} + \frac{1}{2}\delta_{ab}, \qquad (4)$$

$$x = \frac{1 - \cos\xi_{ab}}{2}. \qquad (5)$$

Part III : modelling the waveform

The need for a high-precision template

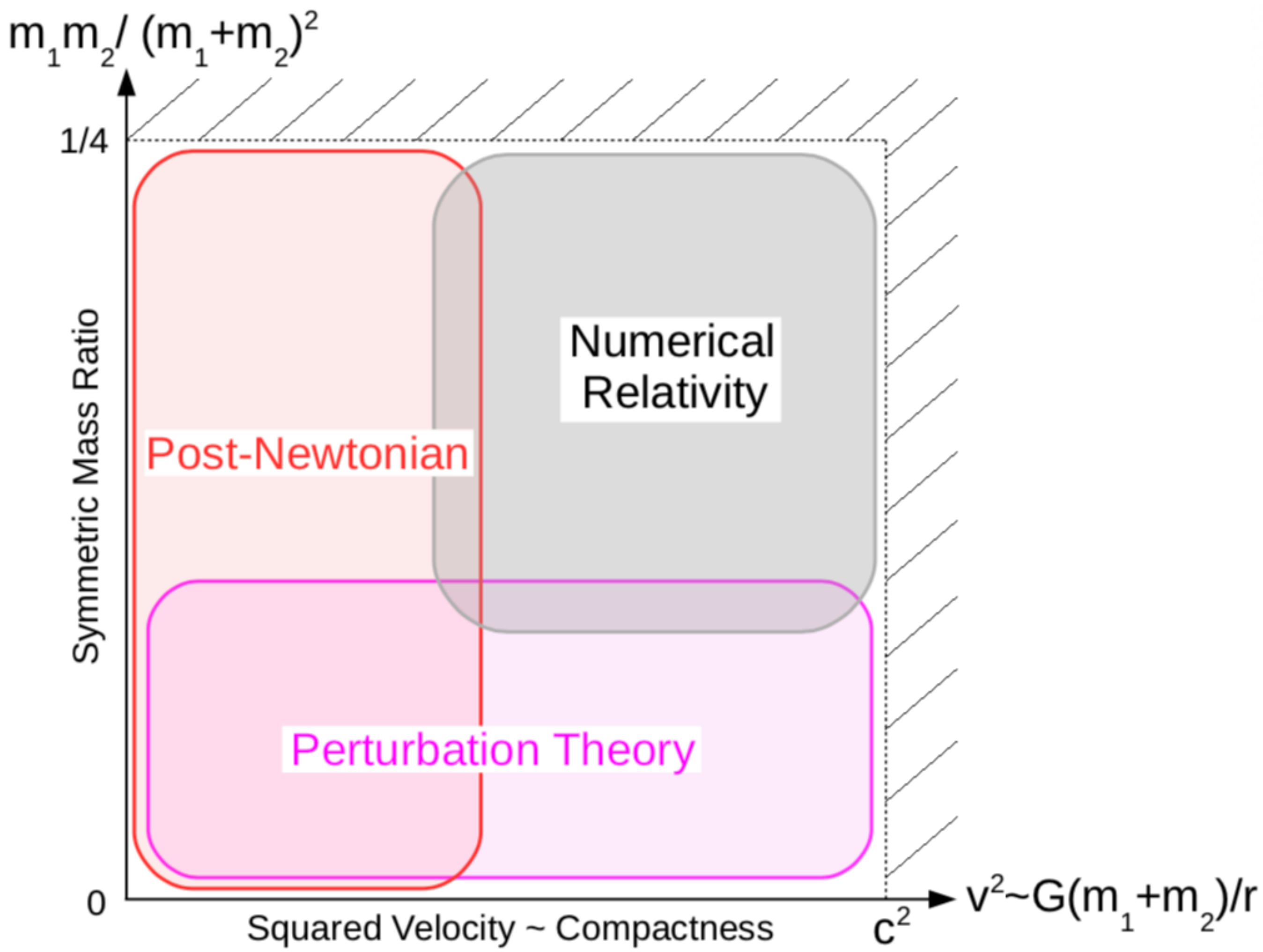


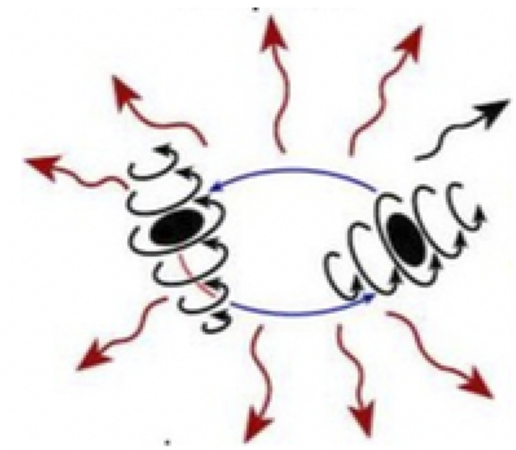
$T \simeq 10^4$ oscillations

Matched filtering analysis

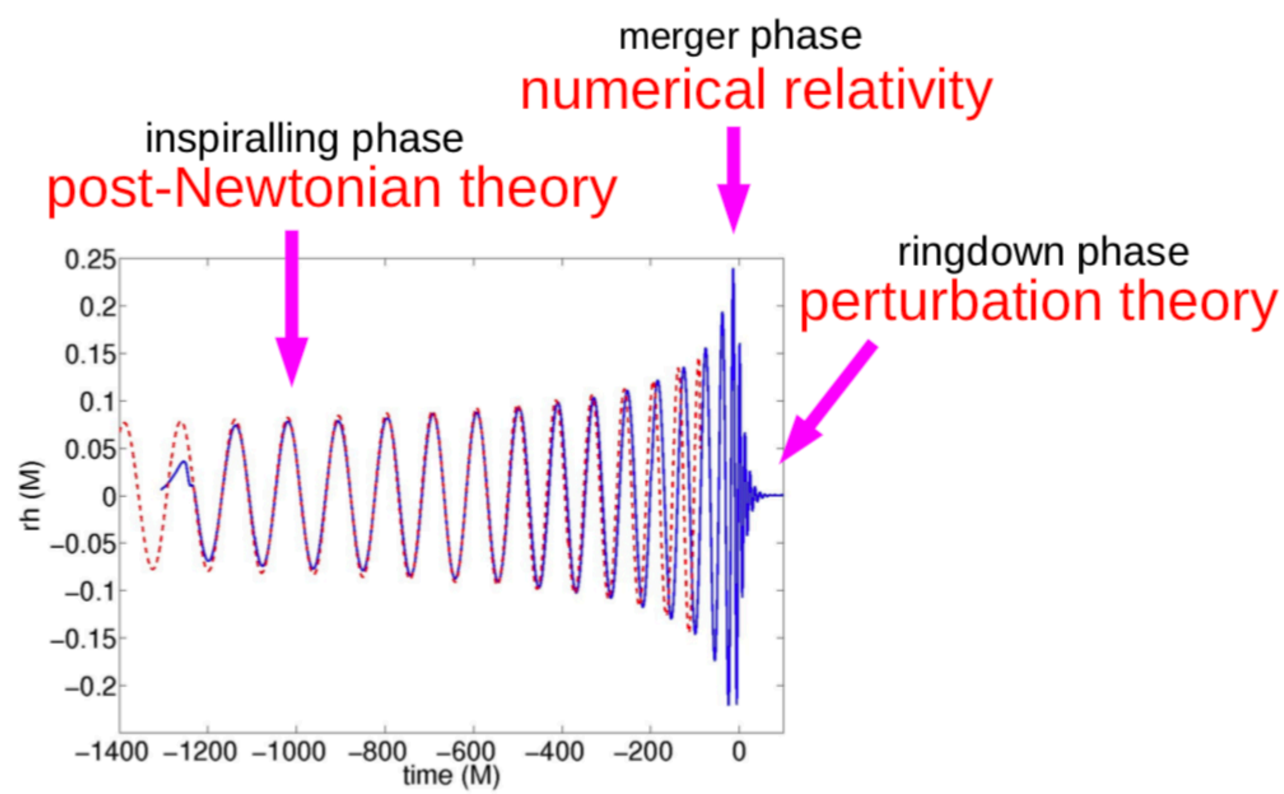
$$\frac{S}{N} \sim \frac{h}{H}\sqrt{T}$$

SOLVING THE TWO-BODY PROBLEM





(Credit : Luc Blanchet)

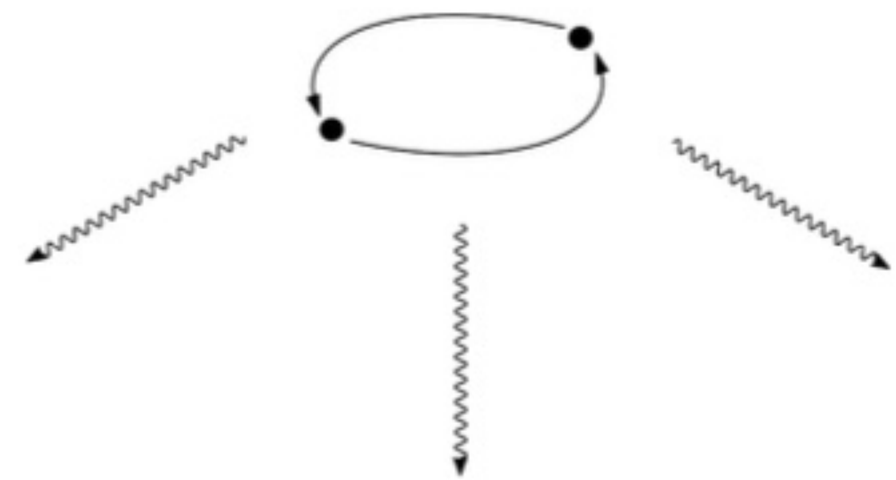


Post-Newtonian approximation

Perturbative solution of the EOM

$$G_{\mu\nu} = 8\pi T_{\mu\nu} \text{ and } g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu},$$

$$h_{\mu\nu} \sim \frac{GM}{r} \sim v^2 \ll 1$$



Matter is modeled by point particles (finite size effects arise only at $\mathcal{O}(v^8)$!) :

$$S_m = -m_1 \int d\tau_1 - m_2 \int d\tau_2$$

Then plug back $g_{\mu\nu}$ in the action and obtain the two-body relativistic Lagrangian :

$$L = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 + \frac{G m_1 m_2}{r} + \text{relativistic corrections}$$

A (SMALL) PART OF THE 3PN ENERGY

$$
\begin{aligned}
E = {} & \frac{m_1 v_1^2}{2} - \frac{G m_1 m_2}{2 r_{12}} \\
& + \frac{1}{c^2}\left\{ \frac{G^2 m_1^2 m_2}{2 r_{12}^2} + \frac{3 m_1 v_1^4}{8} + \frac{G m_1 m_2}{r_{12}} \left(-\frac{1}{4}(n_{12}v_1)(n_{12}v_2) + \frac{3}{2} v_1^2 - \frac{7}{4}(v_1 v_2) \right) \right\} \\
& + \frac{1}{c^4}\left\{ -\frac{G^3 m_1^3 m_2}{2 r_{12}^3} - \frac{19 G^3 m_1^2 m_2^2}{8 r_{12}^3} + \frac{5 m_1 v_1^6}{16} \right. \\
& \qquad + \frac{G m_1 m_2}{r_{12}} \left(\frac{3}{8}(n_{12}v_1)^3 (n_{12}v_2) + \frac{3}{16}(n_{12}v_1)^2 (n_{12}v_2)^2 - \frac{9}{8}(n_{12}v_1)(n_{12}v_2) v_1^2 \right. \\
& \qquad\qquad - \frac{13}{8}(n_{12}v_2)^2 v_1^2 + \frac{21}{8} v_1^4 + \frac{13}{8}(n_{12}v_1)^2 (v_1 v_2) + \frac{3}{4}(n_{12}v_1)(n_{12}v_2)(v_1 v_2) \\
& \qquad\qquad \left. - \frac{55}{8} v_1^2 (v_1 v_2) + \frac{17}{8}(v_1 v_2)^2 + \frac{31}{16} v_1^2 v_2^2 \right) \\
& \qquad \left. + \frac{G^2 m_1^2 m_2}{r_{12}^2} \left(\frac{29}{4}(n_{12}v_1)^2 - \frac{13}{4}(n_{12}v_1)(n_{12}v_2) + \frac{1}{2}(n_{12}v_2)^2 - \frac{3}{2} v_1^2 + \frac{7}{4} v_2^2 \right) \right\} \\
& + \frac{1}{c^6}\left\{ \frac{35 m_1 v_1^8}{128} \right. \\
& \qquad + \frac{G m_1 m_2}{r_{12}} \left(-\frac{5}{16}(n_{12}v_1)^5 (n_{12}v_2) - \frac{5}{16}(n_{12}v_1)^4 (n_{12}v_2)^2 - \frac{5}{32}(n_{12}v_1)^3 (n_{12}v_2)^3 \right. \\
& \qquad\qquad + \frac{19}{16}(n_{12}v_1)^3 (n_{12}v_2) v_1^2 + \frac{15}{16}(n_{12}v_1)^2 (n_{12}v_2)^2 v_1^2 + \frac{3}{4}(n_{12}v_1)(n_{12}v_2)^3 v_1^2 \\
& \qquad\qquad + \frac{19}{16}(n_{12}v_2)^4 v_1^2 - \frac{21}{16}(n_{12}v_1)(n_{12}v_2) v_1^4 - 2(n_{12}v_2)^2 v_1^4 \\
& \qquad\qquad + \frac{55}{16} v_1^6 - \frac{19}{16}(n_{12}v_1)^4 (v_1 v_2) - (n_{12}v_1)^3 (n_{12}v_2)(v_1 v_2) \\
& \qquad\qquad - \frac{15}{32}(n_{12}v_1)^2 (n_{12}v_2)^2 (v_1 v_2) + \frac{45}{16}(n_{12}v_1)^2 v_1^2 (v_1 v_2) \\
& \qquad\qquad + \frac{5}{4}(n_{12}v_1)(n_{12}v_2) v_1^2 (v_1 v_2) + \frac{11}{4}(n_{12}v_2)^2 v_1^2 (v_1 v_2) - \frac{139}{16} v_1^4 (v_1 v_2)
\end{aligned}
$$

(Source : Luc Blanchet)

DISSIPATIVE DYNAMICS

Similarly complicated calculations lead to the final result (for circular orbits) :

$$
\begin{aligned}
\phi = -\frac{x^{-5/2}}{32\nu}\Bigg\{ & 1 + \left(\frac{3715}{1008} + \frac{55}{12}\nu\right) x - 10\pi x^{3/2} + \left(\frac{15293365}{1016064} + \frac{27145}{1008}\nu + \frac{3085}{144}\nu^2\right) x^2 \\
& + \left(\frac{38645}{1344} - \frac{65}{16}\nu\right) \pi x^{5/2} \ln\left(\frac{x}{x_0}\right) \\
& + \left[\frac{12348611926451}{18776862720} - \frac{160}{3}\pi^2 - \frac{1712}{21}C - \frac{856}{21}\ln(16\,x) \right. \\
& \quad \left. + \left(-\frac{15737765635}{12192768} + \frac{2255}{48}\pi^2\right)\nu + \frac{76055}{6912}\nu^2 - \frac{127825}{5184}\nu^3\right] x^3 \\
& + \left(\frac{77096675}{2032128} + \frac{378515}{12096}\nu - \frac{74045}{6048}\nu^2\right) \pi x^{7/2} + \mathcal{O}\left(\frac{1}{c^8}\right)\Bigg\},
\end{aligned}
$$

$$x = (GM\omega)^{2/3} \sim v^2$$

$$\nu = \frac{m_1 m_2}{(m_1 + m_2)^2}$$

	$2 \times 1.4\, M_\odot$	$10\, M_\odot + 1.4\, M_\odot$	$2 \times 10\, M_\odot$
Newtonian order	16031	3576	602
1PN	441	213	59
1.5PN (dominant tail)	−211	−181	−51
2PN	9.9	9.8	4.1
2.5PN	−11.7	−20.0	−7.1
3PN	2.6	2.3	2.2
3.5PN	−0.9	−1.8	−0.8

Note the slow convergence !

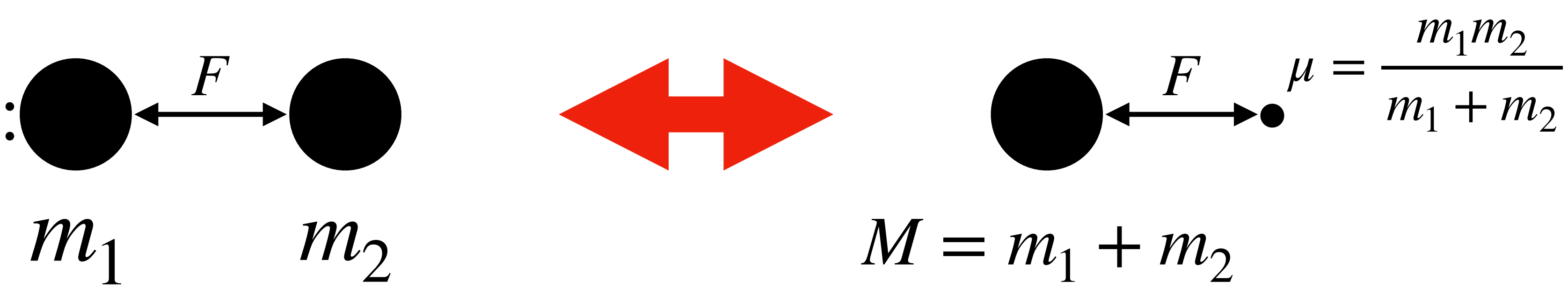
(source : Luc Blanchet)

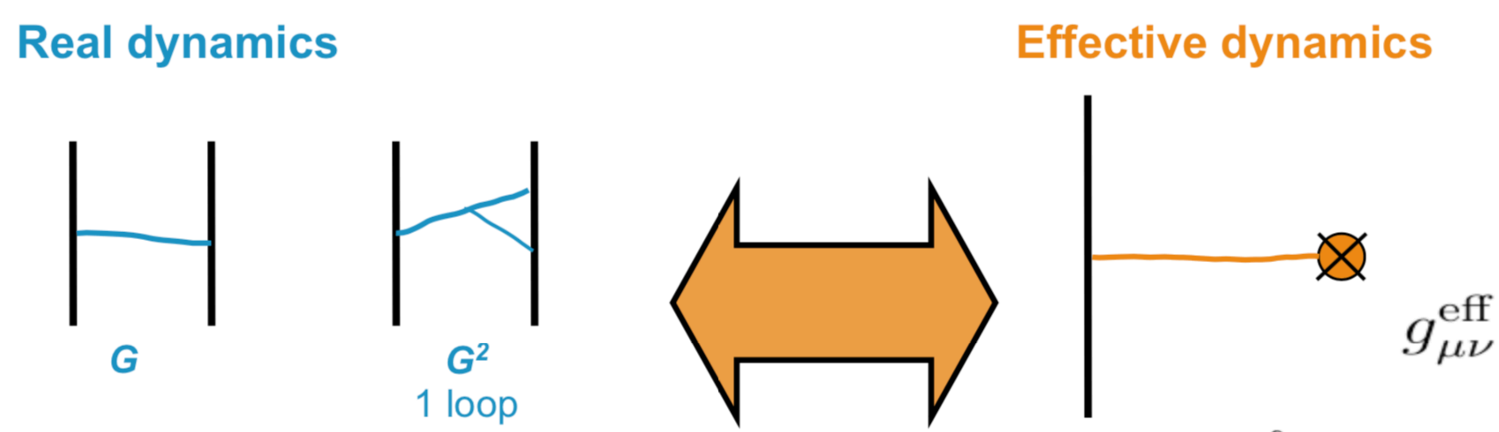
EFFECTIVE ONE-BODY

(Buonanno-Damour 98)

We want to resum as much as possible the (badly convergent) PN expansion

Basic idea :





$$S = -\int \mu ds + \ldots$$

$$H = H_0 + \left(GH_1 + \frac{G^2}{c^2}H_2 + \frac{G^3}{c^4}H_3 + \frac{G^4}{c^6}H_4\right)\left(1 + \frac{1}{c^2} + \ldots\right)$$

$$A(R) = 1 - \frac{2GM}{c^2\,R} + 2\nu\left(\frac{GM}{c^2\,R}\right)^3 + \cdots,$$

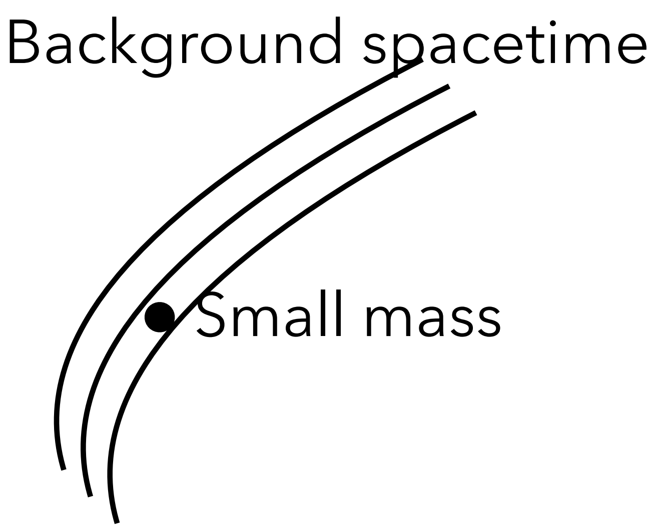
$$B(R) = 1 + \frac{2GM}{c^2\,R} + (4 - 6\nu)\left(\frac{GM}{c^2\,R}\right)^2 + \cdots$$

Effective metric

$$ds^2_{\text{eff}} = -A(r)dt^2 + B(r)dr^2 + r^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right)$$

GRAVITATIONAL SELF-FORCE

EMRI : find the waveform as an expansion in $\epsilon = m/M$. This is far from being achieved.



The tensorial form of the equations of motion is

$$\frac{Du^\mu}{d\tau} = -\frac{1}{2}\left(g^{\mu\nu} + u^\mu u^\nu\right)\left(2h^{\rm tail}_{\nu\lambda\rho} - h^{\rm tail}_{\lambda\rho\nu}\right)u^\lambda u^\rho$$

Deviation from geodesics

where

$$h^{\rm tail}_{\mu\nu\lambda} = 4m\int_{-\infty}^{\tau-\epsilon} \nabla_\lambda\left(G_{\mu\nu\mu'\nu'} - \frac{1}{2}g_{\mu\nu}G^{\rho}{}_{\rho\mu'\nu'}\right)\left(z(\tau), z(\tau')\right)u^{\mu'}u^{\nu'}\,d\tau'$$

(MiSaTaQuWa equations)

Mathematically challenging to go to higher orders, but $\mathcal{O}(\epsilon^3)$ is needed !

CONCLUSIONS

- A new channel of observation for the universe
- We begin to observe the close universe, and we will observe it to cosmological distances in few decades
- Modelling of the signal is analytically and numerically challenging

Il n'y a pas de problèmes résolus, seulement des problèmes plus ou moins résolus