Breakdown of Thermalization in Disordered Quantum Systems: Many Body Localization and its Consequences

The search for a Time Crystal

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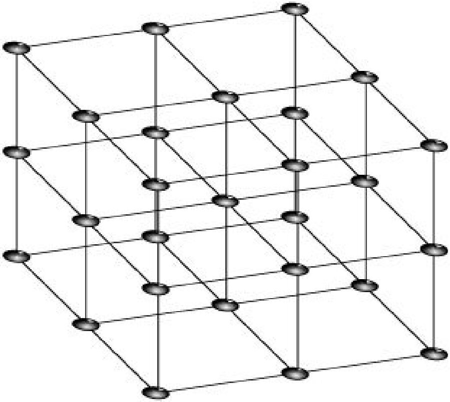
Outline

1. Introduction : What is a Time Crystal ?
2. Driven Systems
    - Time Crystal conditions
    - Floquet spectrum
    - Example
3. Model : Harmonically driven spin chain
    - Hamiltonian and diagonalization
    - Numerical results
4. Conclusion

Space crystal



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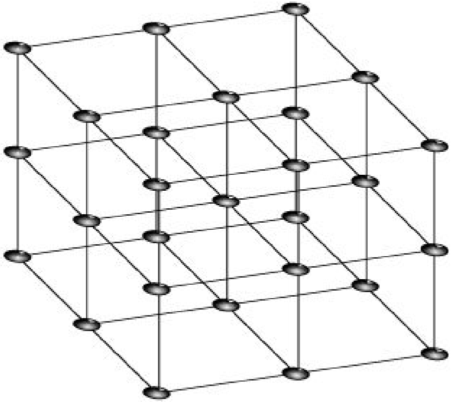

- $\left\langle \hat{O}(\mathrm{x}) \right\rangle = Cst$ (at finite size)

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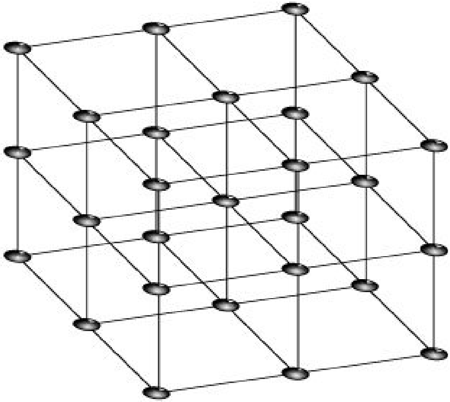

- $\left\langle \hat{O}(\mathrm{x}) \right\rangle = Cst$ (at finite size)
- $\left\langle \hat{O}(\mathrm{x})\hat{O}(\mathrm{x}') \right\rangle = f(\mathrm{x}-\mathrm{x}')$ periodic

Time Crystal in equilibrium

$$\lim_{V\to\infty}\left\langle \hat{O}(\mathbf{x},t)\hat{O}(\mathbf{0},0)\right\rangle = f(\mathbf{x},t)$$

periodic in time (and possibly in space)

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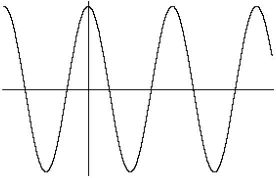

Remark : $\lim_{V\to\infty}\left\langle \hat{O}(t)O(0)\right\rangle = f(t)$ periodic in time not sufficient

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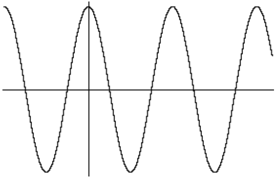

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$\Rightarrow$ Watanabe Oshikawa 2015 : impossible in equilibrium

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Time Crystal in a driven system

$$\hat{H}(t + T) = \hat{H}(t)$$

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- TTSB : $\left\langle \hat{O}(t+T) \right\rangle \neq \left\langle \hat{O}(t) \right\rangle$
- Rigidity : stable to perturbations

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Bloch and Floquet theorems

Crystal : $\psi_\alpha(\mathbf{r}) = e^{i\mathbf{k}_\alpha \mathbf{r}} u_\alpha(\mathbf{r}), \quad u_\alpha(\mathbf{r}+\mathbf{R}) = u(\mathbf{r})$

Bloch and Floquet theorems

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Periodic Hamiltonian : $\psi_\alpha(t) = e^{-i\mu_\alpha t} u_\alpha(t), \quad u_\alpha(t+T) = u(t),$
$\mu_\alpha \in [-\frac{\Omega}{2}, \frac{\Omega}{2}]$

Evolution of an observable

$$|\Psi(t)\rangle = \sum_{\alpha} R_{\alpha} e^{-i\mu_{\alpha} t} |u_{\alpha}(t)\rangle$$

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- Clean system : destructive interferences
- Disorder seems essential

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Example of Time Crystal : Spin-Flip Hamiltonian

$$\hat{H}(t) = \begin{cases} -\sum_{i=1}^{L} h_i \sigma_i^z & \text{if } 0 \leq t < T_1 \\ -\sum_{i=1}^{L-1} J_i \sigma_i^x \sigma_{i+1}^x + B_i \sigma_i^x & \text{if } T_1 \leq t < T = T_1 + T_2 \end{cases}$$

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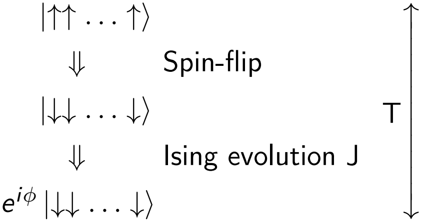

$|\uparrow\uparrow \ldots \uparrow\rangle$

$\Downarrow$ Spin-flip

$|\downarrow\downarrow \ldots \downarrow\rangle$

$\Downarrow$ Ising evolution J

$e^{i\phi} |\downarrow\downarrow \ldots \downarrow\rangle$

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$$\begin{tikzcd}
\left|\uparrow\uparrow\ldots\uparrow\right\rangle \arrow[d, Rightarrow, "\text{Spin-flip}"] & {} \arrow[dd, leftrightarrow, "T"'] \\
\left|\downarrow\downarrow\ldots\downarrow\right\rangle \arrow[d, Rightarrow, "\text{Ising evolution J}"] & \\
e^{i\phi}\left|\downarrow\downarrow\ldots\downarrow\right\rangle & {}
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- Oscillations 2T

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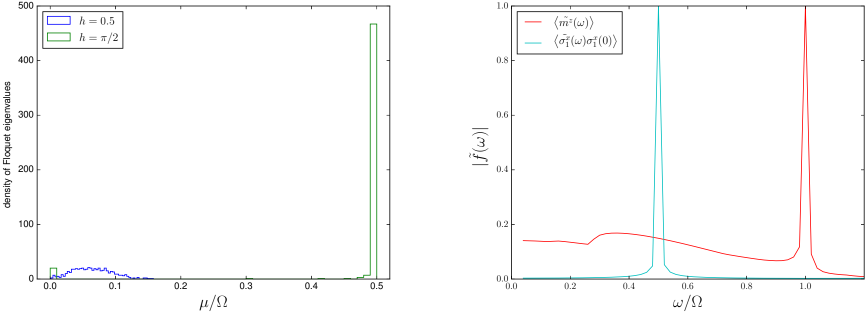
- Oscillations 2T
- $J = 0$: unstable
- $J \neq 0$: stable !

Yao et Al (2017) ; Khemani et Al (2016) ; Experimental : Zhang et Al (2016)

Time Crystal behavior



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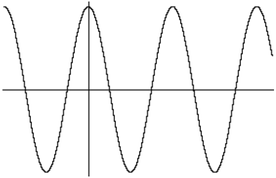
Questions :

- TC with a harmonic drive ?
- Importance of protocol in quantum for period doubling ?



Hamiltonian

$$\hat{H}(t) = -\sum_{j=1}^{L} J_j \left((1+\gamma)\sigma_j^x \sigma_{j+1}^x + (1-\gamma)\sigma_j^y \sigma_{j+1}^y\right) - \sum_{j=1}^{L} h_j(t)\sigma_j^z$$

$h_j(t) = h_j^0 + h_1 \cos(\Omega t)$

Hamiltonian

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$h_j(t) = h_j^0 + h_1 \cos(\Omega t)$

- Bogoliubov transformation : $\{\sigma_i^\alpha\} \rightarrow \{c_i\}$

  $\Rightarrow$ quadratic fermionic Hamiltonian
- Solve Heisenberg eqs of motion

Outline

Typical case

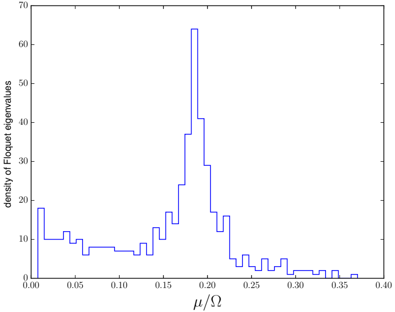


Figure: Floquet spectrum density for parameters : $h_m = 1,\ h_0 = 1,\ h_1 = 1,\ L = 500,\ \gamma = 1,\ J = 1,\ T = 1$ and averaged over 50 realizations.

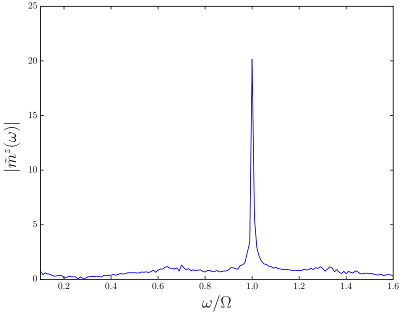
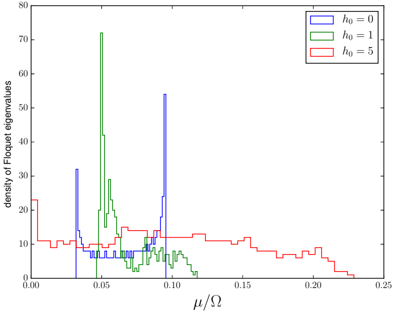


Figure: magnetization for parameters : $h_m = 1,\ h_0 = 1,\ h_1 = 1,\ L = 100,\ \gamma = 1,\ J = 1,\ T = 1,\ t = 100$ and averaged over 20 realizations.

High frequency



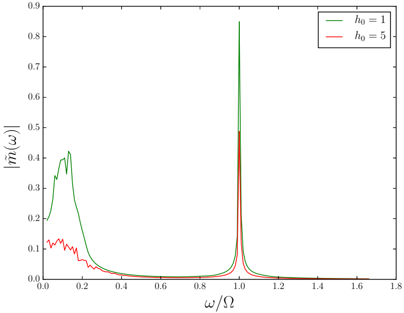


Figure: Floquet spectrum for parameters : $h_m = 1,\ h_1 = 1,\ L = 500,\ \gamma = 1,$ $J = 1,\ T = 0.1$ for different values of h_0 and averaged over 50 realizations.

Figure: magnetization for parameters : $h_m = 1,\ h_1 = 1,\ L = 100,\ \gamma = 1,\ J = 1,$ $T = 0.1$, evaluated using a total time of $t = 10$, and with different values of h_0

Dependance on the observable

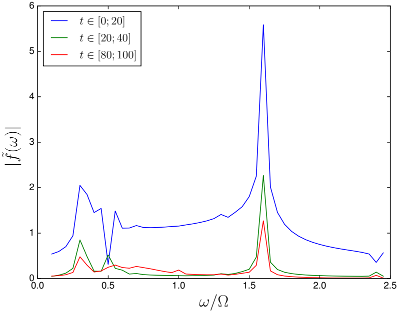


Figure: Clean case, x time correlation function for parameters : $h_m = 3$, $h_1 = 0.3$, $L = 100$, $\gamma = 1$, $J = 1$, $T = 1$

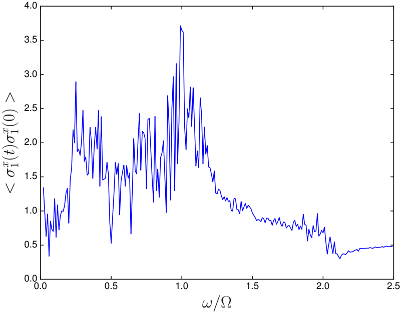


Figure: Disordered case, x time correlation function for parameters : $h_m = 3$, $h_1 = 0.3$, $h_0 = 1$, $L = 100$, $\gamma = 1$, $J = 1$, $T = 1$, $n = 200$

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- We are exploring now the dependance on observables
- A paper is coming

Method of diagonalization

$$
\begin{aligned}
\sigma_j^x &= K_j(c_j^\dagger + c_j) \\
\sigma_j^y &= iK_j(c_j^\dagger - c_j) \\
\sigma_j^z &= (2c_j^\dagger c_j - 1)
\end{aligned}
$$

with

$$K_j = \prod_{i<j}(1 - 2c_i^\dagger c_i) = \exp\left(i\pi \sum_{i<j} c_i^\dagger c_i\right)$$

Method of diagonalization

$\Rightarrow$

$$\begin{aligned}\hat{H}(t) &= -2\sum_{i=1}^{L} J_i(\gamma c_i^\dagger c_{i+1}^\dagger + c_i^\dagger c_{i+1} + h.c) - 2\sum_{i=1}^{L} h_i(t) c_i^\dagger c_i + \sum_{i=1}^{L} h_i(t) \\ &= 2\sum_{\mu} \varepsilon_\mu \left(\gamma_\mu^\dagger \gamma_\mu - 1/2\right)\end{aligned}$$

Then

$$i\frac{dc_i}{dt} = \left[c_i, \hat{H}\right]$$